

# The Practice and Results Analysis for Well Subsidence Monitoring Based on the Similar Single-difference Methodology

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## ABSTRACT:

In order to monitor the deformation, caused by mining coal under ground, of the main and auxiliary wells in the industry square, the monitoring network, composed of 46 monitoring points and 2 base points, is established. The datum session measuring of the monitoring network is adopted GPS static relative positioning method and 3 degree levelling, and the measuring results is used as Subsidence monitoring datum. The observation method of monitoring session is the same as datum session, but the subsidence values is computed by GQuickS software developed on the Similar Single-difference Methodology, abbreviated to SSDM, and compared with that of levelling. Analysis results show that adopting SSDM to subsidence monitoring can up to the requirement of 3 order levelling monitoring.

## 1. INTRODUCTION

When the industrial square of a mine was abandoned in 2008, the main and auxiliary shafts were filled above -42.8m level, and an isolation platform was installed at -250m level and -140m level in the main and auxiliary shafts respectively. According to the mining plans, the protective coal pillar of the industrial square will be mined gradually. In order to mining the protective coal pillar of the industrial square safely, the subsidence monitoring system should be setup nearby the main and auxiliary shafts in the industry square. By monitoring periodically, the laws of surface deformation and shaft subsidence can be controlled and given technical basis for mining the protective coal pillar of the industrial square safely.

Presently, GPS is widely used to monitor all kinds of buildings and constructions' deformations, such as monitoring the well in coal mine, the earth's surface in city, landslides, bridges, high buildings, reservoir dams and so on. According to the deformation features of the monitored objects, there are three different monitoring modes, namely periodical repeating surveying, fixed continuous GPS station array, and the real time dynamic monitoring. The first two modes are suitable for the slow deformation, and commonly the static relative positioning method is used to process the data. The third mode usually fits rapid deformation or slow deformation with sudden change, and the OTF method is mainly adopted in its data processing. When the monitoring surroundings is worse, the GPS signals are generally interrupted, which brings some difficulties of applying OTF method. When using single epoch algorithm, the problem may be solved easily. Many national and international scholars have done more researches on ambiguity resolution at single epoch [1-5], and applied these methods to solve deformable values at single epoch.

Presently, GPS deformation monitoring technique is developing in the direction of high precise, rapid and real time. In order to

fit these developing trends, according to the characteristics of high precise GPS deformation monitoring, and using the first period results as the initial condition, and using the carrier phases in monitoring session as the basic observations, a rapid resolution model at single epoch for high precise GPS deformation monitoring is established by the authors. This model can be used in the conditions, such as few common satellites (e.g. two satellites).

In this paper, the layout of the shaft monitoring networks and the data processing methods for the datum session are introduced firstly, and then, comparing the subsidence obtained by levelling and SSDM, the feasibility of applying SSDM for monitoring subsidence is verified, and some conclusions are derived.

## 2. ESTABLISHING THE SHAFT MONITORING NETWORK

### 2.1 Layout of the shaft monitoring network

Referring Fig.1, The shaft monitoring network is layout according to the following scheme.

1) The shaft monitoring network is composed of 46 monitoring points and 2 datum points, named JZ01 and JZ02. The datum points are located outside the mining affected areas, and the distance between shaft is less than 3 km.

2) The observation pillars of monitoring points possess double-layer marks, the low-layer mark is used for monitoring small subsidence, and the high-layer mark for monitoring large subsidence and horizontal deformation. The height of the observation pillar is 1.5m.

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3) In order to monitor the different directions displacement and deformation, taking the line connecting main shaft center and auxiliary shaft center as the axes line, monitoring points are setup along the eight directions in local coordinate system, namely north, northeast, east, southeast, south, southwest, west, northwest.

4) Respectively, the main and auxiliary well monitoring points are located on four rings centered the shafts' centers, and the distance between rings, from inside to outside, is 15m and 20m.

5) monitoring points coding rules. In favor of field observation, indoor data processing, and analyzing the displacement and deformation of shafts, the monitoring point's code is

unique. Referring Fig.1, according to the following rules, the monitoring point's code is composed of four strings:

The first string is fixed "M" to indicate the monitoring point. The second one signifies the type of shaft, "Z" for main shaft and "F" for auxiliary shaft. The third string is Arabic number for the location of the monitoring point, "0" for the monitoring point in shaft center, the other number from "1" to "4" for the monitoring points on the different ring. The fourth string is Arabic number from "1" to "8", indicating the monitoring point orientation in local coordinate system. For example, "1" expresses north, "2" expresses northeast, and so on.

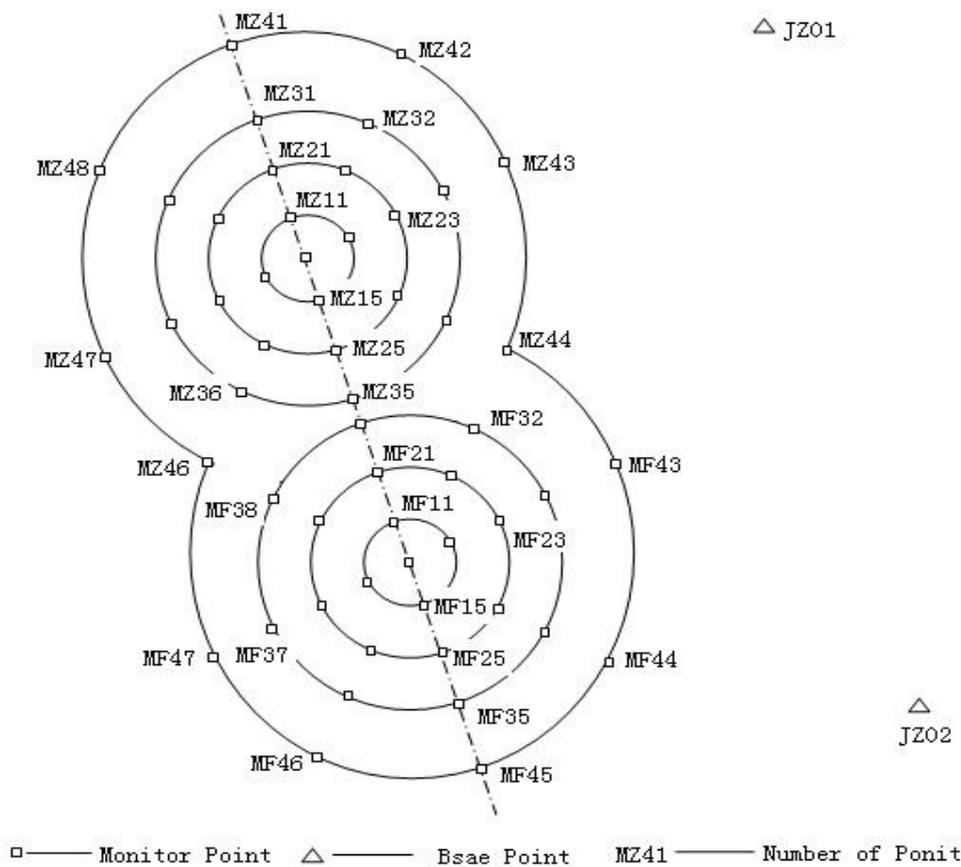


Figure 1. The sketch of the shaft monitoring network

## 2.2 Establishing the deformation monitoring datum

The deformation monitoring datum is composed of plane and height datum, and GPS technology unifies this two datum as a tridimensional datum. Meanwhile, in order to verify the subsidence monitoring accuracy by Similar Single-Difference Methodology, the subsidence monitoring datum is also setup by the third-order levelling. The deformation monitoring datum is established in December, 2010.

Observing the datum session by GPS static relative positioning technology, eight dual frequency receivers, are adopted and 9 sessions are observed. The session length is about 2 hours long, and the sample ration is 15 seconds. Two receivers are fixed on the 2 datum points, named JZ01 and JZ02, and the other 6 receivers are set up on the monitoring point respectively.

Observed the datum session, the baseline vectors are solved by GAMIT precision baseline resolution software and precise ephemeris, and then the GPS datum network is adjusted by GMDPS 2.5 software to obtain the precise deformation analysis datum in WGS-84 system, and this datum is also the foundation of solving the monitoring point's deformation by SSDM.

As for the height monitoring datum setup by the third-order levelling, the measurement is started from mining area bench mark by Lecia DNA03 digital level, and the level network is also adjusted by GMDPS 2.5 software to obtain the precise subsidence monitoring datum.

### 3. BRIEF INTRODUCTION OF SSDM

#### 3.1 The basic mathematical model

Referring Fig.2, in one monitor network, during the course of the monitoring session's observation, and compared with the datum session (the first period observation), the base point  $p_1$  is immovable and the monitoring point  $p_2$  has displaced. The deformed position of  $p_2$  is  $p_3$ , and the deformable value is  $\mathbf{d}$ . The deformable value  $\mathbf{d}$  will be obtained by using the monitoring session's observations.

At the epoch  $t$ , the corrected carrier phase observations for the satellite  $i$  on the base point  $p_1$  and the deformed monitoring point  $p_3$  can be expressed as the following formula

$$\begin{aligned} \bar{\rho}_{pq}^i &= \lambda \varphi_{pq}^i + \lambda N_{pq}^i - c \delta t_{pq} + c \delta t^i \\ &+ h_{pq} \sin \theta_{pq}^i + \rho_{pq}^i \times \dot{\rho}_{pq}^i / c + \dot{\rho}_{pq}^i \delta t_{pq} \\ &- \Delta_{pq,atmo}^i - \Delta_{pq,mult}^i - \Delta_{pq,phase}^i - \Delta_{pq}^i \quad (1) \end{aligned}$$

If  $q = 1$ , then Eq.(1) stands for the observations on the base point  $p_1$ , else if  $q = 3$ , then it stands for the observations on the deformed position of  $p_2$ . In Eq.(1),  $\lambda$  is the wavelength of carrier wave,  $\varphi$  is the carrier phase observation,  $N$  is the initial integer cycles,  $c$  the light velocity,  $\delta t_p$  and  $\delta t^i$  are the clock offsets of receiver clock and satellite clock,  $\rho$  is the geometrical distance between satellite and station,  $\rho$  dot is the distance varying ratio.  $\Delta_{atmo}$  is the atmosphere delay correction,  $\Delta_{phase}$  is the correction of the receiver antenna phase center,  $\Delta_{mult}$  is the correction of multipath effect,  $\Delta_p$  are other corrections such as relativity effect, earth rotation, and so on,  $h$  is the height of the antenna,  $\theta$  is the altitude angle from station to satellite.  $h \cdot \sin \theta$  is the correction that the distance between satellite and the antenna phase center is corrected to the distance between satellite and the station center. After all of the corrections have been computed or some influence have been eliminated, only the initial integer cycles  $N$  is unknown in Eq.(1). For the sake of the conciseness, the terms relative to the corrections in Eq.(1) are ignored.

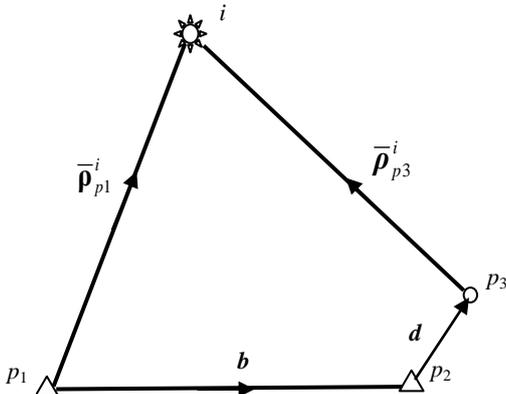


Figure 2. The sketch of the principle

In the spatial quadrangle composed of  $p_1$ ,  $p_2$ ,  $p_3$  and satellite  $i$ , the deformable value  $\mathbf{d}$  of the monitoring point  $p_2$  can be written as

$$\mathbf{d} = \bar{\rho}_{p1}^i - \bar{\rho}_{p3}^i - \mathbf{b} \quad (2)$$

where,  $\mathbf{b}$  is the known baseline vector between  $p_1$  and  $p_2$ , which has been obtained through the first period observation.  $\bar{\rho}$  is the calculated carrier phase observation from the Eq.(1). In order to obtain the deformable value of the monitoring point  $p_2$ , projecting the Eq.(2) to the three coordinate axis directions  $X$ ,  $Y$ ,  $Z$ , and taking the deformation  $dX$  of  $p_2$  in  $X$  axis direction as example, the following formula can be gained,

$$dX = \bar{\rho}_{xp1}^i - \bar{\rho}_{xp3}^i - X_{p1,p2} \quad (3)$$

Substituting Eq.(1) into (3), and considering the direction cosine  $l^i$ , the following formula can be obtained [6-9]

$$\begin{aligned} dX &= -\lambda l_{p1}^i N_{p1,p3}^1 + \{l_{p1}^i (\lambda \varphi_{p1}^i - c \delta t_{p1} + c \delta t^i) \\ &- l_{p3}^i (\lambda \varphi_{p3}^i - c \delta t_{p3} + c \delta t^i) \\ &+ (l_{p1}^i h_{p1} \sin \theta_{p1}^i - l_{p3}^i h_{p3} \sin \theta_{p3}^i) \\ &+ (l_{p1}^i \rho_{p1}^i \times \dot{\rho}_{p1}^i - l_{p3}^i \rho_{p3}^i \times \dot{\rho}_{p3}^i) / c \\ &+ (l_{p1}^i \dot{\rho}_{p1}^i \delta t_{p1} - l_{p3}^i \dot{\rho}_{p3}^i \delta t_{p3}) - X_{p1,p2} \\ &- l_{p1}^i \lambda N_{p1,p3}^{1,i} - l_{p1,p3}^i \lambda N_{p3}^i + (\Delta_{p1,p3,atmo}^i \\ &+ \Delta_{p1,p3,mult}^i + \Delta_{p1,p3,phase}^i + \Delta_{p1,p3}^i) \} \quad (4) \end{aligned}$$

In Eq.(4),  $N_{p1,p3}^1$  is the reference satellite single-difference ambiguity,  $N_{p3}^i$  is the non-reference satellite initial integer cycle unknown on the monitoring point,  $N_{p1,p3}^{1,i}$  is the double-difference ambiguities,  $X_{p1,p2}$  is the known precise baseline vector between the base point  $p_1$  and the monitoring point  $p_2$ , which has been obtained through the first period observation. The Eq.(4) is just the mathematical model of obtaining the GPS monitoring point deformable information. This model is similar to the single-difference model of carrier phase observations, the difference lies in considering the direction cosine. So, this mathematical model is called similar single-difference model, and the corresponding algorithm is called the similar single-difference methodology. This model or methodology will be called SSDM methodology briefly in this paper.

AS for the other two deformable components of the monitor point  $dY$  and  $dZ$ , are similar to Eq.(5), but only the direction cosine is different. So, from Eq.(5), there are only four unknowns for the monitoring point, viz. the three deformable components,  $dX, dY, dZ$  and the reference satellite single-difference ambiguity  $N_{p1,p3}^1$ . These four unknowns have no relationship with the simultaneous satellites.

The main error sources effecting the accuracy of deformation information solved by SSDM are discussed in References [6-13], in order to save space, these will not be introduced in this paper.

#### 3.2 The basic algorithm

At epoch  $t$ , the error equations can be formed from Eq.(4)

$$v_{X,i} = dX + \lambda_{p1}^i N_{p1,p3}^1 - \{\bullet\} \quad (5)$$

where, the  $\{\cdot\}$  is the parts of the  $\{\}$  in the Eq.(4). The initial values of the deformation and SD ambiguity are  $dX^0$  and  $N_{p1,p3}^{1,0}$  respectively, and their corrections are  $\delta X$  and  $\delta N_{p1,p3}^1$ , then the Eq.(5) can be rewritten as

$$v_{X,i} = \delta X + \lambda_{p1}^i \delta N_{p1,p3}^1 - [\{\bullet\} - dX^0 - \lambda_{p1}^i N_{p1,p3}^{1,0}] \quad (6)$$

Then

$$v_{X,i} = \delta X + \lambda_{p1}^i \delta N_{p1,p3}^1 - L_{X,i} \quad (7)$$

the forms of the Y and Z component of the deformation is similar to Eq.(7), then

$$\left. \begin{aligned} v_{Y,i} &= \delta Y + \lambda_{p1}^i \delta N_{p1,p3}^1 - L_{Y,i} \\ v_{Z,i} &= \delta Z + \lambda_{p1}^i \delta N_{p1,p3}^1 - L_{Z,i} \end{aligned} \right\} \quad (8)$$

In Eq.(7) and Eq.(8),  $l, m$  and  $n$  are the direction cosine from station to satellite, and  $L$  is the constant term.

At  $t$  epoch, if  $s$  satellites are observed synchronously, then  $s$  error equations similar to Eq.(7) and Eq.(8) can be formed, and the whole error equations can be written as

$$\mathbf{V} = \mathbf{A} \mathbf{X} - \mathbf{L}, \text{ weight } \mathbf{P} \quad (9)$$

$3s,1$     $3s,4$   $4,1$     $3s,1$     $3s,3s$

where

$$\mathbf{X} = [\delta X \quad \delta Y \quad \delta Z \quad \delta N_{p1,p3}^1]^T \quad (10)$$

According to the Least-Squares estimation theory, the deformations of the monitoring points and SD ambiguity can be obtained from the Eq.(10). In order to ensure the deformations accuracy, it is necessary to compute two times iteratively. If  $k$  epochs have been observed, then the deformation of monitoring point  $p_2$  is the average of all epochs, and its accuracy can be evaluated.

#### 4. SUBSIDENCE MONITORING ANALYSIS FOR SHAFT

##### 4.1 The Scheme of monitoring and data processing

As for the main and auxiliary shafts monitoring network in Fig.1, after established the deformation monitoring datum, in order to test the feasibility of using SSDM to monitoring subsidence in this test, GPS static relative positioning technology and the third-order levelling are used in monitoring session. But for GPS surveying, the session length is only 0.5h.

When using SSDM method to calculate the subsidence, after prepared the spatial rectangular coordinates of base points and monitoring points, RINEX format ephemeris files and observation files, the monitoring points' deformation ( $dX$ 、

$dY$ 、 $dZ$ ) can be solved by GQuickS software developed by SSDM theory, and then the deformation ( $dX$ 、 $dY$ 、 $dZ$ ) were transformed to the deformation ( $dN$ 、 $dE$ 、 $dU$ ) in topocentric horizon rectangular coordinate system, and  $dU$  is the monitoring point subsidence. The monitoring point's theoretical subsidence,  $dh$ , was obtained by third-order levelling.

The accuracy of the monitoring point subsidence solved by SSDM method can be evaluated by the interior coincident accuracy and exterior check accuracy. If  $k$  epochs have been observed in one session, then the interior coincident accuracy of subsidence is

$$m_1 = \pm \sqrt{\frac{\sum_{j=1}^k (dU_j - d\bar{U})^2}{k-1}} \quad (11)$$

where,

$$d\bar{U} = \frac{\sum_{j=1}^k dU_j}{k}$$

the exterior check accuracy of subsidence is

$$m_2 = \pm \sqrt{\frac{\sum_{j=1}^k (dU_j - dh)^2}{k-1}} \quad (10)$$

Where,  $dh$  is the monitoring point's theoretical subsidence obtained by third-order levelling.

##### 4.2 The Numerical results and analyses

For the sake of saving space, for the main and auxiliary shafts monitoring network in Fig.1, partial points' subsidence and accuracy, solved by SSDM method and third-order levelling, are listed in tab.1. In tab.1, " $\Delta$ " means the difference between the subsidence obtained by these two methods.

It can be seen from Tab.1,

1) The ground surface nearby the main and auxiliary shafts occurred displacement and deformation, especially for the main shaft.

2) Comparing the average subsidence solved by GQuickS software with the subsidence obtained by third-order levelling, the deformation trend is identical, the maximum difference is 7.2mm, the minimum difference is 1.6mm, and the RMSE is  $\pm 4.2$ mm. As for the interior coincident accuracy and exterior check accuracy of subsidence series solved by GQuickS software, they are consistent basically. They vary from  $\pm 4$ mm to  $\pm 10$ mm, which shows the subsidence series solved by GQuickS software having good stability and Reliability.

Point Number	$dU$	$dh$	$\Delta$	$m_1$	$m_2$
MZ14	-37.8	-40.9	+3.1	5.5	4.6
MZ21	-41.7	-45.1	+3.4	7.6	6.5

MZ31	-55.9	-52.0	-3.9	6.3	5.3
MZ33	-27.2	-22.2	-5.0	4.5	5.1
MZ35	-35.5	-32.9	-2.6	6.3	6.8
MZ41	-36.8	-51.1	+4.3	5.7	6.5
MZ47	-140.2	-146.6	+6.4	8.2	7.4
MZ48	-182.8	-175.6	-7.2	6.8	9.1
MF22	-4.8	-3.2	-1.6	4.7	6.5
MF26	-7.5	-10.1	+2.6	3.8	5.3
MF35	-8.9	-5.3	-3.6	5.4	5.5
MF36	-11.3	-6.7	-4.6	6.2	4.2
MF46	-7.7	-4.3	-3.4	4.8	5.4

Table 1. Comparison of Subsidence/mm

3) According to the specification of coal mine survey, if there is no Special Requirements, monitoring the ground subsidence caused by mining, third-order and fourth-order levelling are adopted in comprehensive observation sessions and daily observation sessions respectively. According to the law of error propagation, the MSE of monitored subsidence is not more than  $\pm 10\text{mm}$  and  $\pm 20\text{mm}$  respectively. It can be seen from the test data, taking the subsidence obtained by levelling as theoretical values, the average subsidence solved by GQuickS software can meet the third-order levelling requirements basically, and meet the fourth-order levelling requirements totally.

## 5. SUMMARY

According to the characteristics of high precise GPS deformation monitoring, the Similar Single-Difference Model (SSDM) of solving the deformation information at single epoch is built. Since adopting the single epoch algorithm, then the troublesome problem of detecting and repairing cycle slips is avoided. The SSDM can be applied to dynamic and static state monitoring in theory. Since the first period baseline vectors are used as preconditions, the double difference ambiguity resolution becomes easier. Based on the first period observation results of the GPS monitoring network, SSDM Methodology solves monitoring points' deformable values from carrier phases directly. This method can obtain monitoring points' deformable values, and doesn't need to solve baseline vectors and network adjustment. Thus, the data processing speed is improved.

2) It can be seen from SSDM Methodology mathematical model, referenced the Eq.(4), the unknown parameters, which will be solved in one epoch, are constant of 4 and have no relation to the numbers of satellites. The 4 parameters are the reference satellite single-difference ambiguity and the monitor point's three deformed components,  $dX, dY, dZ$ . Since three equations can be established using one satellite, so, theoretically speaking, if the synchronous satellite numbers between stations are not less than two, the monitoring point's deformation information can be solved by GQuickS software. As for other softwares, the four synchronous satellites are prerequisite to solve baseline vectors. Thus, when the observation environment is bad, such as in urban areas or Three Gorge, this characteristics of SSDM Methodology will play key roles.

It can be seen from the test data, the subsidence solved by GQuickS software can meet the third-order levelling requirements basically, and meet the fourth-order levelling requirements totally. This results are on the premise of the forced centring equipment.

With the development of Continuous Operating Reference Station (CORS), computer technology, network technique and Communication Technology, GQuickS software developed upon SSDM methodology can be applied in the following monitoring data processing: reservoir dam, mountain landslide, high building, large bridge, port, dock, embankment, the earth's surface in city, subsidence in mine area, etc.

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