COMPARISON OF CORRECTION MODELS FOR DISTANCE DEPENDENT SYSTEMATIC EFFECTS IN GPS MONITORING NETWORKS WITH LARGE HEIGHT DIFFERENCES

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Abstract: GPS is a promising tool for real-time monitoring of deformations of slopes. However, remaining systematic effects in GPS phase observations (after double differencing and application of apriori models) affect the resulting coordinates and bias the determination of the actual deformations. Therefore, it is vital to remove these network distortions prior to the deformation analysis. Since the magnitude of distance dependent systematic effects (i.e. effects due to tropospheric refraction, ionosphere phase advance and orbit errors) increases linearly with the station separation, it can be shown that the resulting network distortions are affine. Therefore a 12 parameter affine transformation is the general correction model in the coordinate domain. However, due to the special physics of GPS observations only eight independent parameters are sufficient for a correction. Considering GPS networks with large height differences the distortion pattern is dominated by tropospherically induced effects yielding mainly height distortions. For such cases, it was shown in previous work by members of the Institute of Engineering Geodesy and Measurement Systems that a two parameter model (translation and scale in height) seems to be appropriate to correct the height distortions for the special network design of aligned points in direction of the maximum slope.

In this paper, the eight parameter approach is presented and compared with the two parameter approach. A representative 8 h data set from the Gradenbach GPS monitoring network is processed. It is shown that both approaches give almost identical results for the height component for the specific design of the Gradenbach network. Here, about 80% of the height distortions can be corrected to values smaller than 0.5 cm. Furthermore it can be stated that in general the absolute improvements in the horizontal components are rather small compared to those of the vertical component.

1. Motivation

Systematic effects in GPS phase observations distort the network geometry of GPS control networks and thus complicate the separation of actual deformations and pseudo-deformations induced by the systematic effects. For small GPS networks (baseline length < 5 km) distance dependent systematic effects lead to *affine distortions* of the network geometry, [2]. Consequently, the most general correction model is a 12 parameter (affine) transformation. However, due to the special geometry of small GPS networks and the special physics of GPS observations, only 8 out of the 12 parameters can be determined in a meaningful way. These are the six parameters of a two-dimensional affine transformation (two translation (t_x, t_y) and two scale (e_x, e_y) parameters as well as one shear strain and one rotation parameter (e_{xy}, r_z)) and an additional scale and translation parameter (e_z, t_z) in the third dimension, [9].

For small GPS with large height differences (like e.g. monitoring networks of open pit mines [7], bridges or landslides [3]) the distortions are predominated by residual tropospheric effects, yielding mainly height distortions, [3, 4]. Furthermore, if all stations are submitted to the same meteorological regime, theses height distortions depend linearly on the height difference with respect to the reference station. The height distortions are especially well pronounced if all stations are aligned in direction of the maximum slope, cf. [8]. Such networks designs are typical for the installation and maintenance of cable railways.

The GPS monitoring network of the landslide Gradenbach shows a similar design: all network points are quasi located on a line in direction of the maximum slope. Using the example of a 8-h data set from this monitoring network, the performance of two correction models in the coordinate domain is compared: (i) the two-parameter model (translation and scale in height, proposed by [8], and (ii) the eight-parameter model with the above given parameters.

2. Correction strategies in the coordinate domain for deformation measurements

2.1. Two parameter model

We briefly review the two-parameter model proposed by [8]. Its development is based on the following findings from GPS data analysis in mountainous areas. It was stated that the systematic height variations δH_i of the station *i* are linearly related to the station height h_i at an arbitrarily chosen reference epoch by

$$\delta H_i = t_z + h_i \; e_z \,. \tag{1}$$

For the determination of theses two parameters (translation t_z and scale e_z in height), at least two stations with stable heights are necessary, i.e. the stations are not affected by the movements to be monitored. These stations are the reference station and one *calibration station*. The coordinate of the reference station were held fixed ($d\hat{N}_{ref} = d\hat{E}_{ref} = d\hat{H}_{ref} = 0$), while those of the calibration station were estimated in the network adjustment. Assuming that the resulting estimated height correction $d\hat{H}_{cal}$ are predominated by the systematic height variations δH_{cal}, the two parameters can be determined with Eq. (1)

$$\begin{pmatrix} 0 \\ d\hat{H}_{cal} \end{pmatrix} = \begin{pmatrix} 1 & h_{ref} \\ 1 & h_{cal} \end{pmatrix} \begin{pmatrix} t_z \\ e_z \end{pmatrix}, \tag{2}$$

yielding

$$\begin{pmatrix} t_z \\ e_z \end{pmatrix} = \frac{1}{h_{cal} - h_{ref}} \begin{pmatrix} h_{cal} & -h_{ref} \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ d\hat{H}_{cal} \end{pmatrix} = \frac{d\hat{H}_{cal}}{h_{cal} - h_{ref}} \begin{pmatrix} -h_{ref} \\ 1 \end{pmatrix}. \tag{3}$$

Finally, with Eqs. (1) and (3) the corrected estimated height corrections $d\hat{H}_i$ of any monitoring station *i* are obtained by

$$d\hat{H}_i' = d\hat{H}_i - \frac{h_i - h_{ref}}{h_{cal} - h_{ref}} d\hat{H}_{cal} \,. \tag{4}$$

2.2. Eight-parameter model

In the eight-parameter model [9], systematic variations of horizontal coordinates can also be corrected. In this case the apparent coordinate variations $\delta N_i, \delta E_i, \delta H_i$ are expressed by:

$$\begin{pmatrix} \delta N_i \\ \delta E_i \\ \delta H_i \end{pmatrix} = \begin{pmatrix} 1 & 0 & N_i & 0 & E_i & E_i & 0 & 0 \\ 0 & 1 & 0 & E_i & N_i & -N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & H_i \end{pmatrix} \mathbf{p}, \quad \text{with}$$
$$\mathbf{p} = \begin{pmatrix} t_x & t_y & e_x & e_y & e_{xy} & r_z & t_z & e_z \end{pmatrix}^T , \tag{5}$$

where t_x, t_y, t_z denote the 3 translations, e_x, e_y, e_z the 3 scale parameters, e_{xy} the shear strain and r_z the rotation parameter. The coordinates N_i, E_i, H_i are expressed in a common topocentric system with the reference station as its origin. The eight parameter model necessitates at least three "good" GPS stations in stable terrain for the determination of the parameters.

In order to guarantee the identical location of the coordinate system for all epochs, the constraint:

$$t_z = 0 \,. \tag{6}$$

should be added to Eq. (5). If we hold the coordinates of the reference station fixed, with (5) and (6) we obtain directly

$$t_x = t_y = t_z = 0 \,. \tag{7}$$

Using two stable calibration stations with known coordinates, the parameters describing the systematic horizontal coordinate variations are uniquely determined by

$$\begin{pmatrix} e_x \\ e_y \\ e_{xy} \\ r_z \end{pmatrix} = \frac{1}{N_1 E_2 - N_2 E_1} \begin{pmatrix} E_2 & 0 & -E_1 & 0 \\ 0 & -N_2 & 0 & N_1 \\ -\frac{1}{2} N_2 & \frac{1}{2} E_2 & \frac{1}{2} N_1 & -\frac{1}{2} E_1 \\ -\frac{1}{2} N_2 & -\frac{1}{2} E_2 & \frac{1}{2} N_1 & \frac{1}{2} E_1 \end{pmatrix} \begin{pmatrix} d\hat{N}_1 \\ d\hat{E}_1 \\ d\hat{N}_2 \\ d\hat{E}_2 \end{pmatrix} , \tag{8}$$

and that one for the vertical variations by:

$$e_z = \frac{1}{H_1^2 + H_2^2} \begin{pmatrix} H_1 & H_2 \end{pmatrix} \begin{pmatrix} d\hat{H}_1 \\ d\hat{H}_2 \end{pmatrix} . \tag{9}$$

With Eq. (8) and (9), the corrected coordinate estimates of the monitoring station *i* reads:

$$\begin{pmatrix} d\hat{N}_i' \\ d\hat{E}_i' \\ d\hat{H}_i' \end{pmatrix} = \begin{pmatrix} d\hat{N}_i \\ d\hat{E}_i \\ d\hat{H}_i \end{pmatrix} - \begin{pmatrix} N_i & 0 & E_i & E_i & 0 \\ 0 & E_i & N_i & -N_i & 0 \\ 0 & 0 & 0 & 0 & H_i \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_{xy} \\ r_z \\ e_z \end{pmatrix}. \quad (10)$$

3. Application to the GPS monitoring network Gradenbach

Figure 1 shows a digital terrain model of the Gradenbach area (5x5 km^2) and the location of the 6 GPS stations (three stations (R1, R2, R4) in stable terrain (bedrock), and three monitoring stations MA, MC, and MD on the slope). All 6 GPS stations are equipped with Ashtech receivers, and Ashtech choke-ring antennas with SCIS-type radome. The data was recorded all 3 s.

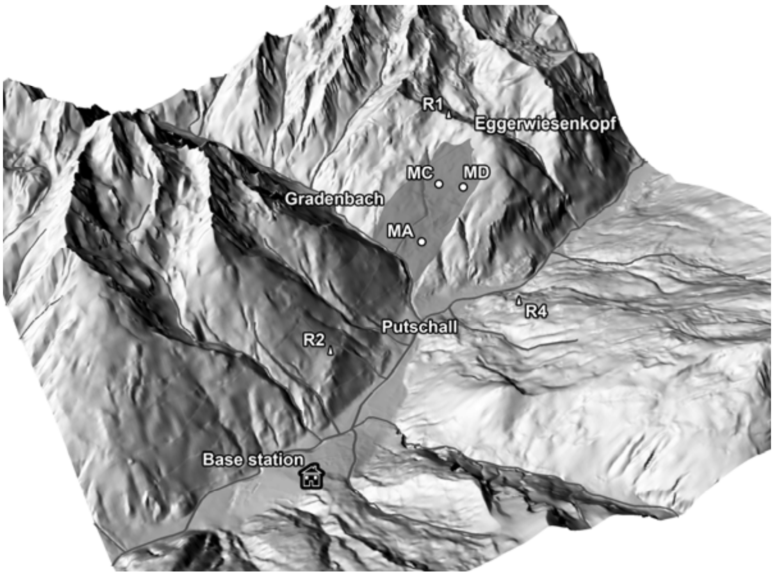

Figure1: Landslide Gradenbach area with GPS stations

The GPS data was processed by a Kalman-filter based Matlab toolbox developed by A. Wieser at the Institute of Engineering Geodesy and Measurement Systems (EGMS). For the processing of a representative 8-h data set (2.10.2004 20:00 – 3.10.2004, 4:00) a cut-off angle of 10° and the SIGMA-ε variance model [5] were applied. As initial coordinates served a 48-h static solution by Bernese 5.0 [6] where zenith wet delays (ZWDs) with a temporal resolution of 2 h were estimated. The baselines were computed with respect to R2, and the coordinates of R2 were held fixed during the adjustment. Table 1 gives the baseline lengths and height differences.

baseline	length [km]	height difference [m]
R2-R1	4.55	899.6
R2-R4	2.71	28.9
R2-MA	2.60	1.4
R2-MC	3.50	391.1
R2-MD	3.54	451.7

Table 1: Overview on the baseline lengths and height differences in the Gradenbach network

Figure 2 shows the obtained coordinate time series for the North, East and Up components with respect to the static Bernese solution, respectively. The height component (Figure 2c) shows a typical height dependent pattern of the tropospherically induced bias: the larger the height differences with respect to the reference station, the larger the bias. For the horizontal components at a first glance, the patterns depicted in Figures 2a and 2b are not well pronounced. However, taking the mean value over the 8-h time series, a bias in baseline direction remains, cf. Figure 2d. Note that unmodelled ionospheric as well as absolute tropospheric effects lead to biases in baseline direction, cf. [1]. Consequently, in this direction, it is most difficult to separate actual and apparent motions. Unfortunately, like in the case of the Gradenbach network, the baseline directions often coincidence with the directions of probable movements. Therefore, for high precision application it is of major interest to correct the network induced distance dependent systematic effects in the GPS observables.

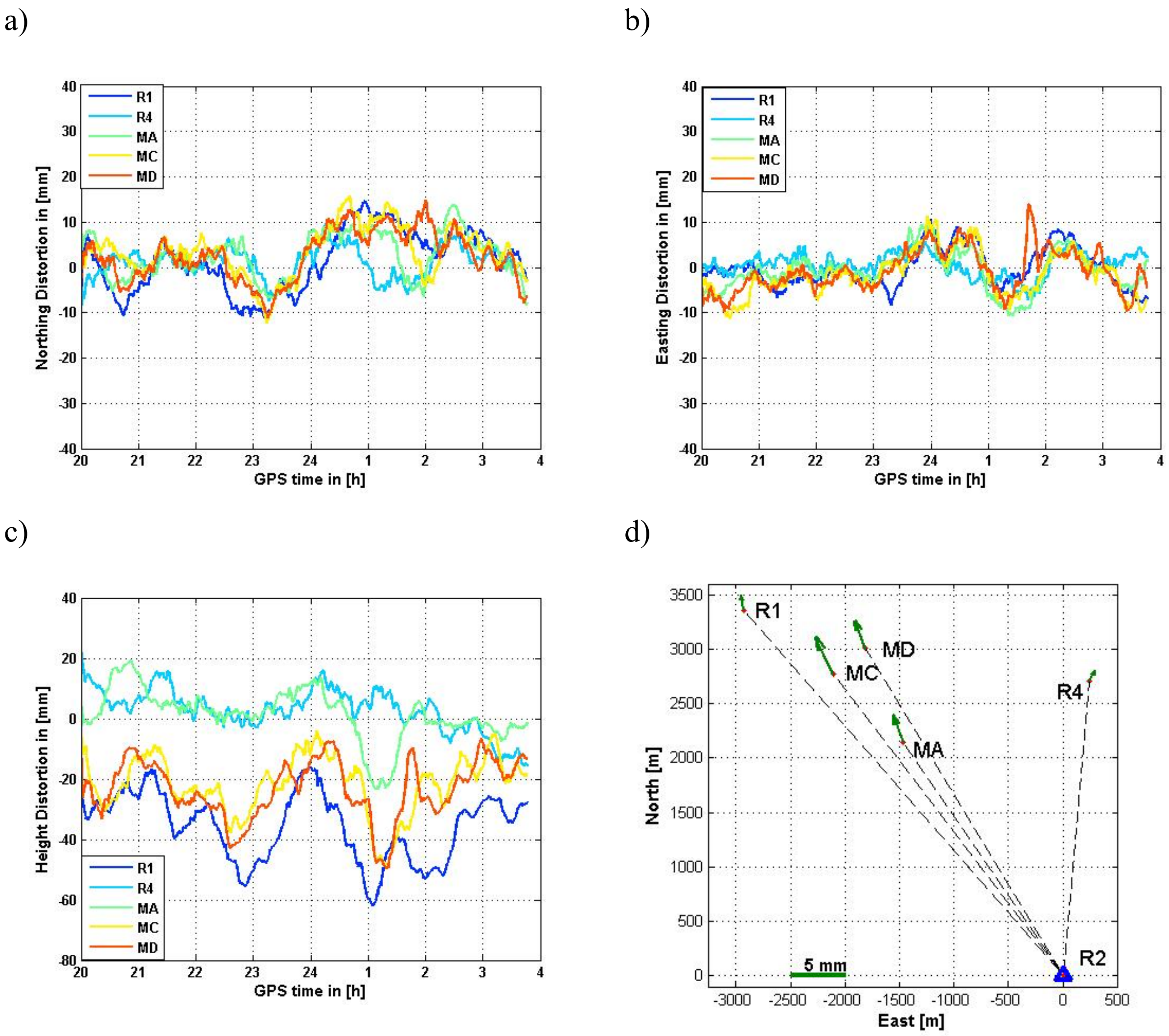

Figure 2: Variations of the coordinate time series with respect to the BENESE 48-h solution

In Figure 3 the corrected height time series obtained after application of both correction models are compared; in Figure 3a the results of the eight parameter model are given, in

Figure 3b those of the two parameter model. Both models are capable to reduce the main part of the bias. In Table 2 and 3 the numerical results in terms of improvement of the median value as well as of the rms are given. For both models the systematic bias is reduced about 75%, i.e., for the stations MC and MD from 24 mm to 5 mm. The coordinates of the station MA remains unchanged since it is located in the same height as the reference station R2, cf. Eqs. (4) and (9).

a) b)

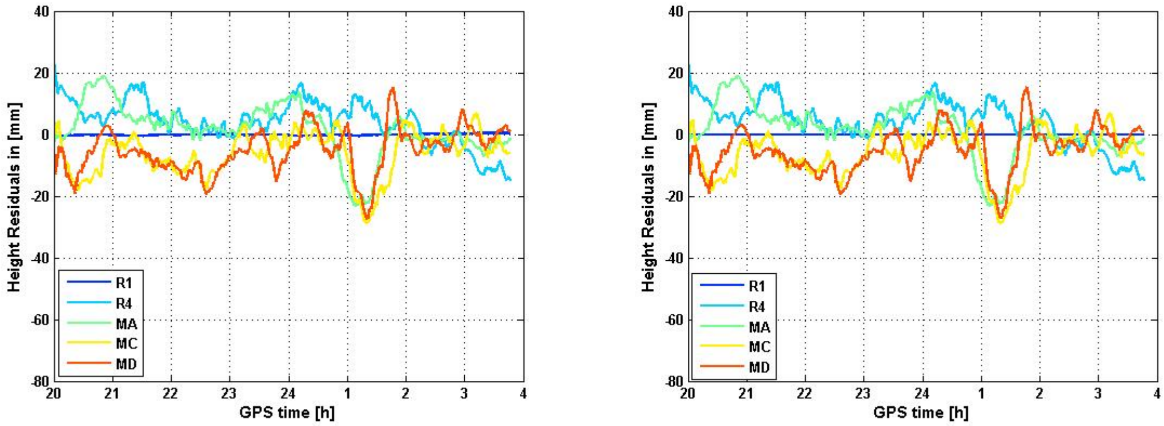

Figure 3: Corrected coordinate time series for the height component: using a) the eight parameter model and b) the two parameter model

The results for the height component of both models are quasi identical. Considering the special design of the Gradenbach network (the stations MA, MC, MD, and R1 are quasi aligned in direction of the slope) these results are expected and reflect just the predominant influence of the network design. The station R4 is located quasi in the same height as the reference stations. Consequently it has hardly any impact. In fact for this special case, the results by Eq. (4) and Eq. (10) are identical for the height component. With $H_{R4} \approx 0$, the corrected height by the eight parameter model

$$d\hat{H}_i' \approx d\hat{H}_i - \frac{H_i\, H_{R1}}{H_{R1}^2} d\hat{H}_{R1} = d\hat{H}_i - \frac{H_i}{H_{R1}} d\hat{H}_{R1} \tag{11}$$

is identical with that one obtained by the two parameter model.

In Figure 4 the results for the horizontal coordinate components corrected by the eight parameter model are given. Regarding the numerical values given in Table 2, the improvement of the median of the time series is about 50% in North for MA and MD, and 28 % for MC. For the east component the bias is reduced by 53% for MA, 36% for MC and 23% for MD. Not only the median of the time series is improved, but also their scatter measured by the rms. Here the reduction is better than 23% for all coordinate components.

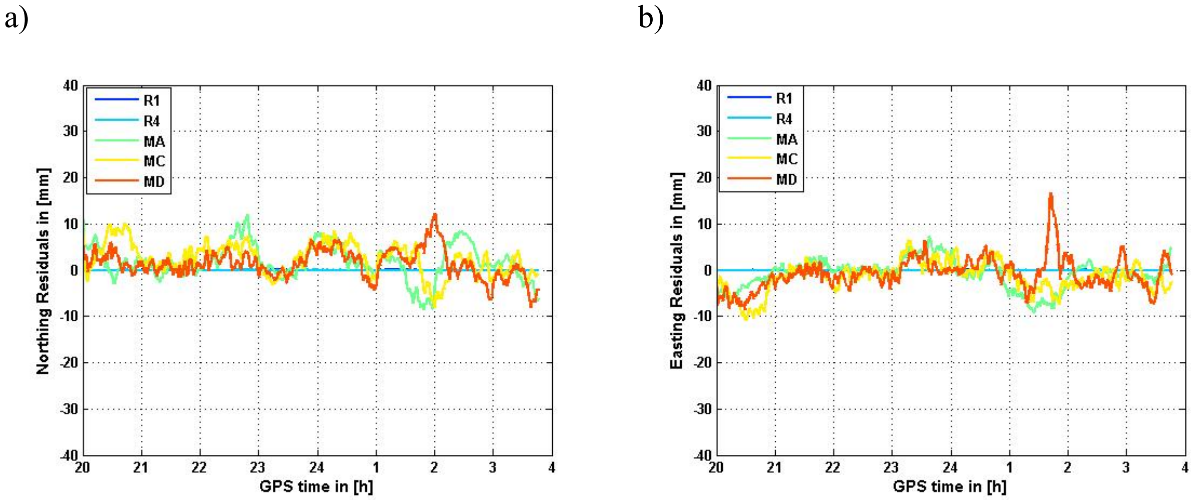

Figure 4: Corrected coordinate time series: of the north (a) and east (b) component obtained by the eight parameter model

	RMS a priori [mm]			RMS a posteriori [mm]			Improvement in %		
	N	E	H	N	E	H	N	E	H
MA	5.4	4.3	9.5	4.0	3.2	8.4	25.5	25.4	0.0
MC	6.3	4.9	23.8	4.0	3.5	9.6	35.7	29.7	59.7
MD	5.9	4.6	25.4	3.3	3.5	9.0	44.5	23.6	64.3
	Median a priori [mm]			Median a posteriori [mm]			Improvement in %		
	N	E	H	N	E	H	N	E	H
MA	2.5	-1.0	1.6	1.2	-0.4	1.7	50.5	53.1	4.1
MC	3.6	-1.9	-20.7	2.6	-1.3	-5.0	27.9	36.5	75.5
MD	2.7	-1.2	-23.9	1.2	-0.9	-5.2	53.5	23.3	78.0

Table 2: Comparison of the numerical results before correction (a priori) and after correction (a posteriori) by the eight parameter correction model

	Median a posteriori [mm]	Improvement in %	RMS a posteriori [mm]	Improvement in %
	H	H	H	H
MA	1.7	4.1	8.4	0.0
MC	-5.0	75.6	9.5	59.9
MD	-5.1	78.6	8.9	64.6

Table 3: Comparison of the numerical results before correction (a priori) and after correction (a posteriori) by the eight parameter correction model

Discussion. Both models are suitable and in their performance equivalent for the correction of the main part of the tropospherically induced height distortions. For the application of the correction models “good” GPS stations (the reference and calibration stations) are needed. Besides their stability (they should be not affected by the movement to be monitored), their position with respect of the vertical network extension is important. In fact, it is preferable that the heights of the calibration station and the reference station are upper and lower bounds to the heights of all monitoring stations in order to avoid extrapolation in Eqs. (4) or (9). Furthermore, an interpolation by Eqs. (4) or (9) reduces the transfer of possible station specific effects of the calibration station, like multipath. In addition the required location of the reference and the calibration station assures that the extreme meteorological conditions are covered by these stations. Taking these considerations into account, the major part of the tropospherically induced height distortion can be reduced. Note that the model (4) can was also successfully applied in the observation domain, cf. [10].

The horizontal coordinate components are less affected by the remaining systematic effects; the bias is one magnitude smaller than the vertical one. Therefore, a trade-off must be made between the additional improvements obtained by the eight parameter model and the necessity and costs of the installation and maintenance of a third “good” GPS station in stable terrain. For many GPS control networks, the baseline direction is the critical direction in which actual point movements are supposed. Therefore, for high precision application this additional logistic and economic burden should be accepted.

4. Conclusion and Outlook

In this paper two correction models in the coordinate domain were compared for the reduction of distance dependent systematic effects in GPS phase observations. Using the example of the landslide Gradenbach it was shown that both models are suitable to reduce most of the predominant, tropospherically induced height bias. With both models up to 80% of the biases can be eliminated.

In addition, the eight parameter model reduces the systematic horizontal network distortions that influence the coordinates mainly in baseline direction of about 35%. This is of special interest for high precision applications, if the critical direction to be monitored coincidence with the baseline directions

The proposed correction models are very flexible. They can be applied for kinematic as well as for static processing. Furthermore, they are software independent and directly applicable to the resulting coordinates processed by any scientific or commercial GPS software.

Further studies should be made on the impact of physical correlation between GPS observables on the bias pattern as well as the impact of station specific effects like multipath.

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