Optimum Weight in Thin Plate Spline for Digital Surface Model Generation

SINA TAGHVAKISH
and
JALAL AMINI
Department of Geomatics Engineering, University of Tehran
Iran

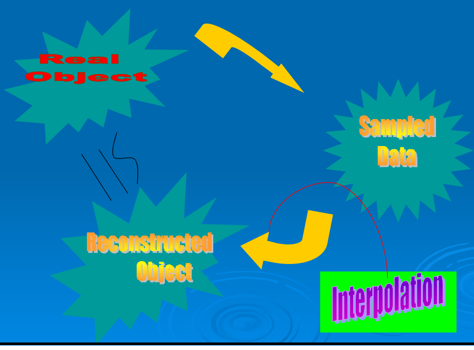
1

Agenda

- Interpolation methods
- Advantages of TPS
- Definition of Thin Plate Spline (TPS)
- Mathematical Model
- Determination of weights
- Experimental results
- Conclusion

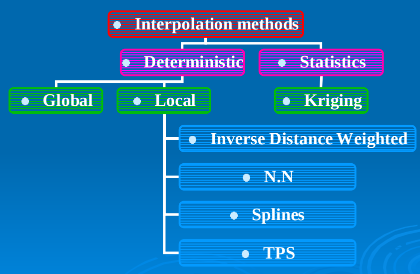
2

Interpolation importance



3

Interpolation methods



4

Advantages of TPS

- Physically based
- Smoothness behavior of TPS in void areas
- No extrapolation
- Straight solution can never be singular
- Fitness to all of the Data

5

Definition of TPS

- A physically based 2D interpolation
  - which represents a thin metal sheet that is constrained not to move at the grid points,
  - free from any external forces relied upon control points,
  - from this sight the bending energy in control points should have been minimized.

6

THE MATHEMATICAL MODEL

$$E_{TPS}=\sum_{i=1}^{n}\mu_i(f(x_i,y_i)-z_i)^2+\iint_{\Re^2} f_{xx}^2+2f_{xy}^2+f_{yy}^2$$

$$\nabla^4 f+\sum_{m=1}^{M}\lambda_m\delta(x-x_m)=0$$

$$\sum_i a_i\phi(r_i(x_j+y_j))+b_0+b_1x_j+b_2y_j=z_j;(j=1,...,n)$$

$$\sum_j a_i=0;\sum_j a_ix_i=0;\sum_j a_iy_i=0;\ \phi(x,x_i)=\phi(r_i)=r_i^2\log(r_i)$$

$$\frac{8\pi a_j}{\mu_j}+\sum_i a_i\phi(r_i(x_j+y_j))+b_0+b_1x_j+b_2y_j=z_j;(j=1,...,n)$$

TPS Straight solution

$$\begin{pmatrix} 8\pi/\mu_1 & c_{21} & \cdots & c_{n1} & 1 & x_1 & y_1 \\ c_{21} & 8\pi/\mu_2 & \cdots & c_{n2} & 1 & x_2 & y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ c_{1n} & c_{2n} & \cdots & 8\pi/\mu_n & 1 & x_n & y_n \\ 1 & 1 & \cdots & 1 & 0 & 0 & 0 \\ x_1 & x_2 & \cdots & x_n & 0 & 0 & 0 \\ y_1 & y_2 & \cdots & y_n & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ b_0 \\ b_1 \\ b_2 \end{pmatrix}=\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \\ 0 \\ 0 \\ 0 \end{pmatrix};$$

$$c_{ij}=\phi(r_i(x_j,y_j))$$

$$a_{1,2,...,n},b_{0,1,2}$$

Determination of weights

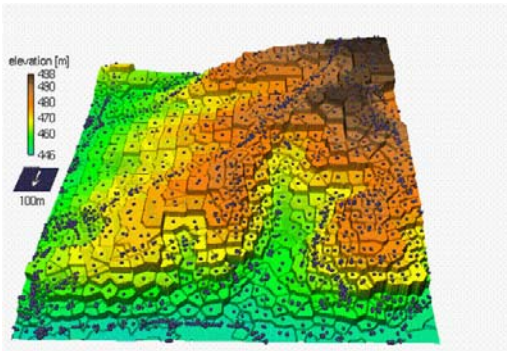
$$E_P=P\sum_{i=1}^{n}r_i(f(x_i,y_i)-z_i)^2+\iint_{\Re^2} f_{xx}^2+2f_{xy}^2+f_{yy}^2$$

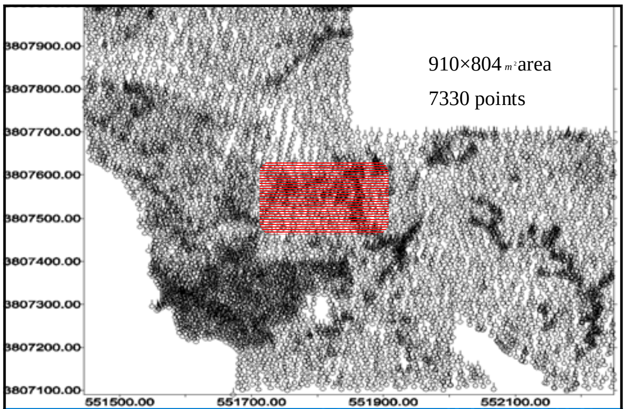
$$P=\sum_i\mu_i,\quad r_i=\frac{\mu_i}{P}$$

a. Normalized Voronoi with fixed P equal to one, μ in edge polygons are assumed to be zero
b. Normalized Voronoi with fixed P equal to one, μ in edge polygons area assumed to be one
c. Normalized Voronoi with fixed weight P=100
d. Normalized Voronoi with fixed weight P=1000
e. Normalized Voronoi with fixed weight P=10000
f. Normalized Voronoi with fixed weight P=100000
g. Voronoi
h. Fixed weight equal to one

$$\mu_i=P\times\frac{A_i}{\sum_{j=1}^{n}A_j};i=1,2,3,...,n$$

Voronoi Polygons





A view of scattering of original data in 2D

Weight Comparison (To be continued)

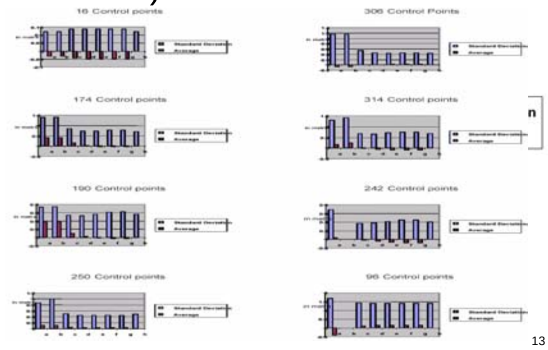
Table 2: Average of datasets shown in figure 4

Method → Data Location	GCP no.	Norm. Voronoi (1 at edge poly.)	Norm. Voronoi (0 at edge poly.)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
5*5 section 11	16.000	-0.032	-0.031	-0.045	-0.048	-0.049	-0.049	-0.049	-0.043
5*5 section 12	174.000	0.411	0.410	0.140	0.033	-0.034	-0.051	-0.051	-0.020
5*5 section 13	190.000	0.385	0.385	0.092	0.032	-0.009	-0.023	-0.025	0.001
5*5 section 21	250.000	0.123	0.124	0.056	0.040	0.040	0.041	0.041	0.035
5*5 section 22	552.000	0.140	0.142	0.088	0.042	0.021	0.008	0.006	0.016
5*5 section 23	306.000	-0.069	-0.068	0.003	0.005	0.017	0.019	0.019	0.012
5*5 section 33	314.000	0.177	0.239	-0.009	-0.054	-0.082	-0.093	-0.094	0.010
5*5 section 43	242.000	0.040		-0.022	-0.044	-0.068	-0.079	-0.080	-0.061
5*5 section 44	96.000	-0.390		0.112	0.133	0.139	0.143	0.143	0.117
From Pnt. 1to1500	1000.000	0.156		0.049	0.029	0.005	-0.006	-0.010	-0.013

Table 1: Standard deviation of datasets shown in figure 4

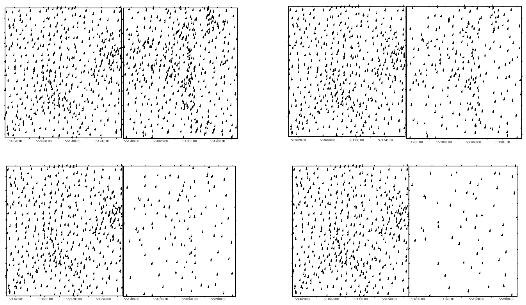
Method → Data Location	GCP no.	Norm. Voronoi (1 at edge poly.)	Norm. Voronoi (0 at edge poly.)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
5*5 section 11	16	0.122	0.122	0.139	0.138	0.138	0.138	0.138	0.122
5*5 section 12	174	1.414	1.415	0.861	0.717	0.740	0.779	0.783	0.710
5*5 section 13	190	0.724	0.736	0.542	0.521	0.568	0.614	0.622	0.564
5*5 section 21	250	0.872	0.992	0.505	0.453	0.454	0.458	0.457	0.483
5*5 section 22	552	1.257	1.258	0.748	0.660	0.683	0.718	0.723	0.663
5*5 section 23	306	1.152	1.152	0.547	0.452	0.437	0.458	0.460	0.432
5*5 section 33	314	1.301	1.403	0.682	0.641	0.710	0.747	0.749	0.682
5*5 section 43	242	0.692		0.373	0.382	0.417	0.444	0.447	0.398
5*5 section 44	96	1.693		1.429	1.410	1.416	1.420	1.420	1.404
From Pnt. 1to1500	1000	1.401		0.867	0.654	0.536	0.507	0.510	0.504

Weight Comparison (To be continued)



13

Sensitivity of TPS relative to control points density



14

Analyzing density effects

Table 4: Average calculated for analysing density effects

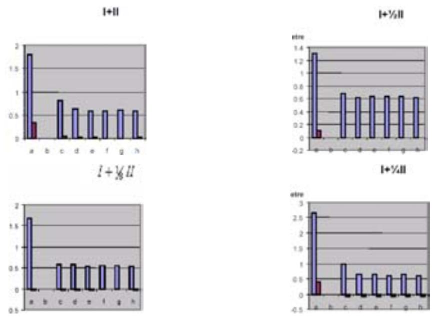
Method / Data Location	Norm. Voronoi (1at edge poly)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
I+II	0.342	0.046	0.021	0.017	0.010	0.003	0.011
I+½II	0.110	-0.014	-0.013	-0.001	-0.001	-0.001	-0.009
I+¼II	0.412	-0.084	-0.090	-0.090	-0.081	-0.077	-0.082
$I+\frac{1}{8}II$	-0.045	-0.038	-0.038	-0.025	-0.017	-0.017	-0.026

Table 3: Standard deveation calculated for analysing density effects

Method / Data Location	Norm. Voronoi (1at edge poly)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
I+II	1.789	0.816	0.637	0.583	0.595	0.609	0.581
I+½II	1.308	0.685	0.623	0.634	0.634	0.635	0.615
I+¼II	2.638	1.004	0.648	0.648	0.622	0.638	0.620
$I+\frac{1}{8}II$	1.690	0.578	0.578	0.534	0.545	0.554	0.531

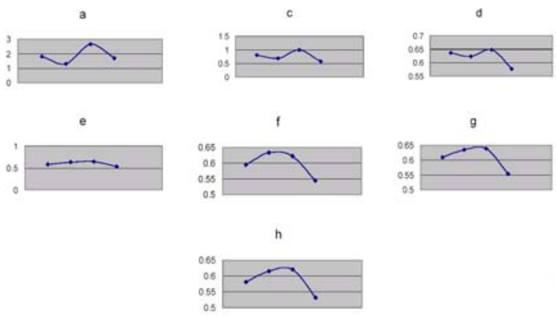
15

Standard deviation and average for density analysing datasets



16

Behavior of every weight definition methods by changing of density



17

Conclusion

- TPS as a density independence method can be used for the construction of the surfaces in the new brand geomatics.
- Using TPS, without using the Voronoi algorithms.
- Because of the density independence mathematical form of straigh method, one can use it in CAD/CAM softwares.
- we recommand the usage of the TPS method for mountainious-flat regions because variation of one zone doesn't affect other zones so much.

18

Where to Get More Information

- Duchon, J. "Interpolation des fonctions de deux variables suivant le principe de la flexion des plaques minces." RAIRO Analyse Numérique 10, 5-12, 1976
- Meinguet, J. "Multivariate Interpolation at Arbitrary Points Made Simple." J. Appl. Math. Phys. 30, 292-304, 1979.
- Billings,Stephen D., Beatsonz, Rick K., and Newsam, Garry N. "Interpolation of geophysical data using continuous global surfaces" GEOPHYSICS, VOL. 67, NO. 6 (NOVEMBER-DECEMBER 2002)
- Bazen, Asker M. and Gerez, Sabih H. "THIN-PLATE SPLINE MODELLING OF ELASTIC DEFORMATIONS IN FINGERPRINTS" 3rd IEEE Benelux Signal Processing Symposium (SPS-2002), Leuven, Belgium, March 21-22, 2002
- Pedersen, Lars "ESTIMATION OF THIN PLATE SPLINE WARP PARAMETERS FROM PROTEIN SPOT POSITIONS IN 2D ELECTROPHORESIS GELS" ,2000
- Gousie, Michael B. "Converting Elevation Contours to a Grid"
- Maillet, Gregoire "DSM reconstruction" Manual of photogrammetry
- http://mathworld.wolfram.com Thin Plate Spline
- Jenkins, David R. "Thin plate spline interpolation on an annulus" ANZIAM J. 42 (E) ppC819{C836, 2000 C819, 7 August 2000
- Boztosun, I.. Chara, A.. M. Zerroukat , and Djidjeli, K. "Thin-Plate Spline Radial Basis Function Scheme for Advection-Diffusion Problems"I. Boztosun et al. / Electronic Journal of Boundary Elements, Vol. BETEQ 2001, No. 2, pp. 267-282 (2002)
- Goncalves,G. Julien P.,Riazanoff,S. Cervelle,B. "Preserving cartographic quality in DTM interpolation from contour lines" ISPRS 56 (2002)210-220
- Sjowall, Aldona. "Short Report on OrthoEngine".Vilnius 2001-04-08
- http://www.bentley.com/products/descartes
- Franke, R., 'Smooth interpolation of scattered data by local thin plate splines', Computing and Mathematics with Applications 8, 273–281. (1982)

19