# ON THE DETECTION OF CHANGE-POINTS IN STRUCTURAL DEFORMATION ANALYSIS

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**Abstract:** In structural deformation analysis the behaviour of the monitored structure is typically described in a dynamic model, by deducing a weighting or transfer function. Its parameters can be estimated from the recorded influencing and deformation signals, using system theory and time series or regression analysis. The analysing functions or the adjustment models can be determined using the entire data set, if the assumption of stationarity up to the $2^{nd}$ order is fulfilled. This is the case if the monitoring activity extends over a long time and the influences on the structure maintain their statistical properties from a long-term point of view.

However due to the higher recording rates made possible by some modern sensors also short-term deformations and influencing factors like wind or traffic, which expose a more irregular pattern, can be registered and included in the investigation of the structures' behaviour. In these cases the stationarity assumption needs a more careful analysis. The data segments with homogeneous statistical properties have to be identified and different model parameters have to be estimated for each of them.

This paper deals with an approach for the detection of the change-points in the statistical properties of the data. The method is based on the likelihood function. In this approach the change-points are estimated by minimising a penalised contrast function. To get a better insight in the behaviour of the structure when several effects overlap, the signals are first decomposed using the Discrete Wavelet Transform. The change-point method is applied to the obtained signal components. The performance of the approach is assessed by analysing simulated and real data, recorded during the monitoring of a wind energy turbine and a vertical lift bridge.

## 1. Introduction

A main objective of deformation analysis is to derive from the recorded influences and deformations the structures behaviour in a system theoretical approach. In the non-parametric modelling approach, to which this paper mainly refers, the system is described by generic weighting or transfer functions. The standard theory of time series analysis used to determine these functions assumes at least weak stationarity of the recorded data series. In a global perspective, after removing the trend, this condition can be regarded as fulfilled when the geodetically monitored processes are slowly-varying and periodic. In this case a single weighting or transfer function describes the properties of the analysed object.

In some particular cases like the variation in magnitude and direction of influences (like in the case of wind or traffic), short-termed monitoring with sensors that facilitates high recording rates (such as GPS, inclinometers or laserscanners) or unusual events occurring during the observation period, the acceptance of weak stationarity requires a more thorough analysis. It will be considered however, that the effects causing non-stationarity are localised in relation to the length of the series and induce a change in the mean and/or variance of the data. Between these changing times the series is assumed to be stationary. Thus, if one identifies the location of the changes, the standard modelling tools can still be used on the homogeneous part of data. Therefore a method, like the one presented in this paper is needed. If the recorded deformation signal is caused by more than one influence, it might be difficult to detect the one causing the change. In this case it is purposed to decompose the signal into frequency components using a Discrete Wavelet Transform (DWT) and to apply the detection method to each one of them.

This paper is organised as follows: chapter two presents the method used for change-point detection. A brief discussion on the DWT in the third chapter is followed by the presentation of the results obtained by applying this purposed approach to deformation signals recorded at a vertical lifting bridge and at a wind energy turbine (w.e.t). The paper ends with a summary and an outlook.

## 2. Detection of Change-Points in Time Series of Observations

The scope of the change-point problem is to detect locations where the statistical properties of the system change and to estimate the magnitude of this change. The detection problem can be formulated either online or off-line depending on the availability of the entire time series for recovering the configuration of the change-points. The method presented in this paper belongs to the second category. It was developed by Lavielle [5],[6] and is based on the likelihood function. The used approach is quite different from other strategies that assess a change by hypothesis testing because it estimates the entire configuration of change-points at a time.

In order to describe the pure estimation process we will begin with considering the simplified situation when the number of changes is known a priori and we need to estimate only their configuration and the parameter values. In the second part of the chapter we will deal with the case of an unknown number of change-points.

### 2.1. Detecting the configuration of a known number of change-points

Let $\{z_1, z_2,...,z_n\}$ be a general set of n observations of a piecewise stationary process **Z** with a density function f depending on some unknown parameters $\theta$. By piecewise stationarity is meant that there are $K<n$ moments at which changes in the statistical properties of the observations occur. Between these moments the process is considered as stationary. The primary goal in this section is to estimate the locations of the K change-points and the parameters $\theta$ according to a minimising criterion. The minimised function is called contrast function $\gamma$ and the obtained estimators are named minimum contrast estimators. This principle is similar to the approaches used in geodetic parameter estimation. In fact minimum contrast estimators are a more general class of estimators that includes least squares (ls), minimum L1-norm or maximum-likelihood (ml). They were introduced in order to derive properties (especially related to convergence) for the above-mentioned estimators in a unified framework. Useful convergence properties have been obtained by Birge et al. [3] under some

necessary assumptions which control the regularity of the contrast function $\gamma$ and the complexity of the parameter space $\Theta$. By expressing the contrast function $\gamma$ in the estimation process with respect to the empirical data, one obtains the empirical contrast $\gamma_n$. The minimum contrast estimator $\hat{\theta}$ is minimising the empirical contrast:

$$\gamma_n\left(f\left(\hat{\theta}\right)\right)=\inf_{\theta\in\Theta}\gamma_n\left(f\left(\theta\right)\right)=\inf_{\theta\in\Theta}\left(\frac{1}{n}\sum_{i=1}^{n}\gamma\left(z_i,f\left(\theta\right)\right)\right) \tag{1}$$

For density estimation problems the log-likelihood function l is a proper choice of a contrast function provided that the density function is continuous. This condition is satisfied by the normal density function which is also appropriate if the changing parameters $\theta$ refers to the mean $\mu$ and/or the variance $\sigma^2$ of the observations. The changing parameters are calculated as the ml-estimates from the observations $Z_k$ belonging to each stationary segment k between the times $t_{k-1}+1$ and $t_k$. The other parameters are calculated using the entire realisation $Z_i$. The configuration $\tau$ of the changes is then obtained by minimising the empirical contrast:

$$\gamma_n\left(\tau\right)=\frac{1}{n}\sum_{k=1}^{K}-l\left(z_{t_{k-1}+1},z_{t_{k-1}+2},\ldots,z_{t_k};\hat{\theta}_k\right)\to\min. \tag{2}$$

When the changes affect the mean and the variance the log-likelihood function in (2) has the following structure:

$$l\left(z_{t_{k-1}+1},z_{t_{k-1}+2},\ldots,z_{t_k};\hat{\mu}_k,\hat{\sigma}_k^2\right)=\frac{n_k}{2}\left[1+\log\left(2\pi\right)+\log\left(\frac{1}{n_k}\sum_{i=1}^{n_k}z_{t_{k-1+i}}^2-\frac{1}{n_k^2}\left(\sum_{i=1}^{n_k}z_{t_{k-1+i}}\right)^2\right)\right] \tag{3}$$

where $n_k$ is number of samples in the $k^{th}$ stationary set. Similar relations can be obtained if the changes affect either the mean or the variance. Lavielle has shown in [6], that if the estimation of parameters $\theta$ is consistent, then the estimation of the change-point configuration is also consistent and converges to the true configuration at the rate of $\mathcal{O}(n^{-1})$. Furthermore, this convergence rate is independent from the covariance structure of the process.

The minimising of (2) can be solved only using a combinatorial approach because $\tau$ is not explicitly in the functional model. The computational burden can be reduced if one adopts a recursive scheme by minimising (2) for subsequent numbers of change-points. Therefore the log-likelihood functions have to be computed for all ordered subsets of the form $\{z_i, z_{i+1},\ldots,z_j \mid 1\leq i\leq j\leq n\}$ in order to detect the current configuration of stationary segments for each subset of the form $Z_i = \{z_1, z_2,\ldots,z_i\}$ with $1 \leq i \leq n$. This approach is also useful in the case of an unknown number of stationary segments which will be discussed in the next section.

### 2.2. Detecting the configuration of an unknown number of change-points

This section deals with a more practical situation in the analysis of real data when the number K of stationary subsets is unknown. In this case the minimisation function (2) has to be extended with a penalisation factor that includes this additional unknown:

$$\gamma_n\left(\tau\right)=\left[\frac{1}{n}\sum_{k=1}^{K}-l\left(z_{t_{k-1}+1},z_{t_{k-1}+2},\ldots,z_{t_k};\hat{\theta}_k\right)\right]+\mathrm{pen}(K)\to\min. \tag{4}$$

Solving this minimisation problem requires some prior knowledge about the maximum number of change-points $K_{max}$. Thus, with a proper choice of the penalisation factor, one may apply the computation procedure described in 2.1 with $K=K_{max}$. The estimation problem is now enhanced by an aspect of model selection. It is known from similar model selection problems that an optimal penalisation factor will compensate for the decrease of the empirical contrast (2) after attaining a plausible number of change-points.

In order to get a better insight to our particular problem of model selection we have analysed a series with 500 independent, standard normally distributed samples for cases when (a) no change of the statistical properties occur and (b) when two changes of the mean occur after 200 and 400 samples respectively. A typical decrease of the empirical contrast (2) for $K_{max}$ taking values from 1 to 10 is represented in Fig. 1.

It can be noticed, that the empirical contrast decreases regularly in the case without changes, while in the right plot a discontinuity clearly shows up corresponding to the correct number of stationary parts of the series. However, for values of $K_{max}$ increased beyond this discontinuity, the decrease of the function is again uniform. This example illustrates, that the choice of the penalty term has to be related to the decrease of the empirical contrast function for values of $K_{max}$ higher than the "true" number of stationary segments, or, to express that in mathematical terms, to the convergence or the risk of the estimate. For the discussed example the choice of a linear model depending on $K_{max}$ seems to be appropriate.

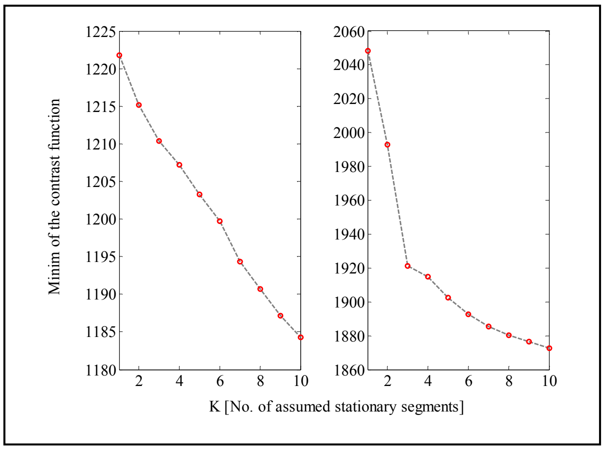


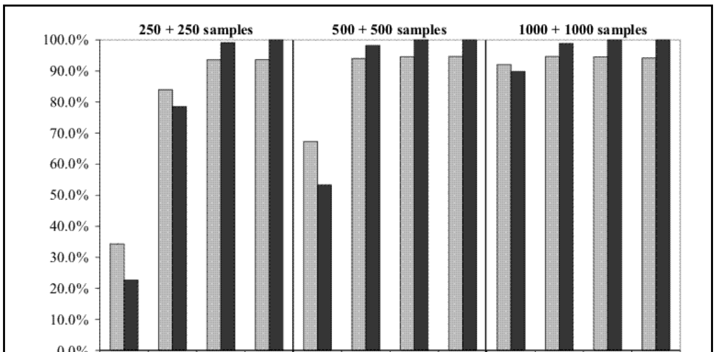

Figure 1: Variation of the contrast function (2) for increasing values of $K_{max}$ for series with no change-points (left) and two change-points (right)

Mathematical foundations of the conclusions previously discussed in a more practical approach were obtained by Birge et al. [4] and Barron et al. [1] using the theory of sieves for contrast estimators. This is the main reason why the well-known ml-estimation was presented in the framework of contrast estimators. The authors have proven that the risk of every empirical contrast estimation is of order $\kappa \cdot L_m \cdot D_m / n$ where $D_m$ is the dimension of the approximating space, $\kappa$ a constant and $L_m$ a weighting function that is chosen depending on whether the approximating space is built in a regular or irregular approach. In the first case, the approximating space is obtained by adding successively one element to the ordered set of already selected change-points. Thus a weighting function chosen as $L_m = 1$ represents the only possibility to obtain the space of a given dimension. In the irregular case, the D-dimensional approximating spaces are generated from every possible combination of D members out of an N-dimensional set. From the number of possible combinations $C_N^D$ one gets a weighting function of the type $L_m = 1+\log(N/D)$ (see [1] for more details).

In order to get a picture of the sensitivity of the change-point method it was first applied on synthetic signals. This research deals only with the detection of a single sudden change in the mean and does not claim completeness. The detection was done using both types of weighting functions. In the first case the model is referred as LinBib and in the second case as LinLog. The breakpoint in the variation of the contrast function (Fig. 1) was detected by assessing the significance of the prediction in a progressive regression, with the penalty term as a functional model. In the case of the linear model, beside the standard ls-estimation a robust approach based on the BIBER estimator [9] was applied to prevent a greater influence by the end points. The differences between the identification rates were less than 1% but those obtained in the robust approach were systematically better. This is why the following presentations will refer to this one only.

The analysed series were obtained from uncorrelated, standard normally distributed variables. Their lengths were of 500, 1000 and 2000 samples when the change occurred in the middle and of 275, 550 and 1100 when only 10% of the data in the first part followed after the change. The magnitude of change was set at 0.3σ, 0.5σ, 0.8σ and 1.0σ. For each combination of the factors 10,000 signal samples were generated and analysed.

A dependence of detection performances on the changes magnitude and on the length of the series was clearly noticeable for both locations of the changes. In the case of small changes the choice of a linear penalty term has led to better results while for larger changes and data lengths the linear-logarithmic performed better. For jumps of the order of noise variance the logarithmic penalisation led to ideal detection rates while the performances of the linear model seem to stagnate at a level of 95% in spite of an increased data length. Furthermore, these results show that the ability to detect small changes with the discussed approach, an important aspect in geodetic monitoring activities, achieves practical relevance only if it is applied to a large data basis. Fig. 2 presents the results of the change in the mid of the series.




| | 0,3s | 0.5s | 0.8s | 1.0s | 0,3s | 0.5s | 0.8s | 1.0s | 0,3s | 0.5s | 0.8s | 1.0s |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LinBib | 34.3% | 84.0% | 93.6% | 93.7% | 67.2% | 94.0% | 94.6% | 94.7% | 92.1% | 94.7% | 94.6% | 94.2% |
| LinLog | 22.6% | 78.5% | 99.1% | 100.0% | 53.3% | 98.2% | 100.0% | 100.0% | 89.9% | 98.9% | 100.0% | 100.0% |

Figure 2: Automatic detected changes in 10.000 runnings with the jump occurring in the mid of the series.

This analysis of synthetic signals, although referring to just one specific kind of change, gives a good basis for understanding advantages and weaknesses of the discussed method. However these results can be of little help, if the recorded deformation signal contains more than one frequency component because the allocation of a detected change to a certain component might be difficult. One way to handle this is to separate the signal into frequency components and to apply the change-point detection method on each of them. This task could be accomplished with a wavelet-transform that is briefly presented in the following section.

## 3. The Wavelet Transform for Time Series Analysis

The DWT is a tool for analysing the signal in the time-frequency domain. It decomposes the analysed signal in high- and low-frequency components by passing it through an orthogonal two-channel filter bank and retaining every other value [2]. The high- and low-pass filters of the bank are called wavelet and scaling filter, and the corresponding outputs are the wavelet and scaling coefficients, v and u respectively. In a following decomposition step j the previously obtained scaling coefficients $u_{j-1,k}$ are the new input to the filter-bank:

$$u_{j,n} = \sum_{k \in Z} h_{k-2n} \cdot u_{j-1,k} \quad \text{and} \quad v_{j,n} = \sum_{k \in Z} g_{k-2n} \cdot u_{j-1,k} \tag{5}$$

with h and g representing the scaling and wavelet filters. The cascading filtering operation according to equation (5) can be expressed as a single low-pass and band-pass filter respectively with pass-bands $0\text{-}1/2^{j+1}$ and $1/2^{j+1} - 1/2^{j}$ [8]. Therefore the $j^{th}$ decomposition level is referred by the scale $2^j$ representing the spectral components of the signal contained in the wavelet coefficients.

A powerful characteristic of the DWT is its energy preserving property allowing the extension of Parseval's relation to the time-frequency domain. Since energy and variance are proportional measures, the DWT decomposes also the variance over the scales. Thus the set of wavelet coefficients $\{v_j\}$ represents the contribution to the total energy of the signal due to changes with frequencies in the corresponding pass-band of the scale $2^j$ and express the build-up in time of this variance component. It is therefore possible, to assess the variance homogeneity of individual signal components by applying a method for variance change detection on the corresponding series of wavelet coefficients and to calculate only the respective model parameters on homogeneous intervals [7].

On the other hand a relation between the wavelet coefficients and the signals mean cannot be established because all wavelet coefficients have expected value equal to zero by the definition, thus being centred on zero regardless of the signals level. Therefore, the information about the mean must be recovered from the scaling coefficients. However, a straightforward detection of mean changes from the scaling coefficients is not possible, because local characteristics of the signal appear different in the coefficients of a DWT, depending on where it "breaks" into the signal. Thus, the shape or pattern of the signal can look completely changed in the scaling coefficients. This translation dependency of the DWT is caused by the downsampling step performed at each level and has obviously negative influence on locating the change in mean.

One way to circumvent the downsampling step is to control the resolution level at which the signal is analysed by introducing a zero between every term of the wavelet and the scaling filters. This upsampling operation is denoted in the following equation by the symbol $\left(\uparrow 2\right)$. In this case, the relation (5) for the $j^{th}$ decomposition level turns to:

$$u_{j,n} = \sum_{k \in Z} \left( \left(\uparrow 2\right)^{j-1} h \right)_k \cdot u_{j-1,n-k} \quad \text{and} \quad v_{j,n} = \sum_{k \in Z} \left( \left(\uparrow 2\right)^{j-1} g \right)_k \cdot u_{j-1,n-k} \tag{6}$$

These relations correspond in structure to a convolution, and thus fulfil the condition of translation-invariance. The downside of this approach is the abandonment of the orthogonality property. Nonetheless, Percival et al. [8] have proven that the energy preserving property also holds in the case of the Maximal Overlap DWT (MODWT), as they named the modified transformation (6). Thus, the method for detecting variance changes can be applied to the wavelet coefficients obtained by (6) too. Additionally, the local characteristics in the signal and in the resulting coefficients can be lined up such that the method for detecting changes in the mean can be applied to the scaling coefficients.

In this study both kinds of transformations were used to decompose the signals, depending on the type of expected changes. The change point detection method was used to assess for variance changes in the wavelet coefficients and mean changes in the scaling coefficients. The main results are presented in the next chapter.

## 4. Automatic Change-Point Detection for Structural Deformation Analysis

A first application of the discussed change-point method aims to study the deformations of a vertical lift bridge due to influences of traffic. This is a trivial task if it is performed on a manageable amount of data. However, as this bridge is an aged structure built in 1934 an increased number of verification measurements might be necessary to assure its safety and functionality. Hence, the amount of data can increase rapidly. In this case some degree of automation can support their evaluation.

The analysed time series were recorded with a Schaevitz servo-inclinometer that is highly resistant against vibrations. The sensor was placed next to the carriage way, 16.6 m far from the end of the lifting part of the bridge. Its analogue output signal was sampled at a rate of 50 Hz. The measurements were done at an air temperature of 3°C. The traffic was logged by video.

Fig. 3 illustrates exemplarily a deformation signal recorded over a time span of 2.5 minutes and its smooth version filtered with a moving average filter with a length of 50 values. The step-line indicates the traffic load. A load caused by a vehicle is indicated during its total passing time.

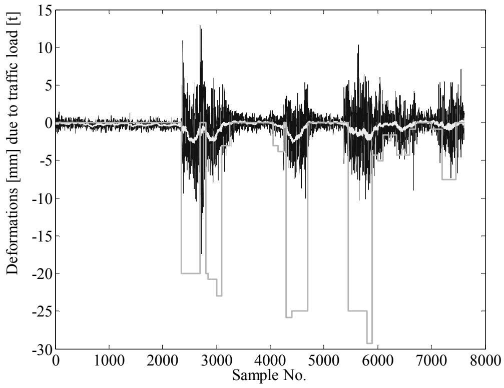



Figure 3: Deformation of the bridge due to traffic load

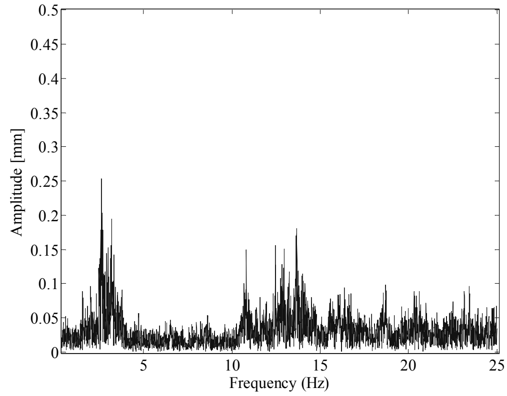



Figure 4: Amplitude spectrum of the series

The deflections produced by the passing vehicles vary between 0.8 and 2.6 mm. They are additionally superimposed by an increased variance. The amplitude spectrum of the data is shown in Fig. 4. It indicates an increased variability with frequencies in the band between 2.5 and 3.7 Hz and a higher noise level above 10 Hz. However it is not possible to ascribe certain features to the states of the structure under various loads. Therefore the recorded signal is decomposed by a wavelet-transform using the Haar-filters. These filters were chosen because of their good properties for time-localisation. The focus was set on the analysis of the wavelet and scaling coefficients obtained after the fourth decomposition level because no further significant variance components were indicated beyond the fourth level and the previous coefficients did not contain any information about the deformation behaviour.

The detection method discussed in Chapter 2 was applied to assess for changes in the mean of the scaling coefficients. The configuration of change-points shown in Fig. 5 was detected automatically using a linear penalisation. Note that the deformations obtained from the wavelet and scaling coefficients are scaled approx. by 4 due to the gain of the used filters. As can be seen the method performs reasonably in detecting the time segments when deformations occurs. However, in more complex situations, when the deformations succeed at a short time they are detected as a single segment or remain undetected as in the case of the two smaller deflections at the end of the series. For a logarithmic penalisation the number of segments increased to eleven, but all the new bounds are set within the second segment. In this case the logarithmic penalisation is too sensitive because it is detecting segments that are of no practical use. This happens in general when the parameter is not changing sharp enough.

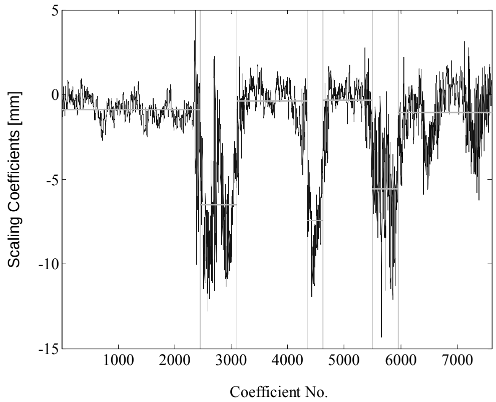



Figure 5: Deflections of the bridge detected as changes in the mean

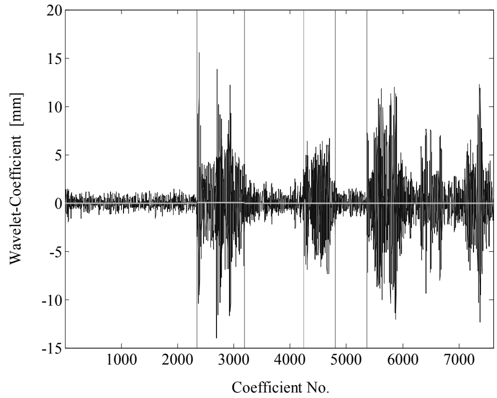



Figure 6: Influences of the traffic load detected as changes in the variance

The detection of deformations is a good support for estimating their magnitude by fitting polynomial or exponential functions that are certainly more adequate than the mean. Further the elastic character of the deformation can be checked at any time. The structures behaviour under traffic load can be related also to temperature by repeating the monitoring at different atmospheric conditions and comparing the lengths of similar deformations.

Additionally to the deflections, the traffic causes also oscillations of the structure. The change-point detection method was applied to the wavelet coefficients to search for variance changes because the variability information is mainly contained in them. In this case the linear and the logarithmic penalisation led to the same number of bounds. Comparing the change-point configuration shown in Fig. 6 with the estimation from the scaling coefficients one observes the good agreement between the right-most five segments. In the complex situation at the end of the series fewer change-points were identified, although from the visual analysis three more bounds seem plausible. If the number of change-points is increased manually the new bounds are placed at the expected locations of 5905, 6702 und 7107. Fig. 7 shows exemplarily an amplitude spectrum obtained for the data in the segments with increased variability. The significant amplitude at a frequency of 3.1 Hz occurs due to oscillations most probably induced by the handrail of the bridge.

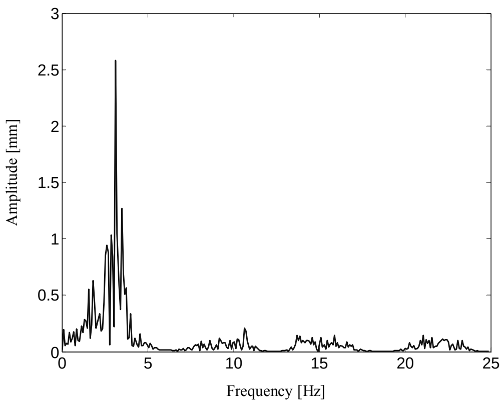



Figure 7: Amplitude spectra of the data in the 4th segment with homog. variance

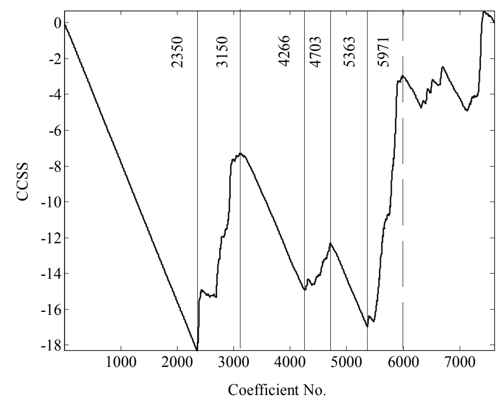



Figure 8: Centred cumulative sum of squares of the wavelet coefficients

If one uses the centred cumulative sum of squares (CCSS) of the wavelet coefficients to assess for variance homogeneity as done in [7], the iterative process for the automatic detection will not converge after a reasonable number of iterations. The intervals in Fig. 8 are obtained by setting the bounds manually at the points where the slope of the CCSS-line changes. They agree very well with the segments in Fig. 6.

The features identified in the deformation signal can be used in a refined model. The performances of such a refined model are briefly shown in a second example that refers to the deformations of the pylon of a w. e. t. due to wind loads. A detailed presentation of this study was given in [7]. The analysed data were recorded with a sampling rate of 6.1 Hz using the same Schaevitz-inclinometer. During this time the w.e.t. had a nearly constant power output of 110 KW and a rotor velocity of 12 rpm. Due to the changing wind direction the orientation of the nacelle and the blades' pitch varied. The main periodicities contained in the data are on the first and second eigenfrequency of the pylon and on rotation induced frequencies like the rotors frequency of 0.2 Hz, the blade frequency of 0.6 Hz and higher harmonics of the latter. If all amplitudes and phases of the dominant frequencies are estimated in a common adjustment model, the spectrum of the residuals shown in Fig. 9 is obtained.

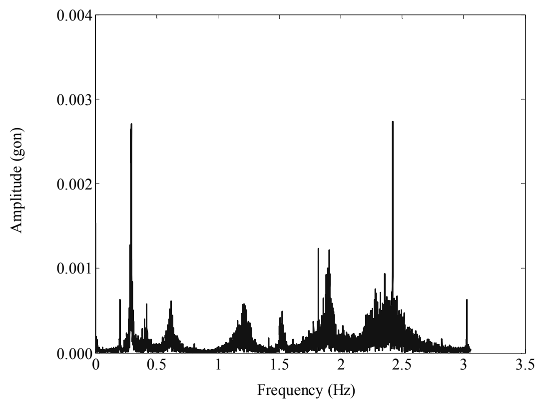



Figure 9: Amplitude spectra of the residuals of the global model

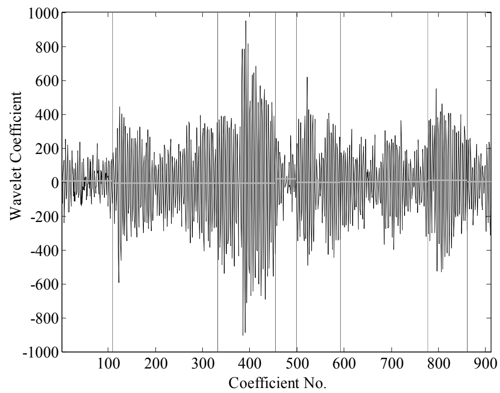



Figure 10: Intervals of homogeneous variance detected from the wavelet coefficients

Fig. 9 shows that there are still periodicities on the first and second eigenfrequencies which are not modelled satisfactory. A possible reason for this can be small variations of the frequency and/or the amplitudes. In order to get a better insight, the series was decomposed into spectral components using a DWT with Daubechies filters of the $4^{th}$ order (db4). The higher filter order was chosen to improve the separation between the dominant frequencies in the decomposition over scales. The presented change-point detection method was used to detect intervals of homogeneous variance in the series of wavelet coefficients. Fig. 10 shows the obtained bounds for the wavelet coefficients in the $2^4$ scale. The number of homogeneous intervals was set correspondingly to the ones detected with the CCSS algorithm. Their distribution of the bounds agreed to a good part but there were also some small differences at single locations. Future studies will deal with the cause for these differences.

The amplitudes and phases were estimated separately on each homogeneous interval of wavelet coefficients. The modelled wavelet coefficients were recomposed to an overall modelled signal using the inverse DWT. The wavelet transform and its inverse caused no information loss because of its orthogonality. Thus the residuals between the modelled and the measured signal reflect only the accuracy of the model. The standard deviation of these residuals is only 30% of the one obtained for the global model. This improvement is also visible in the spectrum of the residuals that contains only reduced amplitudes at the dominant frequencies:

## 5. Summary and Outlook

The method used in this paper to detect changes of the mean and/or variance of time series is based on the maximisation of the likelihood-function. It uses a penalised function if the number of stationary segments is unknown. The choice of the penalty factor depends on the complexity of the search space for the bounds. The performances of various penalty functions were assessed on simulated signals with different configurations of the change-points.

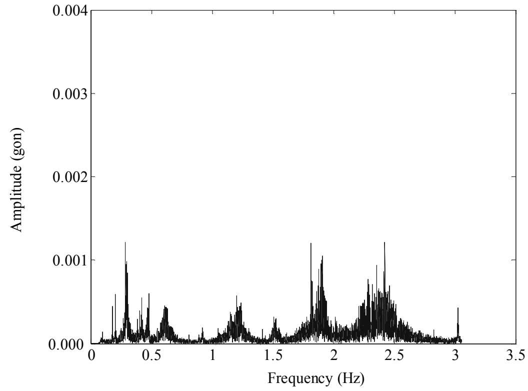



Fig. 11: Amplitude spectra of the residuals of the refined model

By applying the detection method directly to the original signal the identification of the specific influence causing the change becomes more complicated. Therefore it was proposed to decompose the signal by a wavelet-transform and check the wavelet and scaling coefficients for changes in the variance and mean respectively. The advantages of this approach were shown by detecting specific features of the behaviour of a bridge when influenced by traffic and of a pylon of a w.e.t. when influenced by wind.

In this analysis the data were treated as being uncorrelated and normally distributed. Accounting for existing correlations in the minimising procedure, the handling of other distributions and comparisons with other change-point detection methods are tasks for future work.

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