Determination of an Orthometric Height Profile in the Okavango Delta Using GPS Levelling

Heister, H., Lang, M., [1]
Merry, C. L. and Rüther, H. [2]

[1] University of the Bundeswehr Munich, Geodetic Institute,
85577 Neubiberg, Germany
[2] University of Cape Town, Dept. of Geomatics,
7700 Rondebosch, South Africa

Summary

A multidisciplinary research project to examine hydrology, sedimentology and plant ecology of the wetlands of the Okavango Delta was initiated in 1993. One key research area is the determination of precise orthometric height differences along the Okavango/Jao/Boro river system. Traditional methods (geodetic and trigonometric levelling) to determine orthometric heights within the delta is prevented due to the wide swampy areas. Local geoid models do not have the required accuracy to transfer ellipsoidal heights derived by GPS to orthometric heights. Premission analysis revealed that the determination of a local geoid, derived by a combination of GPS and levelling determined height differences, is the most suitable approach. This special method, called GPS levelling, is presented in detail. A discussion of the numerical results and the achieved accuracies, particular in comparison to the most actual geoid models (UCT95A and EGM96) is given.

Abstract

Seit 1993 werden im Rahmen eines interdisziplinären Forschungsprojekts ökologische, hydrologische und sedimentative Vorgänge im Okavangodelta untersucht. Einen Schwerpunkt bildet dabei die Bestimmung eines orthometrischen Höhenprofils entlang des Okavango-Jao-Boro Flußsystems. Herkömmliche Meßverfahren zur Bestimmung von orthometrischen Höhen, wie z. B. Nivellement und trigonometrische Höhenmessung können in den Sumpflandschaften des Deltas nicht angewandt werden. Lokale Geoidmodelle liegen nur mit eingeschränkter Genauigkeit vor, so daß mit GPS bestimmte ellipsoidische Höhen in der Regel nicht in orthometrische überführt werden können. Praktische Versuche haben gezeigt, daß die Bestimmung eines lokalen Geoids unter Verwendung einer Kombination von GPS bestimmten ellipsoidischen und nivellierten Höhenunterschieden, die praktikabelste Methode in solch einem Fall darstellt. Dieses Verfahren, als GPS-Nivellement bezeichnet, wird eingehend erläutert. Die Ergebnisse der Messungen im Okavangodelta werden hinsichtlich der erzielten Genauigkeit und der Übereinstimmung mit Geoidmodellen jüngeren Datums (UCT95A und EMG96) diskutiert.

1. Introduction

The Okavango Delta in northwestern Botswana (Fig. 1) forms the largest wetland in southern Africa covering about 25000 km^2. Surrounded by dry and desert-like areas proposals have been made to abstract water upstream (PALLETT, 1997) and from the delta (STANDISH-WHITE, 1972; SCUDDER, 1993) to support the regional development often threatened by water shortages. Despite intensive research the complex hydrology of the delta is not known in all details yet. Recent satellite images have shown a changing pattern of the water distribution within the delta. This illustrates that only a solid knowledge of all parameters contributing to the hydrological situation is the key to the long-term conservation and use of this unique wetland. On condition of that a removal of water from the Okavango River system can be carefully managed. This is necessary to avoid irreversible damage to one of the most pristine wilderness areas in the world.

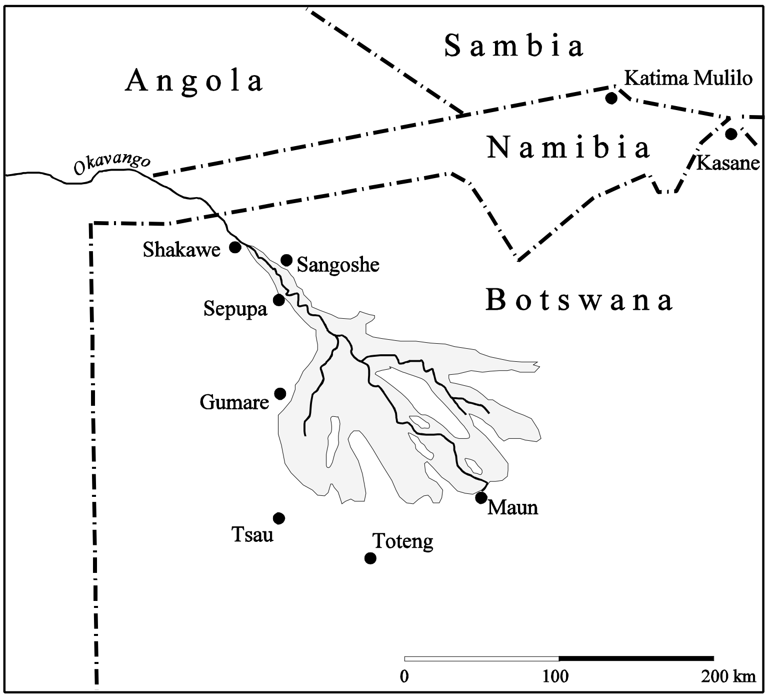

Fig. 1: Okavango Delta, Botswana

Beginning in 1994 GPS surveys have been conducted in the Okavango Delta to interconnect the isolated water gauges to enable wide area comparison of the current water level. In order to convert the GPS derived ellipsoidal heights (h) to orthometric heights (H), the geoidal height (N) at each point must be known:

$$H = h - N \tag{1}$$

Various global (OSU91, EMG96) and local geoid models (UCT95A) are available. Since the gradient of the Okavango River in the research area is rather small (about 1:4000) it is doubtful whether these models provide sufficient accuracy. For further research a local orthometric height profile along the Okavango/Jao/Boro river system was established in 1996 using the new method GPS levelling (HEISTER ET AL., 1991).

2. General considerations

Four types of observations can be used for the determination of the geoid:

- gravimetric,
- astronomical,
- satellite or
- geodetic

observations. In general a combination of different types is useful. Traditionally most of the existing geoid models are based on gravimetric and astronomical observations.

Gravimetric observations require expensive and sensitive instruments and time consuming observation procedures like the leap-frog method. In addition the observations must cover a large area for reliable geoid modelling. All these factors characterize that method as too time consuming and too expensive to be used in a hardly accessible environment like the Delta.

Astronomical observations are extensive as well and have to be carried out during night. Even though the observation process can be partly automated, the „productivity“ is restricted under these environmental conditions.

GPS observations and geometric levelling in local networks can be carried out literally on every island in the Delta. Trigonometric levelling can even be performed from one island to another. To cancel out the refraction simultaneous reciprocal zenith angles have to be measured. The considerations of minimizing the equipment and elementariness of observation method were given room, thus a combination of GPS measurements and geometric levelling was finally chosen for the determination of a local geoid.

3. GPS Levelling

3.1. Theoretical background

Orthometric heights (H) refer to an equipotential reference surface, e. g. the geoid. The orthometric height of a distinct point on the surface of the earth is the distance from that point to the geoid, measured along the plumb line normal to the geoid. Due to the fact that equipotential surfaces are not parallel, this plumb line is a bend line. Orthometric heights can be derived using geometric or trigonometric levelling.

Ellipsoidal heights (h) refer to a reference ellipsoid, e. g. the WGS-84 ellipsoid. In analogy the height of a point is defined as the distance from the ellipsoid measured along a normal to the reference ellipsoid. Ellipsoidal heights can be derived from geocentric cartesian coordinates provided by GPS observations. The difference between both heights is defined as the geoid height (N).

Considering two distinct points with known heights in both height systems (Fig. 2), formula (1) can be written as

$$H_2 - H_1 = h_2 - h_1 - N_2 + N_1 \Rightarrow dN = dh - dH \,. \qquad (2)$$

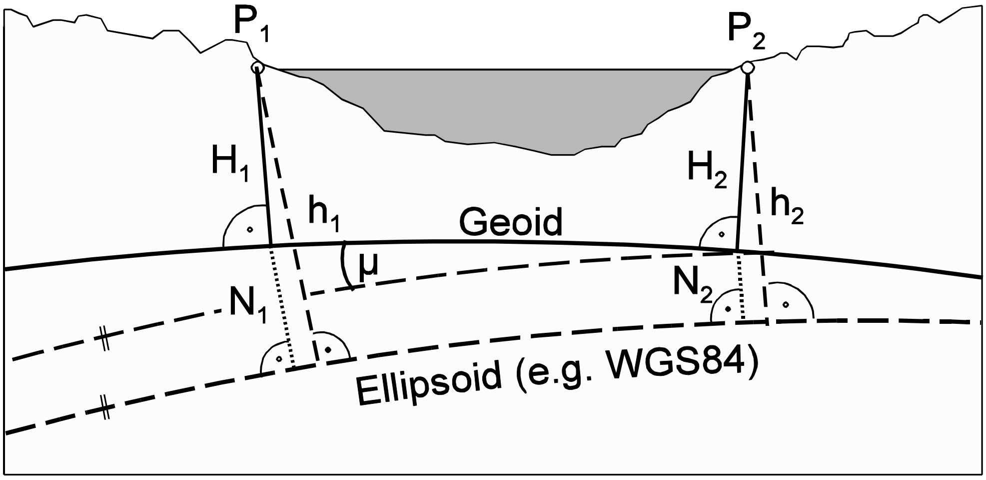

Fig. 2: Ellisoidal (h), geoidal (N) and orthometric (H) height

Taking the distance d between both points into account the deflection of the vertical μ is

$$\mu = \arctan(dN/d) = (dh - dH)/d \cdot \rho'' \tag{3}$$

Using the meridian (ξ) and the prime vertical component (η) the deflection of the vertical between two points P_1 and P_2 can be finally written as

$$\mu_{12} = \xi_1 \cdot \cos t_{12} + \eta_1 \cdot \sin t_{12} \, , \tag{4}$$

where t_{12} is the azimuth of the line P_1P_2.

Formula (2)-(4) provide numerous advantages: First, the knowledge of the absolute values in either height system is not necessary for the derivation of the local components of the deflection of the vertical. Second, the differential nature of (2) will cancel out most of the errors in the height determination affecting nearby points in a similar way, e.g. atmospheric influences in GPS measurements. Third, the determination of the deflection components ξ and η allows computation of the deflection of the vertical in any azimuth.

The combination of GPS derived ellipsoidal heights with geoidal information for the purpose of orthometric height determination is called „GPS levelling“.

The accuracy of geoidal heights or vertical deflections derived by this new approach is mainly limited by the accuracy of the GPS observations. Orthometric height differences dH can be easily determined with standard deviations less than 1 mm/km, whereas the accuracies for GPS-derived ellipsoidal height differences dh will be significantly bigger.

3.2. Measurements (OKAV96)

In 1994 a precise GPS survey was already carried out to determine the ellipsoidal heights of 21 stations along a profile between Shakawe and Maun (RÜTHER, 1994; HEISTER, STERNBERG, 1995). The aim of the campaign OKAV96 was the derivation of the orthometric height profile traversing the Okavango Delta with an homogenous accuracy of $\sigma_{dH} < 1$ cm/km. It was planned to determine local deflections of the vertical in eight locations of the profile each about 25-30 kilometers apart. To be able to depict the tilt of the geoid four points forming a local network (a star which rays intersect under an angle of 120° (Fig. 3)) were temporarily marked using 50 cm long iron peaks. In general these stars have an extent of about 1 x 1 km^2.

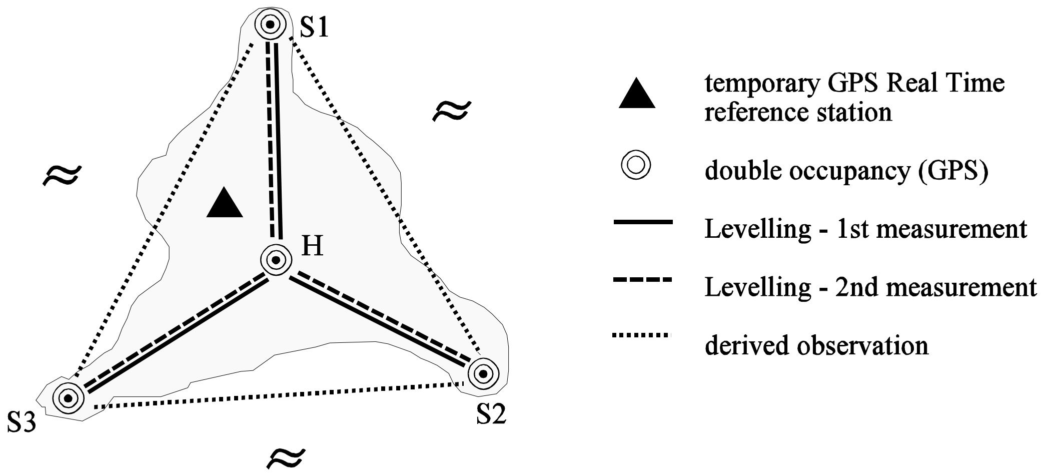

Fig. 3: Scheme of the observations carried out

Intensive premission investigations revealed that the real-time GPS measurements are advantageous for that type of project: On the one hand the time for the post-processing of the data in the field can be minimized and on the other hand information about the quality of the coordinate determination is already available during the measurement process. To eliminate errors in determination of instrumental heights - a common error in GPS surveys - a temporary reference station was set up on a tripod near the middle of the star. All points forming the star (H, S1, S2 and S3) were observed with the same setup (external antenna mounted on a survey pole of constant height). Using the height differences in the course of the evaluation procedure, the instrumental height of the roving antenna is eliminated. To enable independent control of the GPS derived ellipsoidal height differences and to minimize the influence of the satellite constellation all points were occupied twice. In general the first and second occupation of one point were carried out some hours apart to guarantee different satellite constellations. Pre-mission investigations revealed an observation time of 45 minutes per session as suitable. This session length represents a good compromise between observation effort and accuracy; a standard deviation of an ellipsoidal height of less than 5 mm can be expected. As well detailed studies in the aftermath confirmed those findings (HEISTER ET AL., 1997). All real time GPS observations were carried out with a sampling rate of 5 seconds resulting in at least 540 coordinate triples for each point in one session.

To eliminate common errors during the geometric levelling a digital code level was used. This kind of instrument provides objective measurement results and stores all data in the internal memory making reading and writing errors obsolete. The expected strong influence of the refraction was partly averaged out using automatic multiple readings by the instrument. All rays were levelled on a way there and back to have immediate control. Due to the good accordance rays levelled twice no additional levelling was necessary.

3.3. Data processing

In a first step each type of observation was treated separately. The means and the related standard deviations of the GPS coordinate tripels of one session were calculated. In the following an analysis to remove outliers was applied and refined means and standard deviations determined. In a next step significant trends, as far as they could be found in the data sets were removed. Due to multi-path effects spectral analysis was used to determine amplitude and frequency of the dominant oscillations of the ellipsoidal height time series. Nearly every data set had oscillations with frequencies of about 4 cycles per hour and amplitudes up to 2 cm (Fig. 4). Putting together the discovered oscillations to one function, its mean axis was estimated and taken into account to determine the improved final ellipsoidal height of a particular point in one session. These ellipsoidal heights were used to derive six height differences, three along the rays HS_i and three connecting the star points S1 to S3 (Fig. 3). The final local ellipsoidal heights were calculated using a least square adjustment.

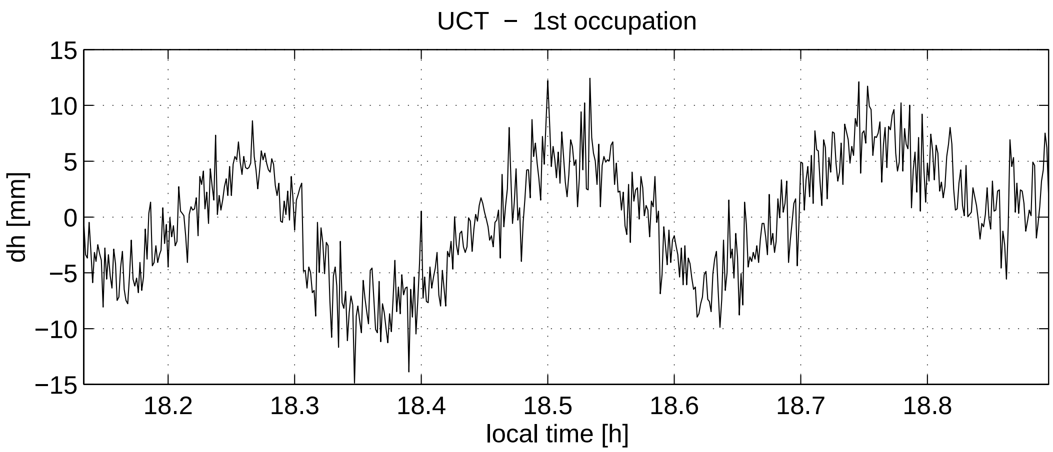

Fig. 4: Characteristic variations of the ellipsoidal height

The derivation of final levelled local heights was done too by an adjustment procedure. The three surplus observations from the double levelled lines were used to derive height differences connecting the star points S1 to S3.

Using the final local ellipsoidal and orthometric heights the wanted local geoidal heights were calculated. For determination of the final local deflection components ξ and η the related local geoid height differences of the star points were used in a least square adjustment. To get an independent check for the estimated accuracy the height for the forth point of the star was calculated using the adjusted local deflection components. A comparison with the directly „measured“ dN, calculated from the adjusted ellipsoidal and orthometric heights, proved a high degree of accordance.

Using formula (4) the local deflection of the vertical between two points P_i and P_k can be interpolated to

$$\mu_{ik} = \xi_i \cdot \cos t_{ik} + \eta_i \cdot \sin t_{ik} \,. \tag{5}$$

Rearranging (3) yields

$$dN_{ik} = \mu_{ik} \cdot d_{ik} / \rho'' \tag{6}$$

and in connection with (2) the final orthometric height difference is

$$dH_{ik} = dh_{ik} - dN_{ik} \,. \tag{7}$$

The lower than usual flood wave in 1996 prevented the determination of deflections of the vertical in the southern part of the Delta. Thus and the fact that the distance between the „geoid determination points" (GDP) is too far a modified approach was used for the determination of the orthometric heights along the Okavango River profile. A kind of weighted interpolation is applied: For a point j within the first third of the distance d_{ik} between two GPDs formula (8a), in the middle third (8b) and in the last third (8c) is used:

$$\mu_{ij} = \mu_{ik} \,, \tag{8a}$$
$$\mu_{ij} = (\mu_{ik} - \mu_{ki})/2 \,, \tag{8b}$$
$$\mu_{ij} = - \mu_{ki} \,. \tag{8c}$$

The ellipsoidal heights, corrected by the undulations dN (formula (6) and (8)) yield the orthometric heights of all points along the profile. These values are plotted in Fig. 5.

3.4. Results

Despite the sandy surface on the islands in the Delta the accuracy of the levelling data is excellent. The standard deviations for the adjusted heights determined by means of geometric levelling range from 0,1 mm to 0,8 mm with a mean of 0,4 mm.

The mean standard deviation for the adjusted local ellipsoidal heights of 1,6 mm is remarkable and shows the potential of GPS measurements for heighting applications. The single values range from 0,6 mm up to 2,7 mm.

Taking into account that the baselines the height differences were derived from is about 1 km an accuracy of about 0,8″ (≅ 4 mm/km) for the deflection of the vertical can be expected. The mean of the adjusted local values of 0,94″ confirms that.

The uncertainty for the orthometric heights results from the accuracy the local geoidal height differences and the ellipsoidal heights were determined. The interpolation of the geoidal heights, which can't be done without hypothesis adds an additional fraction. Whereas the first two accuracies are statistical quantities and can be calculated from measurements, the last one a value has to be estimated for. The RMS for the geoidal height differences is 4 cm, whereas the accuracy of ellipsoidal heights of the points along the Okavango river profile is 1,4 cm (HEISTER, STERNBERG (1995)). Thus leads to an uncertainty of an orthometric height difference of 5 cm and 22 cm for the total height difference Shakawe - Maun.

A comparison of the derived local geoid model along the profile with existing geoids -

global and local - is restricted in general to the geoidal height difference Shakawe - Maun (MERRY, 1995; MERRY ET AL., 1997). For the UCT87A geoid only, geoid heights along the profile were available. Whereas the more recent models (UCT95A and EMG96) are in good accordance with the results presented here (Tab. 1) a detailed comparison with the UCT87A geoid reveals local discrepancies ranging up to more than 50 cm (Fig. 5).

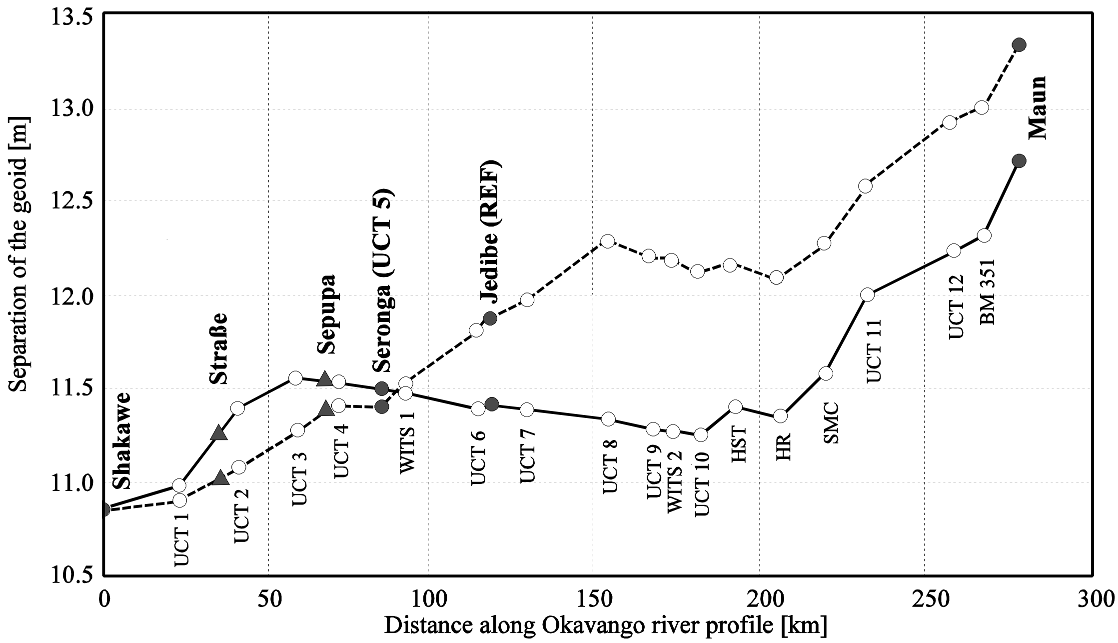

Fig. 5: Local geoid along the river profile, derived from UCT87A (dashed line) and OKAV96 (solid line)

Geoid model / point		UCT87A (1)	UCT95A	EGM96	OKAV96 (2)	ΔN (1) - (2)
Shakawe (BPS 320)	N[m]	10,83	14,12	13,70	10,83*	0,00
Straße	N[m]	11,07	-	-	11,29**	-0,22
Sepupa (UCT 13)	N[m]	11,41	-	-	11,52	-0,11
Seronga (UCT 5)	N[m]	11,40	-	-	11,50	-0,10
Jedibe (REF)	N[m]	11,86	-	-	11,43	0,43
Maun (BP 084)	N[m]	13,33	15,94	15,47	12,72	0,61
	Delta N[m]	**2,50**	**1,82**	**1,77**	**1,89**	**0,61**

Tab. 1: Geoid heights and height differences for profile points observations were carried out during the OKAV96 campaign
(* Value taken from UCT87A, ** Point „Straße“ belongs not to the profile, UCT87A value is extrapolated)

4. Resumee

Local ellipsoidal height differences in combination with local orthometric height differences were used to determine the separation of the geoid in order to convert ellipsoidal heights along a profile of the Okavango Delta into orthometric heights. The determination of the local geoid profile revealed accuracies of 5 mm/km. Assuming a distance of 20 km between the areas local geoidal height differences are determined, accuracies of about 5 cm of orthometric heights can be achieved even in hard to access areas like the Okavango Delta were continuous levelling is impossible.

The main advantage of the presented approach of GPS levelling can be seen in the fact, that orthometric heights can be determined in an environment which is either not suitable for continuous geometric or trigonometric levelling or national survey data (including the knowledge of the geoid) are not available. The use of standard surveying techniques still provides the possibility for accurate and cost efficient orthometric height determination.

5. Acknowledgements

Many people and institutions supported this field campaign contributing their labor, experience and equipment. It was Prof. Dr. Spike McCarthy, Department of Geology, University of Witswatersrand, Johannesburg who started off this project. Mr. Walter Volkmann, Windhoek shared is invaluable knowledge about surveying in such an environment and loaned his equipment, Dr. Jonathan Davey acted as team doctor and Dr. Scott Mason assisted with field work.

Financally support was given by the Universities of Witswatersrand and Cape Town, the Foundation for Research Development and the Deutsche Forschungsgemeinschaft (DFG), Germany.

6. References

HEISTER, H.; HOLLMANN, R., LANG, M.: Multipath-Einfluß bei GPS-Phasenmessungen: Auswirkungen und Konsequenzen für praktische Messungen. *Allgemeine Vermessungs-nachrichten* 104(1997), S. 166-177 (in German)

HEISTER, H.; KRACK, K., SCHÖDLBAUER, A.: GPS 3D point positioning: the height problem, different approaches and numerical results. *South African Journal for Surveying and Mapping* 21(3), pp. 129-132

HEISTER, H., STERNBERG, H.: *GPS-Messungen zur Bestimmung eines Höhenprofils des Jao/ Boro Flußsystems, Okavango Delta*, *Botswana*. Neubiberg, Universität der Bundeswehr München, Institut für Geodäsie, Neubiberg, 1995 (in German)

MERRY, C. H.: *A local geoid for the Okavango delta*. Proceedings of the XXIst IUGG General Assembly, 3 - 14 July, 1995, Bolder, Colorado

MERRY, C. H., HEISTER, H., BARRY, M., ELLERY, W. N., MCCARTHY, T. S., RÜTHER, H., STERNBERG, H.: GPS Heighting in Okavango Delta. *Journal of Surveying Engineering*, November 1998, pp. 145-155

PALLETT, J.: *Sharing water in southern Africa.* Desert Research Foundation of Namibia, Windhoek, Namibia, 1997

RÜTHER, H.: *Report on Okavango Delta GPS Survey, GPS Campaign July 1994.* Cape Town, University, Department of Surveying and Geodetic Engineering, Cape Town, 1994

SCUDDER, T. (ED.).: *Review of the Southern Okavango Integrated Water Development Project.* IUCN, Switzerland, 1993

STANDISH-WHITE, D. W.: Watch on the Okavango. *Optima*, December, 1972.