Highly accurate distance measurement with a frequency comb laser

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Abstract. We introduce a scheme for highly accurate absolute distance measurement based on interferometry with a laser that emits many wavelengths simultaneously. This approach allows for distance determination that is absolute, i.e. without the need to generate an uninterrupted, continuous displacement from an initial start position. As such, it enables for large remote distance ranging, also for targets that are not practically accessible for near proximity measurements or referencing, e.g. due to terrain obstacles. A femtosecond frequency comb laser is a very suitable source for this purpose, because it emits thousands of stable wavelengths simultaneously, providing a wealth of information. As a consequence, the range of non-ambiguity is on the level of tens of centimeters which can easily be measured with an electronic distance meter. This is a significant advantage compared to single wavelength interferometry, which requires a precision linear guidance and uninterrupted beam path. Using the frequency comb as a source, the output of a Michelson interferometer is measured with a high resolution-spectrometer, which is able to spectrally resolve the individual wavelengths on a camera. From the measured interference patterns, acquired with a fast single shot image, a distance is determined. We have compared the measurement results that were obtained with the frequency comb to the measured displacement of a counting laser interferometer for distances up to 50 m. An agreement between both methods on the sub-micrometer level was found ($< 10^{-8}$ at 50 m).

With the prospect of frequency-comb lasers becoming smaller, cheaper and easier to operate, this measurement scheme opens up the possibility for field applications. These may not only be found in the field of surveying or measurement of large structures, but also in space applications, like distance measurement between satellites.

Keywords. EMRP JRP SIB60 Surveying, Laser ranging, Remote sensing, Absolute distance measurement, Frequency combs

Introduction

Already since 1983 the meter has formally been related to the second in the SI systems of units, by defining the meter as the distance that is travelled in vacuum in 1/*c* second. Here *c* is the speed of light in vacuum, which is fixed at $c = 299\ 792\ 458$ m/s. Although this definition can directly be applied for measuring very long distances, simply by measuring the time that a pulse of light needs to travel to reach a certain target (e.g. the moon), it is not very practical to measure short distances with such a 'time of flight' method. Alternatively, interferometry has proven a very powerful and accurate method for measuring distances or displacements. Here the wavelength of laser light serves as a 'ruler' for measuring the path-length difference of an interferometer. The distance is expressed in terms of an integer number of wavelengths plus a wavelength fraction. To trace back this measurement to the definition of the meter, the wavelength of the laser needs to be known. With the speed of light being fixed, the wavelength λ is related to the optical frequency via the simple relationship $\lambda = c / f$, with *f* the optical frequency. To obtain traceability of length measurement to the SI second, the optical frequency f thus needs to be measured with respect to a time standard, e.g. an atomic clock. With the invention of the optical frequency comb at the beginning of this century the measurement of optical frequencies has been simplified tremendously. Being the national metrology institute of the Netherlands, VSL operates such a frequency comb on a regular basis for optical frequency measurements.

In this article, however, we will focus on another application of the frequency comb, which is distance measurement with the comb directly. We exploit the frequency comb as a source that emits thousands of wavelengths simultaneously and perform interferometry with all these wavelengths in parallel. The strength of this method is that it enables the measurement of absolute distances with high accuracy, without the need to generate a displacement. This is a large advantage compared to single-wavelength interferometry, which is usually based on fringe counting and requires a linear guidance and an uninterrupted beam path during the measurement. To access the information that is available from the thousands of interfering wavelengths, we have developed a high resolution spectrometer that is able to separate the closely spaced wavelengths. By analyzing the output of an interferometer with the spectrometer, a distance can be derived [1]. Recently we have applied this method to the measurement of distances up to 50 m [2]. This paper is largely based on [2].

Femtosecond frequency comb

An optical frequency comb laser is a pulsed laser that emits ultrashort (femtosecond) pulses with a repetition frequency of typically 100 MHz to 1 GHz. The optical spectrum of such a laser contains a large number of frequencies, which are equally spaced (see Fig. 1). This is where the name ‘frequency comb’ comes from.

The frequency difference between neighboring laser frequencies is equal to the repetition frequency of the laser. At VSL we operate a frequency comb that is based on a titanium-sapphire laser, emitting pulses at a 1 GHz repetition frequency. A photograph of the VSL frequency comb laser is shown in Fig.2.

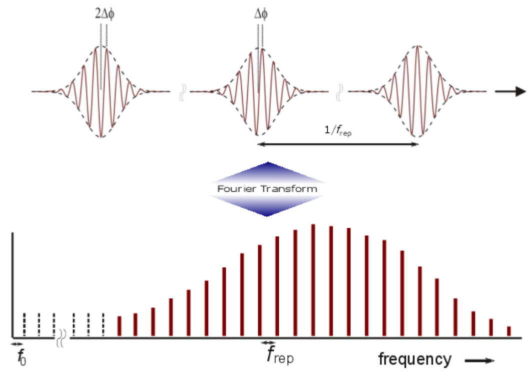

Fig. 1 Visualization of the pulse train emitted by a femtosecond laser and its corresponding spectrum of optical frequencies. In reality the spectrum contains ten thousands of optical frequencies (wavelengths).

The optical spectrum of the Ti:Sapphire laser ranges roughly from 810-830 nm and consists of about 9000 individual wavelengths. The repetition frequency of the laser is stabilized with respect to the atomic clock, which fixes the mutual spacing between the comb frequencies. A single frequency in the comb can be described as $f_p = f_0 + p\, f_{rep}$, with p a large integer number (10^5-10^6) and f_0 an offset frequency. Both f_{rep} and f_0 are stabilized on the level of 10^{-11} in 1 second averaging time, which means that all emitted wavelengths are stabilized at that level as well.

Fig.2 The VSL frequency comb in operation. The colored fibers broaden the spectrum that is emitted by the Ti:Sapphire laser over the full visible spectrum, which is used for optical frequency calibration. For distance measurement the original Ti:Sapphire spectrum is used, ranging from 810-830 nm.

High resolution spectrometer

In order to fully exploit the large number of wavelengths that are emitted from the frequency comb laser for distance measurement, it is required to spectrally separate them. For this purpose we developed a high-resolution spectrometer, based on a 'virtually imaged phase array' (VIPA) and a grating, which is inspired on VIPA applications in telecommunications and high-resolution spectroscopy [3,4]. A VIPA is an etalon with high-reflectivity coatings, which generates an angular dispersion in the vertical plane. The free spectral range of the etalon is 50 GHz, which means that optical frequencies with a frequency difference of 50 GHz are emitted from the VIPA at the same angle. Therefore a grating is introduced to spectrally separate these frequencies in the horizontal plane. Subsequently, the light is imaged onto a camera with a lens, revealing the individual laser modes of the frequency comb as dots. The VIPA spectrometer setup is illustrated in Fig.3. The full comb spectrum is reconstructed by stitching the vertical lines together. In order to do this correctly, a few reference markers are generated on the camera by sending the beam of a tunable single mode laser along the same path, while simultaneously measuring the wavelength with an independent wavemeter. The accuracy of the wavemeter is within 100 MHz, which is accurate enough to unambiguously determine the absolute wavelength of each dot.

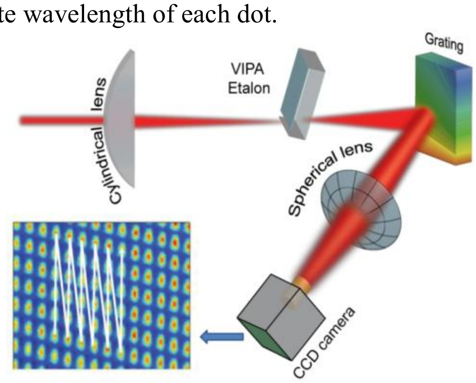

Fig. 3. High-resolution spectrometer based on a VIPA and a grating for unraveling the frequency comb spectrum. The white arrow indicates how the full comb spectrum is reconstructed by stitching vertical lines together. Here only a small part of the camera image is shown for clarity, in reality a vertical line contains about 50 unique dots.

Massively parallel interferometry

For the distance measurement we have constructed a Michelson-interferometer with a measurement arm that can be changed over a distance of 50 m by moving a carriage along a straight linear guidance. The displacement is not only measured with the frequency comb, but also with a conventional counting laser interferometer for comparison. The setup is shown in Fig.4.

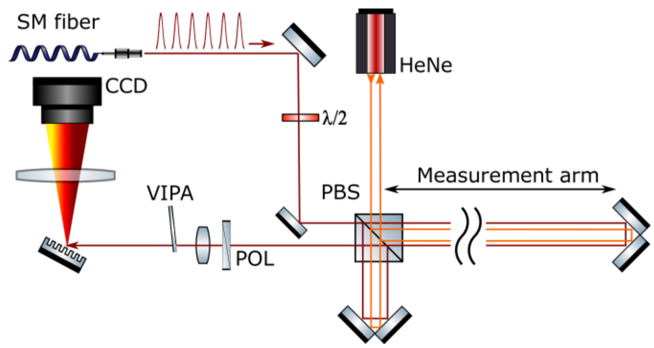

Fig. 4 Schematic overview of the measurement setup for comparing distance measurement up to 50 m with a counting helium-neon laser and a frequency comb. The comb light is delivered to the setup with a single mode (SM) fiber. The HeNe laser (orange line) and comb laser (red line) measure the displacement quasi-simultaneously. PBS: polarizing beam splitter, POL: polarizer, λ/2: half-waveplate, CCD: charge-coupled device camera, M: planar mirror, RR: hollow retro reflector, CL: cylindrical lens, SL: spherical lens

The output of the Michelson interferometer is spectrally resolved with the VIPA spectrometer. As shown in Fig.5, simultaneous interference at many wavelengths occurs. We have measured such interference patterns at various distances ranging from 0 to 50 m. On the right hand side the reconstructed spectra are shown. From the measured phase change as a function of wavelength a distance can be derived, as we will discuss below.

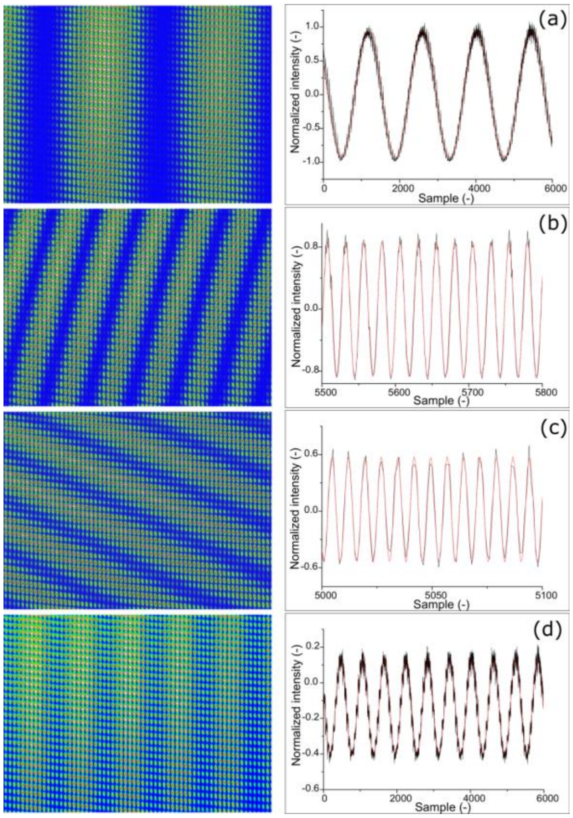

Fig. 5 Interferometry with thousands of wavelengths simultaneously for distance of ≈ 0 m (a), 5 mm (b), 20 m (c) and 50 m (d), respectively. Spectral interferometry images are shown on the left. For clarity only 1/4 of the CCD chip area has been selected. On the right side the reconstructed spectra are shown. The X-axis with the sample number, has been scaled differently for these graphs to clearly visualize the fringes.

Spectral interferometry

Measuring the wavelength dependent interference is called spectral interferometry. From the phase change as a function of wavelength a distance is derived, which has already demonstrated in various configurations [5,6]. The interference term can be written as:

$$I(\lambda) = I_0 \cos\left(\frac{2\pi \cdot 2L \cdot n}{\lambda}\right), \qquad (1)$$

with I_0 the intensity of the light sent into the interferometer, L, the path length difference of the interferometer arms (single path), λ the vacuum wavelength and n the refractive index of the medium, usually air. The total accumulated phase can be written as:

$$\phi = \frac{4\pi L n}{\lambda} = \frac{4\pi L n f}{c}, \qquad (2)$$

with f the optical frequency and c the speed of light in vacuum. In Fig. 6 we show simulated spectra for a delay of 5 and 6 mm respectively, to illustrate the interference and phase change as a function of wavelength.

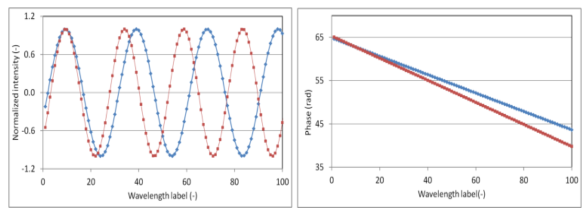

Fig. 6 Simulated spectral interferometry for a distance *L* equal to 5 mm (blue) and 6 mm (red) respectively. The left graphs shows the simulated intensity, the right graph the phase. The absolute phase ($4\pi L/\lambda$) has been offset by an arbitrary multiple of 2π for both distances. For clarity only 100 wavelengths are shown. Note that at zero delay the absolute phase would equal zero for all wavelengths.

The phase change as a function of optical frequency can be written as:

$$\frac{\mathrm{d}\phi}{\mathrm{d}f} = \frac{4\pi L}{c}\left[n + f\frac{\mathrm{d}n}{\mathrm{d}f}\right] = \frac{4\pi L}{c} n_g, \qquad (3)$$

With n_g the group velocity refractive index of air. This leads to the following expression for the distance L:

$$L = \frac{\mathrm{d}\phi}{\mathrm{d}f}\frac{c}{4\pi n_g}. \qquad (4)$$

In practise we determine the phase change as a function of frequency by fitting a cosine through the measurement data.

Due to the periodicity of the pulse train, the interference patterns are repeating themselves: pulse-overlap occurs when the total path length difference of the interferometer equals a multiple of the pulse-to-pulse distance L_{pp}, with $L_{pp}= c/(f_{rep}\, n_g)$. An arbitrary distance L_t is then written as:

$$L_t = \frac{1}{2} m L_{pp} + L \qquad (5)$$

To determine L_t, the integer m needs to be known. Therefore it is necessary to know the distance to be measured with an accuracy better than $L_{pp}/2$. This is a relaxed requirement, with $L_{pp}/2 \approx 30$ cm for our system, which can easily be fulfilled by measuring the approximate distance with a simple electronic distance meter, time of flight measurement or even a measurement tape.

Homodyne interferometry

So far, the absolute wavelength of each dot has not yet been used, but we know the phase of each wavelength from its relative position on the cosine fit. This information can be used to determine the distance for a particular wavelength λ_p via

$$L_p(\lambda) = \left(q_p + \frac{\phi_{p,fit}}{2\pi} \right) \frac{\lambda_p}{2n_p}, \qquad (6)$$

with q_p an integer number, and n_p, the (phase) refractive index of λ_p. The phase $\phi_{p,fit}$ is obtained from the cosine fit. Note that $q_p >> m$, since $\lambda_p << L_{pp}$. The measurement uncertainty resulting from Eq. 6 is expected to be smaller than the measurement uncertainty resulting from Eq. 5, since the uncertainty on the phase is multiplied by λ_p instead of L_{pp}. For a practical measurement other contributions to the measurement uncertainty arise, like wavelength stability and, dominantly in our case, the stability of the interferometer itself.

Measurement results and discussion

Based on the measurement data presented in Fig.5 and the analysis described above, we have determined the distance for measurements up to 50 m. The results are summarized in Fig.7, showing the difference between the counting laser interferometer and the comb-meassurement. An agreement between both methods is found within 1 μm over the full length of the measurement bench for each individual measurement and < 500 nm when averaged over 5 measurements. At a distance of 50 m this is a relative agreement within 10^{-8}.

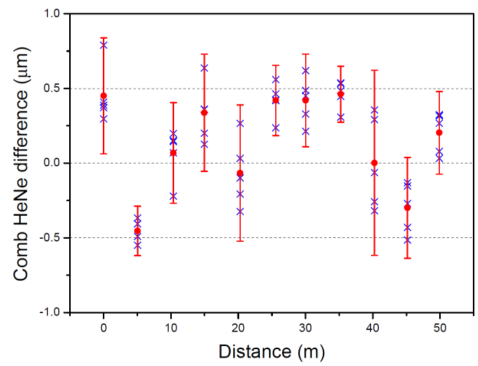

Fig.7 Observed differences between the frequency comb distance measurement, based on spectral interferometry and the counting laser interferometer. The error bars indicate twice the standard deviation over the 5 measurements.

Remarkably, the agreement between the measurements with HeNe and frequency comb and the corresponding standard deviation are independent from the distance that is measured. This is explained by environmental vibrations that are coupled to the moving carriage of the 50 m measurement bench, dominating the measurement uncertainty, independent from the position of the carriage on the guidance.

We have analyzed the measurement data to derive a distance based on homodyne interferometry as well, following Eq. 6. Although the measurement results based on homodyne interferometry are expected to be more accurate than those based on spectral interferometry, the results are very similar for these measurements. Both methods agree with each other within 100 nm for all distances. The expected superior performance of homodyne interferometry is hidden by the mentioned environmental vibrations that dominate the measurement uncertainty of this comparison. The total estimated measurement uncertainty for a coverage factor k=2 (corresponding to a 95% coverage interval) is 0.75 μm for spectral interferometry, against 0.69 μm for homodyne interferometry. This is in good agreement with the observations in Fig. 7.

Conclusion

We have shown that a frequency comb can be exploited as a powerful source for absolute distance measurement. Due to the wealth of information available from the thousands of wavelengths present in the comb, the range of non-ambiguity is huge compared to single-wavelength interferometry. This allows for absolute distance measurement with high accuracy, without the need to generate a displacement. With the prospect of frequency-comb lasers becoming smaller, cheaper and easier to operate, the presented method may find a wide range of applications, e.g. surveying applications or distance measurement in space. Based on the uncertainty of the phase measurement, an accuracy on the level of a few nm may be feasible in vacuum and a vibration-free environment. The measurement range and best achievable accuracy will ultimately be limited by the coherence length of the light and thus the accuracy of the reference clock.

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