

## The fourth precise levelling campaign of Poland in 1999 – 2003

Adam Łyszczolowicz  
 University of Warsaw and Mieczysław Cichoń, Poland  
 adam@mosti.uw.edu.pl  
 Marcin Leptizycki  
 Head Office of Geodesy and Cartography, Warsaw, Poland  
[marcin.leptizycki@pki.gov.pl](mailto:marcin.leptizycki@pki.gov.pl)

## Outline

- Introduction
- The fourth precise levelling campaign
- Accuracy estimation
  - by Lallemand's formulas
  - Vignal's formulas
  - After adjustment
- Statistical distribution of discrepancies
- Variance analysis
  - Lines
  - Observers
  - Instruments
- Summary and conclusion

5th FIG Regional Conference, Accra, Ghana, 0 - 11 March, 2006

2

## Introduction

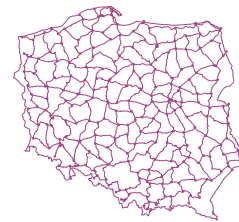
- Spirit levelling seemed to be one of the most accurate techniques in height difference determination,
- However, errors originating from instruments, ambient circumstances and observer, have such character that it is very difficult to remove them from observations,
- Also assessment of leveling accuracy is not an easy task,
- In 1912 at the Hamburg meeting of IAG Lallemand proposed hypothesis that levelling was affected by the two kind of errors:
  - random errors  $\propto \sqrt{l}$
  - Systematic errors  $\propto l$
- In 1936 Vignal, proposed a different classification of the levelling errors
- In 1955 Wassef, have proposed the application of mathematical statistics to study levelling error in levelling networks.

5th FIG Regional Conference, Accra, Ghana, 0 - 11 March, 2006

3

## The fourth levelling campaign

- Number of lines: 382
- Total length of lines: 17 516 km
- Average length of line: 46 km
- Number of sections: 16 132
- Average length of section: 1.1 km
- Number of nodal points: 245
- Number of loops: 135
- Instruments:
  - Zeiss Ni 002, Zeiss DiNi 11,
  - Topcon NJ
- 23 observers



5th FIG Regional Conference, Accra, Ghana, 0 - 11 March, 2006

4

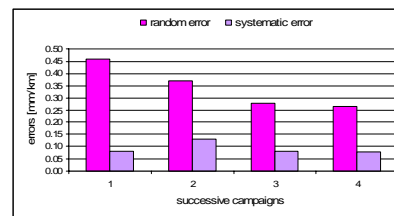
## Accuracy estimation

- Accuracy can be estimated from :
  - sections discrepancies  $\Delta$ ,
  - lines discrepancies  $S$ ,
  - loop discrepancies  $\phi$

5th FIG Regional Conference, Accra, Ghana, 0 - 11 March, 2006

5

## Accuracy estimation by Lallemand's formulas

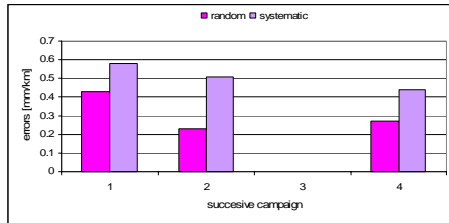


- Fourth campaign:  $\eta = \pm 0.27 \text{ mm/km}^{-1/2}$ ,  $s = 0.08 \text{ mm/km}$ ,  $\sigma = \pm 0.28$

5th FIG Regional Conference, Accra, Ghana, 0 - 11 March, 2006

6

### Accuracy estimation by Vignal's formulas

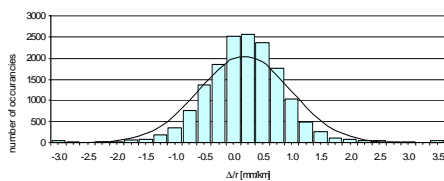


- Fourth campaign:  $\eta = \pm 0.27 \text{ mm/km}^{-1/2}$ ,  $s = 0.44 \text{ mm/km}$ ,  $\sigma = \pm 0.52 \text{ mm/km}$

### Accuracy estimation from adjustment

- Empirical variance factor:
  - $s_0 = \pm 0.90 \text{ mm/km}$

### Statistical distribution of discrepancies (1)



- Histogram of section discrepancies  $\Delta/r$ 
  - Number of discrepancies: **16 132**
  - Mean value: **+0.07**
  - Std dev:  **$\pm 0.78$**
  - Skewness: **-0.78**
  - Kurtosis: **79.8**

### Statistical distribution of discrepancies (2)

- Line discrepancies S/L
  - Number of discrepancies: 379
  - Mean value: **+0.07**
  - Std dev:  **$\pm 0.16$**
  - Skewness: **0.11**
  - Kurtosis: **2.85**

### Statistical distribution of discrepancies (2)

- Loop discrepancies  $\phi/L$ 
  - Number of discrepancies: 133
  - Mean value: **+0.00**
  - Std dev:  **$\pm 0.06$**
  - Skewness: **0.12**
  - Kurtosis: **1.4**

### Variance analysis (1)

- Mathematical background:
  - $\Delta_i$  variables is taken from the general population which has normal distribution,
  - variables  $\Delta_i$  are classified in r groups due to one factor,
  - hypothesis: mean values computed for each group are equal.

## Variance analysis (2)

---

- variable  $\Delta$  are classified in respect ;
  - Lines:
    - hypothesis - **rejected**
  - Observers:
    - hypothesis - **rejected**
  - Instruments:
    - hypothesis - **accepted**

## Summary and conclusions

---

- Lallemand's formula
  - In successive levelling campaigns random errors **decrease**,
  - While the systematic errors stay **almost** the same,
- Vignala's formulas
  - Random error almost the same like from Lallemand's formulas,
  - Systematic error almost four time bigger that from Lallemand's formulas
- Discrepancies  $\Delta$  do not show any **outliers**, while numerous discrepancies  $\Delta/r$  apparently have **outstanding** values,
- Section discrepancies  $\Delta$  are **significantly correlated** with the length of section  $r$ ,
- line discrepancies  $S$  are **less correlated** with the line length  $L$ ,
- while loop misclosures are **almost independent** from the loop length.
- Variance analysis **confirm** existence of systematic errors in levelling observations

---

**Thank you for your attention**

---