Ways of Determining the Orthometric Heights Using GPS Technology

Octavian ROMAN, Romania

SUMMARY

The work refers to the possibilities of causes the orthometrics height. There are presented: general notions concerning the utilization GPS for the determination of ellipsoidal height, models of geoid calculation, the utilization of GPS for the determination relative altitudes and finally of orthometrics altitudes. These can be determinated through the many methods, each of them offering the different accuracy for results obtained.

The author suggests to find quick possibilities, much more precise and efficient for the determination of orthometrics height otherwise than through geometrically levelling.

SUMAR

Lucrarea se refera la posibilitatile de determinare a altitudinilor ortometrice. Sunt prezentate: notiuni generale privind utilizarea GPS pentru determinarea cotelor elipsoidale, modele de calcul a geoidului, utilizarea GPS pentru determinarea altitudinilor relative si in ultima instanta a altitudinilor ortometrice. Acestea pot fi determinate prin mai multe metode, fiecare dintre ele oferind precizii diferite pentru rezultatele obtinute.

Autorul isi propune sa gaseasca posibilitati rapide, cat mai precise si cat mai eficiente pentru determinarea altitudinilor ortometrice altfel decat prin nivelment geometric.

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1. SYSTEMS OF ORTHOMETRIC HEIGHTS

Defining a system of heights consists mainly of:

- Choosing a reference surface
- Adopting a definition, with physical or geometrical significance, whereby the position of the points on the surface of the Earth is described as against the reference surface. The level surfaces are not parallel. The basic equation can be written in every space point:

$dW = - g \cdot dn$

whereby we can establish the dependence between **dn** distance and **dW** potential difference existing between the two close infinite level surfaces.
The system of orthometric heights is defined as that system in which the geoid is the reference surface and the orthometric height is the force line segment included between the position of the point on the surface of the Earth and the geoid, respectively.
In most cases geodesists are interested in the orthometric height as being the measured one above a reference surfac4e, identified as geoid. The surface of the geoid is one of a whole family of surfaces or equipotential levels of the gravity field of the Earth. Most geodesic measurements, by virtue of their connection with the local reference plane, are influenced by the gravity field of the Earth. The level surfaces, as their name indicate, are surfaces with constant gravitational potential.

The gravity vector or the direction of the vertical of any point is perpendicular to this level surface, passing through that point.
The orthometric height has a more “physical” meaning than the “geometrical” one of the ellipsoidal height. The orthometric height was traditionally determined, by levelling technique whereby height increases were obtained by intersecting the sight line of a levelling instrument tangentially on the level surface, on two graded levelling staff as it is illustrated in fig.1. Knowing the orthometric height is necessary for accurate engineering operations such as dams, pipes, tunnels, which operate with fluids and their flow.

During the last century it was admitted that the average surface of the oceans was a good approximation of the level surface of the gravity field of the Earth and this surface was chosen as the reference surface for heights. The geoid became a concept very used in practical determination of the orthometric heights and wording such as “heights above the average sea level” or “heights above the geoid” are considered equivalent in the context of most measuring applications.

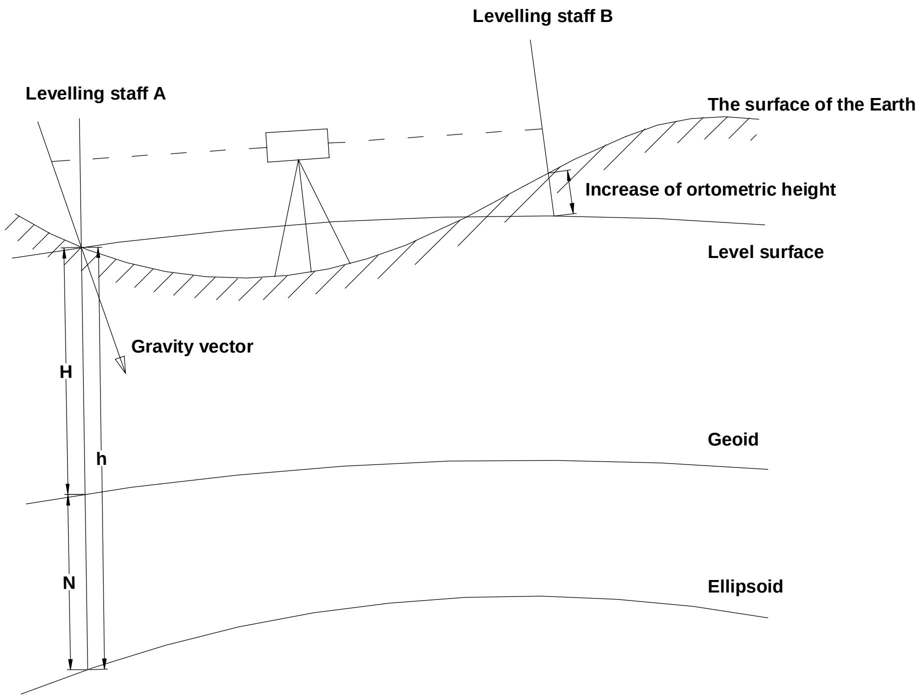

Fig. 1. Geoid, Ellipsoid, The surface of the Earth and Levelling

It seems that not only the traditional geodesic measuring and photogrammetry methods are decisively influenced by GPS possibilities but also physical geodesy within the determination of orthometric heights, which is closely connected – as it is known - to the definition of the geoid as a reference surface.

Software models and several examples of this problem are briefly presented below.
Regarding the “height” the following question can be asked: ”Which way does water flow?” This question can be answered only by means of **ΔW** gravitational potential difference

$$\Delta W = W_j - W_i = -\int_{P_i}^{P_j} g \bullet dn = -\sum_{P_i}^{P_j} g \bullet dn \qquad (1.1)$$

which results from the combination of the traditional levelling (**dn** level difference) and the relative gravity measurement (**g** gravity).

The so-called differences of gravitational potential are named (**C**) geopotential heights. To get a better determination of the height, the geopotential heights from gravimetric space are fixed in the geometrical space, namely by dividing to the determined value of the gravity:

$$\bar{H} = \frac{C}{\bar{g}} \qquad (1.2)$$

$\bar{g}$ is chosen as average of the gravity values along Pi – Pj force line (in the sense of a generalized balanced average)

$$\bar{g} = \frac{1}{H} \bullet \int_{o}^{H} g \bullet dH \qquad (1.3)$$

where: $\bar{H} = H_{orth}$ is the orthometric height.

Standard gravity **Y** has been used in more countries, instead of real gravity **g**, that leads to a definition of the standard-orthometric or Helmert height.

On the contrary, GPS supplies accurate ellipsoidal altimetric differences $\mathbf{\Delta h = h_j - h_i}$, in the field 0,1 – 1 ppm.

The difference between ellipsoidal height **h** (the size measured by GPS) and orthometric height **H** (the required size) is the height of the geoid or the undulation of the geoid **N**, simply given by:

$$H = h - N \qquad (1.4)$$

and in its differential form:

$$H_j - H_i = (h_j - h_i) - (N_j - N_i) \qquad (1.5)$$

now playing a main role in the determination of the relative orthometric height.

Now the physical geodesy has received with this a challenge to determine the height difference of geoid $\mathbf{N_j - N_i}$ with the equivalent accuracy of GPS. The application of relation (1.4) is above all not possible because the absolute height of the geoid can not be calculated due to the low accuracy of the global model of the Earth with an accuracy higher than 0,5 m. The orthometric height does not strictly refer to the geoid because:

- standard gravity was used in relation (1.3) (standard orthometric correction)
- various water level measuring devices were not used
- the heights were calculated in the most simple way, without taking into consideration the orthometric corrections.

Thus the orthometric heights refer to zero level (***NN***), a reference surface close to the geoid. The deviations between the real geoid and zero level (***NN***) should be within 0,4 m limit.

2. WAYS OF DETERMINING THE ORTHOMETRIC HEIGHTS BY USING GPS TECHNIQUES

2.1 Procedure of Determination the Orthometrics Altitudes through GPS Ignoring the Effect Non-Linear of Geoid Surface

The fact as the GPS system measures coordinates or relative coordinates in the three-dimensional space presents numerous advantages. Reduction the ellipsoid altitudes coming directly through the GPS system in orthometrics height, concerning of waviness definition isn't always possible. We can solve this problem creating un surface model of waviness named "the geoid map " that request time and middles. Besides, for obtain the secure quotas of sea level it should be this map sufficient dense and it should be take into account during interpolation by the non-linear effect of geoid surface.

The idea is to apply a model of transformation the plans of reference for defining of orthometrics altitudes. This method requires at least three common points from the two systems with heights ellipsoid, but a better solution shall be obtain with supplementary points. It doesn't request the absolute values of waviness and either the fixed longitudes and latitudes, because we shall deal with the differential values of waviness.

2.1.1. Turning an Ellipsoid into a Geoid

Turning an ellipsoid into a geoid requires minimum three common points with heights in both surfaces, but as it was mentioned before, more common points with heights in both surfaces are desired. We must specify that the geometry of the points and the locating of the known points are very important. Let us suppose that the GPS project was achieved and more than three points of the GPS network have orthometric values.
The procedure is to be accomplished as follows:

- The calculation of the vectors among all the points (it is usually achieved by the standard software that is connected at the receivers)
- The calculation of the grid by a minimum constrained compensation
- Turning the datum of the source system into the target system. In the source system we will find the common points originating in the GPS net, each point described by latitude, longitude and ellipsoidal height. In the target system each point will be described by the same two components from the source system (e.g. the same latitude and longitude) but the third component will be the orthometric height.

The parameters, which are to be obtained from each transformation, will turn the other points of GPS net having only ellipsoidal heights into orthometric heights. Each procedure was checked and examined in statistical reports from the compensation of the surfaces with minimum constraint and procedures of transforming the datum.

2.1.2. Mathematical Model Approach

We use the mathematical model to transform the elements, based on turning the seven parameters source data into target data. We used a model that decreases the number of unknown quantities to four, giving "pseudo-values" to the points relative to the geometrical center of the configuration of the known points. Then we eliminated the solution necessity from the three translations.
These are the equations defining the whole transformation:

$$X_1 = x_1 + dX_1 + x_1 \cdot d\lambda + 0 + x_3 \cdot a_2 - x_2 \cdot a_3$$
$$X_2 = x_2 + dX_2 + x_2 \cdot d\lambda - x_3 \cdot a_1 + 0 + x_1 \cdot a_3$$
$$X_3 = x_3 + dX_3 + x_3 \cdot d\lambda + x_2 \cdot a_1 - x_1 \cdot a_2 + 0$$

Where:

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}$$ - the geocentric coordinates of the point in the target system

$$\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$ - the geocentric coordinates of the point in the source system

$$\begin{vmatrix} a_1 \\ a_2 \\ a_3 \end{vmatrix}$$ - the angles of rotation

$$\begin{vmatrix} dX_1 \\ dX_2 \\ dX_3 \end{vmatrix}$$ - translations

$d\lambda$ - scale factor

The above equations calculate the "pseudo-values" ($\Delta X_i, \Delta x_i$):

$$\Delta X_i = X_i - SX_i$$
$$\Delta x_i = x_i - sx_i \qquad (i = 1,2,3).$$

Where:
SX - the geometric center of X system
sx - the geometric center of x system

After applying this procedure we can use the above equations which define only four unknown quantities (3 rotations and the scale factor):

$$\Delta X_1 - \Delta x_1 = \Delta x_1 \cdot d\lambda + 0 + \Delta x_3 \cdot a_2 - \Delta x_2 \cdot a_3$$
$$\Delta X_2 - \Delta x_2 = \Delta x_2 \cdot d\lambda + \Delta x_3 \cdot a_1 + 0 + \Delta x_1 \cdot a_3$$
$$\Delta X_3 - \Delta x_3 = \Delta x_3 \cdot d\lambda + \Delta x_2 \cdot a_1 - \Delta x_1 \cdot a_2 + 0$$

We think that by using this procedure the solution for the restrained areas is more real and less sensible regarding the mathematical model.

2.1.3. A Brief Discussion about the Method.

We find a fault of this procedure from the conceptual point of view: ignoring the non-linear effect of the geoid surface. This procedure assumes a transformation of heights accomplished from the flat surface of the source (ellipsoid) on the flat surface of the target (smooth geoid). The idea existing in this assumption is not entirely correct because the geoid is a physical surface that is not smooth and it can not be described by a pure mathematical model (unlike the ellipsoid). The non-linear terms of the geoid surface can be introduced into the calculation by adding a rectification of the arrangement model or only by a polynomial model. We have to specify that such procedures require more points or “a map of the geoid” that is not always available. For many technical reasons where the size of the area is restricted and topography is not rough the addition of the non-linear effect is not necessary. In fact we can say that such procedures are unrealistic in some cases because they can create non-flatness of the geoid surface, while in reality , in certain areas the geoid is close to a smooth surface. We have to specify that in restricted areas where such projects can be completed it is desirable to use the procedure that decreases the number of unknown quantities from 7 to 4. Such a model is better suitable for small areas.

2.2. One-Dimensional Transformation with Known Height Points

We assume that Cartesian coordinates X and their orthometric heights are known for the common points. Now from these data we want to determine the transformation parameters which could allow passing from the ellipsoidal heights generated by GPS measurements to orthometric heights for the new points. To solve this, we suggest the following script:

– Corrections $\mathbf{X_o,Y_o,Z_o}$ will be applied to GPS coordinates (translation between the origins of the coordinate system), which have to be known with an accuracy of scores of meters. As a rule values are obtained from the national geodesic funds. In case of great distance nets non-application of such corrections can lead to some deformations of the net;
– The ellipsoidal coordinates $\mathbf{(B,L,h)_{GPS}}$ will be calculated from the corrected Cartesian coordinates with the following relations:

$$\tan B = \frac{Z}{\sqrt{X^2+Y^2}} \bullet (1 - e^2 \bullet \frac{N}{N+h})^{-1}$$
$$\tan L = \frac{Y}{X} \tag{2.1}$$
$$h = \frac{\sqrt{X^2+Y^2}}{\cos B} - N$$

– Now we will have ellipsoidal heights **h** and orthometric heights **H** for the common points. With these we can determine the three parameters of a one-dimensional transformation. The starting relation will be (Hofmann - Wellenhof 1994)

$$H_i = h_i + \Delta h - y_i \cdot d\alpha_1 + x_i \cdot d\alpha_2 \tag{2.2}$$

where the introduction of a proper scale factor was given up. Height translations $\mathbf{\Delta h}$ and the two small angles of rotation $\mathbf{d\alpha_1}$ and $\mathbf{d\alpha_2}$ - representing rotations around the coordinate axes of the system in which the planimetric position of the levelling points are defined - appear here as unknown quantities. The plane coordinates of the known points of heights - which can be interpolated on a map or can be determined from GPS coordinates - were noted with x_i and y_i. From geometrical point of view the height translation **Δh** can be regarded as the negative value of the undulation of the geoid in the origin of the coordinate system and the rotation angles $\mathbf{d\alpha_1}$ and $\mathbf{d\alpha_2}$ as tipping angles around the coordinate axes. Each item offers an equation similar to (2.2), thus at least 3 common points being necessary for solving the problem. If n > 3 the system becomes inconsistent and corrections have to be introduced; solving can be accomplished by compensation calculations.

$$H_i = h_i + \Delta h - y_i \cdot d\alpha_1 + x_i \cdot d\alpha_2 + v_i \qquad (2.3)$$

or

$$v_i = - \Delta h + y_i \cdot d\alpha_1 - x_i \cdot d\alpha_2 - (h_i - H_i) \qquad (2.4)$$

– Now the ellipsoidal heights of the new points can be turned into orthometric heights by means of the three known transformation parameters in accordance with relation (2.2). The undulation of the geoid does not have to be known when using this way of approach in transformation of the heights. The undulation of the geoid in the new points is obtained by a linear interpolation. If there are great variations of the geoid undulation in the area of the determinations and if there are more than three levelling items a high order surface can be accepted for interpolation.

3. CONCLUSIONS AND RECOMMENDATIONS

Although the accuracy of the global undulations of the geoid improves rapidly, a long time is to pass until a centimeter level as accuracy can be attained because of so many limitations such as:

– unavailability of the necessary gravimetrical data as well as their heterogeneous distribution on the surface of the Earth;
– processing of these data for the calculation of the global undulations of the geoid.

That is why an improvement of the values of the global undulation of the geoid is suggested, as they are available as digital data for the whole Earth. The results of the tests indicate high accuracy in this case. These results can be additionally improved by using more accurate values of the open air gravity anomaly for a near-by surface.
Getting the data of open air gravity anomaly is a difficult and expensive process. That is why an integrated approach is recommended to optimize the models of the geoid for any region of the world:

– establishment / observation of the relative gravimetrical net (open air) for a 5’ x 5’ grid;
– calculation of open air gravity anomaly using models of digital elevation (DEM) and the gravimetrical data of point 1) for detailed grids;
– the accurate calculation of the undulation of the geoid using the data of points (1) and (2);
– comparing / testing the models of the geoid using GPS and the data of the existing vertical net.

Simultaneous determinations of orthometric heights and heights of the geoid by integrated geodesic compensation.

The integrated geodesic compensation starts from a model

$$\underline{l} = \underline{A} \bullet \underline{x} + \underline{R} \bullet \underline{t} + \underline{n} \qquad (3.1)$$

in which:

$\underline{x}$ the vector of unknown quantities

$\underline{t}$ the vector of the sizes of the gravity field

$\underline{l}$ the height of the geoid from all the observations placed at disposal;

$\underline{n}$ is the background noise of the observations that occurs normally;

$\underline{A}$ and $\underline{R}$ the proper design matrices

The GPS observations can be processed as linear basis vectors $\Delta \underline{x}_{ij}^{obs}$ with a proper variance-covariance matrix $\underline{C}_{\Delta x \Delta x}$:

$$\Delta \underline{x}_{ij}^{obs} = (1+k) \bullet \underline{R}(\varepsilon) \bullet \Delta \underline{x}_{ij}, \underline{C}_{\Delta x \Delta x} \qquad (3.2)$$

where:

k is a scale factor;

$\underline{R}(\varepsilon)$ is rotational matrix between the national system and WGS-84 system.

The orthometric heights or the heights of the geoid already known can be taken into consideration in model (3.1) as “pseudo-observations” with the corresponding variant. This takes place by introducing relation (3.3) in which T is the perturbing potential and Y is the standard gravity:

$$h = H + N = H + T/\gamma \qquad (3.3)$$

In this case the orthometric height is taken into consideration in the determined area $\underline{x}$ while T the height of the geoid can be extracted from $\underline{t}$ signal area.

The gravimetrical observations as well as other observations can be combined, without problems, with the linear basis vectors and they can be processed in a model. On the whole the integrated geodesic compensation offer the following advantages:

- the integrated automated accomplishment;
- utilization of various observations;
- interpolation by means of gravity when the heights of the geoid or / and the orthometric heights are known;
- possibility of taking into consideration the correlation and accuracy of geometrical and gravity observations;
- taking into consideration the local and regional deformations, depending on height;
- inclusion of GPS compensation of the linear basis vectors;
- gravimetrical observations from a data bank can be simply connected to the specialized software;
- possibility of making any combination with observations of the Earth.

4. REMARKS

The carried out experiments had the following results: the orthometric heights were calculated by means of GPS and gravity data by integrated geodesic compensations with an accuracy of 1,4 cm for over 5 km, getting to 3 cm for over 30 km. For comparison, an accurate geometrical levelling does not provide more than one centimeter for over 30 km.
The integrated compensation as an interpolation method obtained, in comparison with other procedures, such as Stokes integration, a far better result. It goes without saying that we have to take into consideration for each area to be tested that the field is relatively flat with small level differences (< 300 m).

But one can consider that when using the new digital surface models with a high resolution the establishment of the gravimetric field of the geoid can be substantially improved in areas with higher height differences.

CONTACTS

Work Chief Dr. eng. Octavian Roman
Dunarea de Jos University
Galati
ROMANIA
Tel. + 40 722363824
Fax + 40 239677688
Email: oroman@ugal.ro