

History of Angle Measurement

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SUMMARY

The paper covers the period from the time when angles were measured using rather primitive methods and charts the requirement for increasing accuracy, which was driven by scientist and instrument makers, employing their skills in devising ways of achieving the best possible measurements.

In particular the paper summarises the development of angle reading systems over the last century, making it possible to reliably measure angles to within a fraction of a second of arc, both optically and electronically and traces the thought behind each of the advances in instrument design.

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1. EARLIEST TIMES

In 1936 a clay tablet was excavated at Shush (Khuzistan region of Iran) some 350km from the ancient city of Babylon on which was inscribed a script that was only translated as late as 1950, The text provided confirmation that the Babylonians' measured angles using the figure of 360 to form a circle The inscription on the tablet shows the ratio of a perimeter of a regular hexagon to the circumscribed circle i.e.

Six sides of a hexagon times their base of 60 = 360.

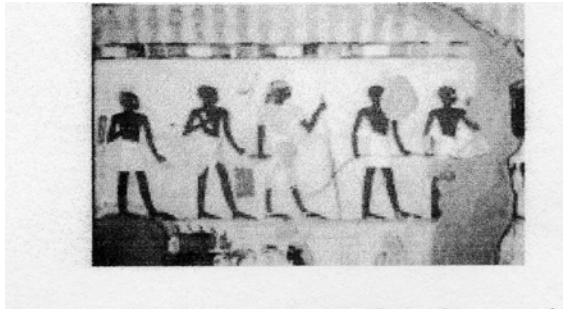


Clay Tablet from Shush, depicting mathematical symbols

The Babylonians' knew that the perimeter of a hexagon was exactly equal to six times the radius of a circumscribed circle, a fact that was evidently the reason why they chose to divide the circle into 360 parts. The tablet gave $\text{Pi} = 25/8 = 3.125$

This proved that the Babylonians used the sexagesimal system based on 60 rather than the centesimal system based on units of 10.

The history of the mathematical measurement of angles, possibly dates back to 1500BC in Egypt, where measurements were taken of the Sun's shadow against graduations marked on stone tables, examples of which can be seen in the Egyptian Museum in Berlin. The shadow was cast but a vertical rod (Gnomon) along the length of the markings on a stone tablet, enabling time and seasons to be measured with some degree of accuracy.



Scene from the Tomb of Djoserkeresonb Scene from the tomb of Menna (Schultz, 1998).

The first known instrument for measuring angle was possibly the Egyptian Groma an instrument used in the construction of massive works such as the pyramids. The Groma consisted of 4 stones hanging by cords from sticks set at right angles; measurements were then taken by the visual alignment of two of the suspended cords and the point to be set out. It was rather limited in its application due to the fact that it was only able to be used on fairly flat terrain and its accuracy limited by distance.



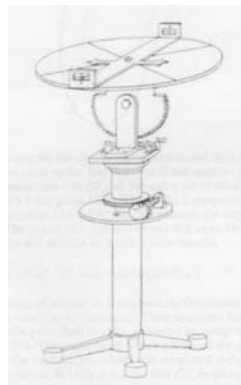
Model of Egyptian Groma

The Groma continued to be the instrument used for setting out right angles for many thousands of years, the Roman's in their construction works, in particular roads, which were always set out in straight lines for which the Groma was ideally suited. There is a carving on a tomb near Turin dating from the first century AD, which depicts such an instrument. The name on the tomb is that of Lucius Aebutius Faustus who was said to be an "Agrimeter" an early term for Land Surveyor.

When Pompeii was excavated in 1912 versions of the Groma were discovered, these being constructed from an iron support with plumb bob made from bronze. This type of crude instrument for setting out right angles has perpetuated to the present day, with the Cross-Staff Head and its successor the Optical Square.

Possibly the first instrument that could be called as such was the Dioptra (The Greek word meaning instrument to see through), which possibly dates from about 150BC. Heron of

Alexandra (10-70AD) described this instrument and from his description one can say it was the forerunner of the Theodolite.

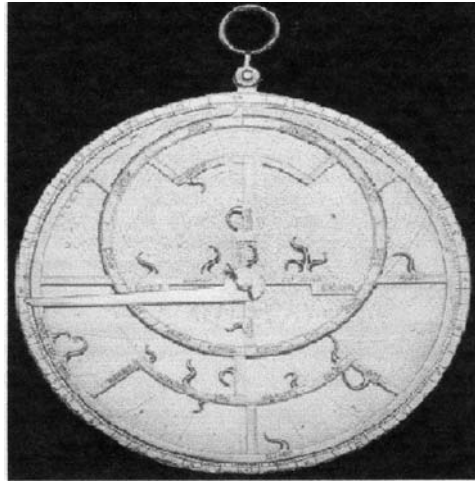


Model of Dioptra

The Dioptra described by Heron was an instrument consisting of a circular metal plate, which could be rotated horizontally through 360 degrees by a toothed gear and another plate that could be tilted in the vertical plane, again by a worm drive. To these plates could be attached a levelling device, consisting of two interconnected tubes containing water, the level line being the alignment of the two menisci of the liquid in the upper parts of the tubes. The Dioptra had a large horizontal circular plate, which was marked out in angles, these could be aligned with a rotatable metal straightedge to which was fitted with a pair of open sights the complete instrument was mounted on a heavy wooden stand.

Claudius Ptolemy the Greek astronomer and geographer (85-165AD) made many astronomical observations in Alexandria, some of which were described as being made with an instrument called a Triquetrum, which means in Greek "Three Legged" It was a sighting device with three hinged sections, one section was planted in the ground while the other two could be rotated to form an isosceles triangle, together with the fixed vertical rod. The rod, which was directly hinged to the vertical rod, carried two sighting devices or pinholes through which one could sight the stars and planets. Zenith angles were measured from lines marked on the third rod. In more modern times, this type of instrument was employed by the military for some aspects of field surveys.

The first hand held navigation instrument designed to measure angles of any magnitude was the Astrolabe used by the navigators of the Middle East in their quest to navigate the oceans. These sophisticated instruments were primarily designed for taking bearings to the stars and planets to determine position at sea as well as an aid to mapping the land and the shoreline. It could also be applied to the setting out of civil works



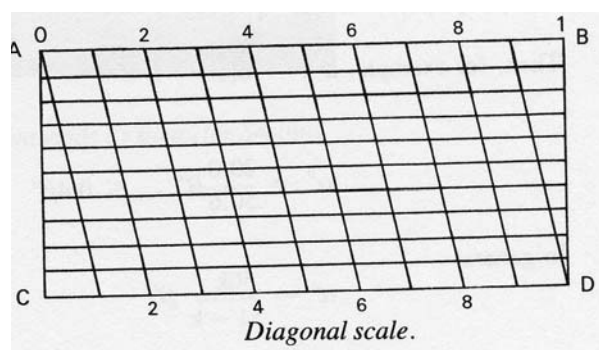
Early Spanish Astrolabe

The principles of the Astrolabe were known before 150 BC. But recognisable instruments date from about 400AD. They were highly developed in the Moorish world by 800AD. It was introduced into Europe from Islamic Spain (Andalusia) in the early part of the 12th century. It was the most popular astronomical instrument until about 1650 when more specialised and accurate instruments such as the Sextant replaced it.

As instruments improved, a need was found for reading circles more accurately. Observations and reading graduations by the unaided eye could be expected to no better than 10 or more minutes of arc.

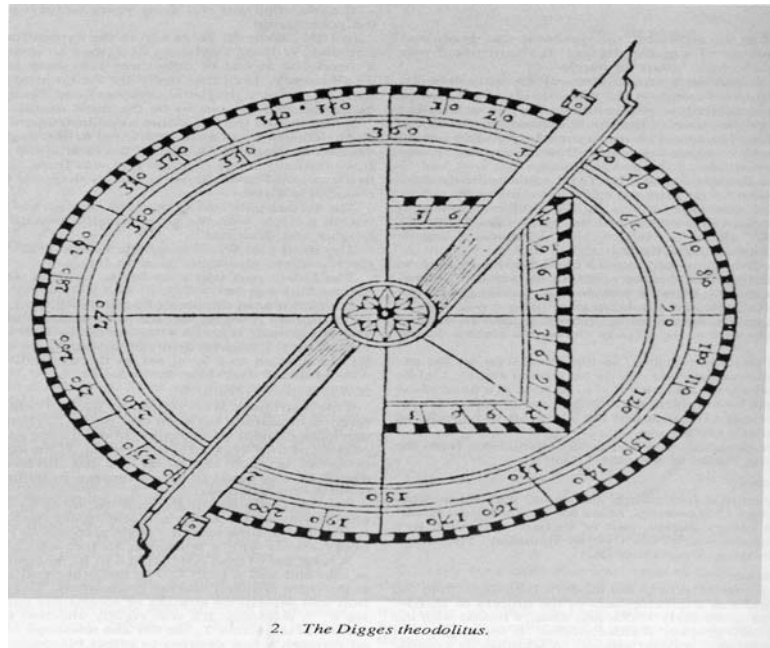
The first approach to reading subdivisions is uncertain. Some references give credit to Peurbach and Regiomontanus of the 15th century or Richard Chanzler in the 16th, yet others put forward that attempts were made in the 14th century to read angles more accurately.

The earliest attempts at subdivisions were akin to the modern diagonal scale and its introduction is often credited to Levi Ben Gershon. Originally the idea was almost certainly one applied to straight scales where it is simple to apply yet accurate.



2. THE THEODOLITE

The first mention of an instrument called a Theodolite was in 1571 by Leonard Digges in his book "Pantometria" in which he describes an instrument called a "Theodolitus". In principal the instrument had a graduated horizontal circle mounted on a vertical column to which was fixed a graduated vertical semicircle. Angles were taken via a pair of sights mounted on a rotating ruler with a reading index.



Drawing of Digges Theolitus

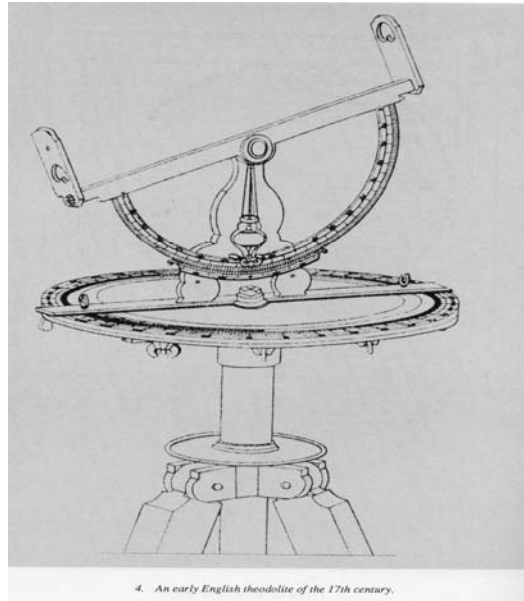
In 1653 William Leybourn described an instrument called a Theodolite thus: -

The graduated circles divided in 360 degrees and subdivided as much as the instrument would allow.

The horizontal circle should be about 12" to 14" in diameter, the subdivisions determined by diagonal lines between concentric circles.

The instrument should have four main parts: -

- 1) The horizontal circle 12" to 14" diameter in 360 degrees with subdivisions.
- 2) Optional geometrical square drawn within a circle with sides equally divided and used for the determination of heights and distance.
- 3) A magnetic needle at the centre of the horizontal circle.
- 4) A socket so the total instrument could be mounted or screwed to a pole.



4. An early English theodolite of the 17th century.
Drawing of William Leybourn's Theodolite

3. THE VERNIER

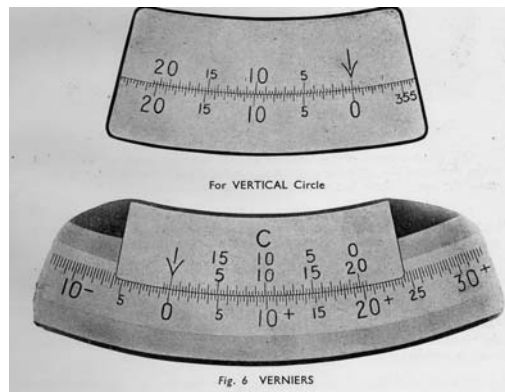
The next stage in the evolution of the Theodolite was the invention of the Vernier, a system described and named after Pierre Vernier in 1631. Vernier's development was taken from a description of dividing an arc of different lengths into 60 parts.

At Brussels in 1631, he published and dedicated to the Infanta of Spain, the treatise "*La construction, l'usage, et les propriétés du quadrant nouveau de mathématiques*", describing this ingenious device.

To a quadrant with a primary scale in half degrees Vernier proposed to attach a movable sector, thirty-one half degrees in length but divided into thirty equal parts (each part consisting then of a half degree plus one minute). In measuring an angle, minutes could be easily reckoned by noticing which division line of the sector coincided with a division line of the quadrant. Christopher Clavius (q.v.) had mentioned the idea but had not proposed to attach permanently the scale to the alidade.

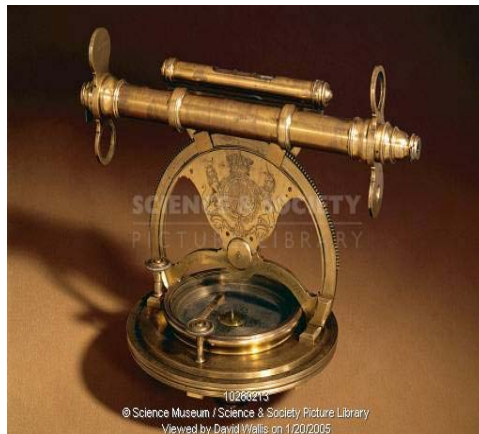
Vernier adapted a Quadrant reading to 30 minutes and attached a scale that spanned 2 degrees, which was divided into 30 divisions, thereby enabling the user to determine the angle to a value of 1 minute by observing which division on the Vernier scale coincided with one of the divisions on the circle graduations.

The Vernier reading system did not come into universal acceptance until the 18th century when in 1759 a better and a more accurate form of sighting were introduced with the achromatic telescope invented by John Dolland of London. Although originally invented by Chester Hall in 1733 it was kept secret for 26 years.



Example of modern Vernier reading system

In 1725 an instrument, recognisable as a Theodolite, was produced by the workshop of Jonathan Sissons. It was constructed in brass and had a 4 1/2" horizontal circle with three vernier's that enabled reading to be taken to an accuracy of 6 minutes. It was fitted with a vertical semicircle with a 70-degree movement above and below the horizontal; the whole instrument was levelled by four footscrews.



Early 18th Century Theodolite
(Science Museum, London.)

4. DIVIDING

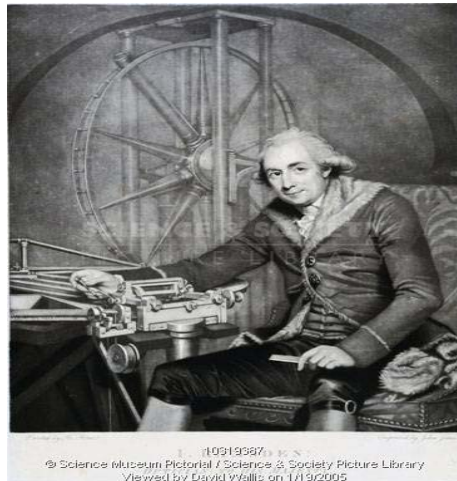
Dividing circles to a high order of accuracy was a continuing problem. In 1793 it was said that from the time of Ptolemy to that of Copernicus the best that could be expected was 5 to 10 minutes of arc. Tycho Brahe reduced this to 1 minute and Hevelius to 15 to 20 seconds. Flamstead achieved 10 seconds and the workshops of George Graham in 1740 achieved 7 to 8 seconds.

The method used to divide circles was very imprecise and time consuming, with a very doubtful outcome. In the early 18th century Olaus Roemer was using dividers set at the linear equivalent of 10 minutes, to mark off divisions around an arc. This method had many disadvantages compared with that employed by Robert Hooke, who was using a mechanical

worm drive to mark around the circumference of the circle or scale of an arc. As the worm drive could not be reliably calibrated, errors of up to 1 minute were quite common in the final result.

5. DIVIDING ENGINE

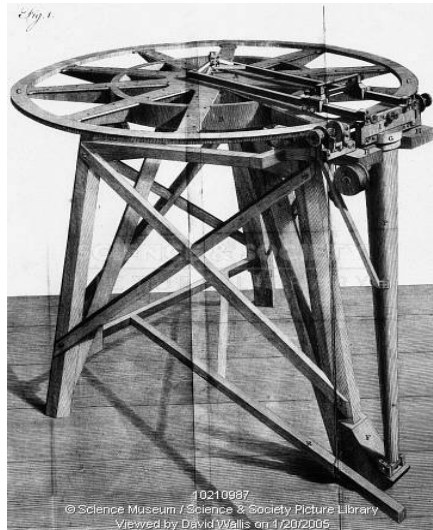
The next and most important development was the invention in 1773 by Jesse Ramsden of the precision-dividing engine. Ramsden, who was born in Yorkshire, was the foremost instrument maker of his age. He produced a wide range of high quality instruments, ranging from eyeglasses to large astronomical telescopes as well as a range of physics apparatus.



Jesse Ramsden (Science Museum, London)

He was a very gifted and skilful designer and at the age of 27 started his own business in London after serving his apprenticeship with leading London instrument maker.

By 1773 Ramsden had built a reputation for his great skill in designing instruments that were far and above that available from any other instrument maker anywhere in Europe. He enjoyed royal patronage and was made a Fellow of the Royal Society in 1786. His greatest achievement was probably the invention of the circular-dividing engine, which enabled, for the very first time, surveying and astronomical instrument to leap forward in terms of their accuracy.



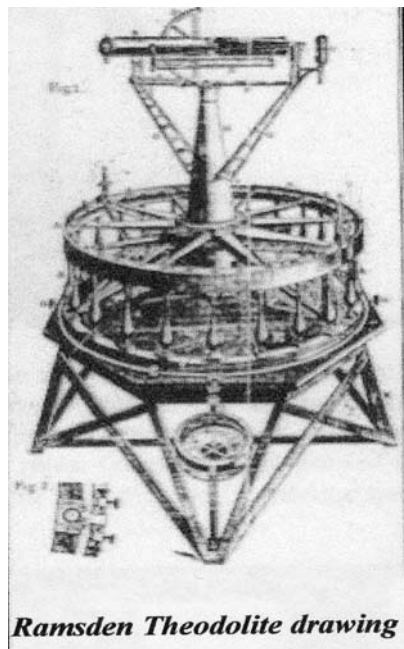
Ramsden's circle Dividing Engine
(Science Museum, London)

Ramsden was able to divide circles to a repeatable accuracy of 3 seconds of arc and as his dividing engine became more refined it was said that it eventually could produce a reliable accuracy of one second.

In 1782 Jesse Ramsden commenced one of his greatest commissions, that of building the Great Theodolite for the triangulation between the observatories of London and Paris, to be carried out by General Roy.

The project took much longer than estimated, as many problems had to be overcome to fulfil the required specification.

The completed instrument had a horizontal circle of 36 inches in diameter with 6 micrometers each capable of reading directly to one second of arc. The total weight of the assembled instrument was 200 lbs. (90Kg)

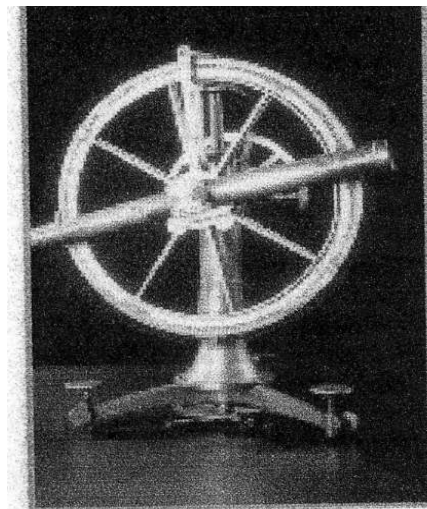


Ramsden Theodolite drawing
(Science Museum, London)

The use of a micrometer to subdivide the graduations of a circle or arc was the most popular method for obtaining a high order of accuracy. The work on the development of the micrometer date back to the 1660's with the astronomers Auzout and Picard. Before that William Gascoigne had made a working micrometer in 1639 but unfortunately died at the Battle of Marston Moor in 1644.before receiving due credit for his invention. It was therefore some 26 years later that an instrument was produced that combined the micrometer with an astronomical telescope.

The first application of micrometers was in astronomy, to measure the diameter of heavenly bodies. Early models employed a knife-edge moved by a fine threaded lead screw, to which was connected a graduated drum divided into 100 subdivisions of the main scale divisions positioned alongside the lead-screw.

In about 1659 Robert Hooke replaced the knife-edge indices with fine hairlines. Circa 1662 Cornelio Malvasia introduced grid lines with thin silver wires. Auzout and Picard introduced one fixed and one moveable thread, such that the movement of the thread across the object could be recorded on the drum. Later they introduced more threads across the field of view. In the middle of the 18th century Tobias Meyer expounded a method of angle repetition or the multiplication method for increasing the accuracy of determine angles. However it was 20 years before this method was applied to an instrument, which was produced by the French manufacturer E. Lenoir on the basis of a design by the geographical engineer. J.C. Borda.



Repetition Circle Theodolite by Jaworski

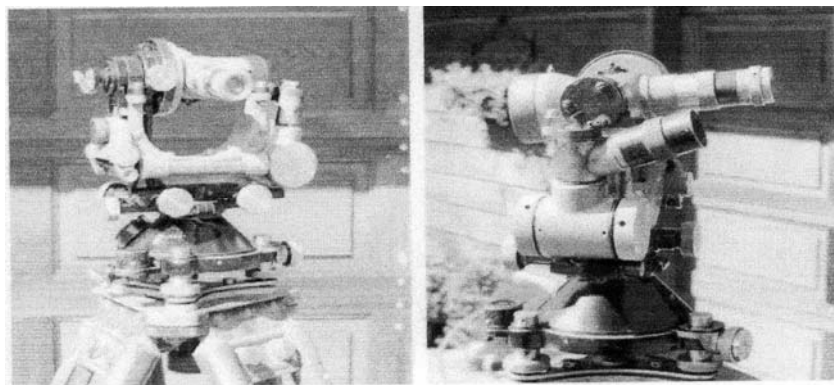
The angle multiplication or repetition method involves repeating the measurements of the same angle many times, without returning to zero, but using the starting point for each measurement as the end point of the previous reading. The total number of angle repartitions then provides the final calculated angle by averaging out the total number of readings. In this way, it was believed it would make it unnecessary to have very fine divisions on the circles and errors in dividing would be reduced.

In the early 19th century Andreus Jaworski of Vienna designed and produced the first double circle Theodolite, which enabled systematic errors to be compensated by rotating the circles in such a way as to take readings from different parts of the divisions, which also compensated for centring errors. His instrument had 4 vernier's, which could produce an accuracy of 4 seconds of arc. Carlini in Milan used this instrument to determine several fundamental astronomical parameters.

Theodolites became smaller and lighter in weight as instrument makers were able to divided circles to greater accuracy, thereby rendering it unnecessary to have large circles with well-spaced divisions. The Vernier became the standard method for determining angles for all general purpose Theodolites. The micrometer was adopted for instruments used for first order geodetic surveys. These types of instruments continued to be produced up until the late 20th century.

6. GLASS CIRCLES

However the introduction of the glass circle Theodolite in the 1920's designed by Heinrich Wild when working for Carl Zeiss of Jena marked the dawn of a new era in Theodolite design.



Carl Zeiss TH1 Theodolite, 1921

In the spring of 1908, Heinrich Wild moved his wife and 5 children from Switzerland to Jena, Germany and accepting employment with Carl Zeiss as manager of their new department, Geo Carl Zeiss. Prior to this time, Zeiss had produced no geodetic instruments.

He first designed and fabricated instruments for levelling, later, Transits and Theodolites were produced, His goal was to make available small, lightweight, stable instruments that maintained their adjustments over long periods of time, and were easy to understand and operate.

These instruments were to prove very successful and Wild became well respected and recognised, worldwide.

During his time with Zeiss, Heinrich Wild devised many new features and improvements including: -

- The anallactic telescope of constant length with internal focusing
- Device for centring the bubble on levelling-instruments by making apparent coincidence of its two ends as viewed in a prism
- Cylindrical steel main axis
- Dust-tight arrangement for foot screws
- The plane-parallel glass plate as an optical micrometer for geodetic instruments.
(First applied to levels)

At the close of the First World War in 1919, living conditions in Germany became very difficult and Wild elected to return to Switzerland. He resigned his permanent employment with Zeiss but agreed to continue with them until 1921 as a contract consultant.

The Theodolite TH1 produced by Zeiss of Jena came on the market in the early 1920s. It read directly to 2 seconds of arc by optical micrometer. It was the smallest and lightest Theodolite ever produced, with the exception of the Kern DKM1, also designed by Heinrich Wild, which only weighed 1.9kg. The sale of the Zeiss glass circle Theodolite grew in popularity as its advantages were fully recognised by surveyors working in the field.

The basic design of the Wild optical reading Theodolite was to have the circle graduation read each side of the circle in the same window, thereby eliminating circle centring errors as well as those associated with graduation errors by always taking the mean reading of the circle graduations.

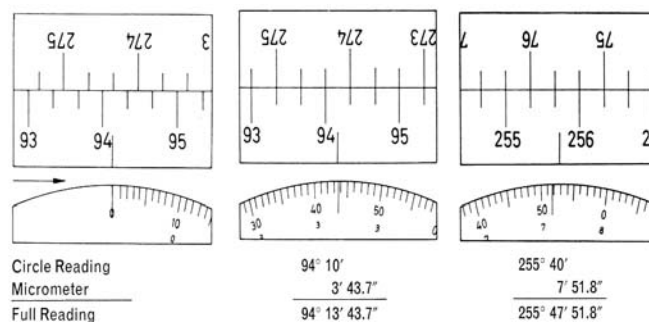
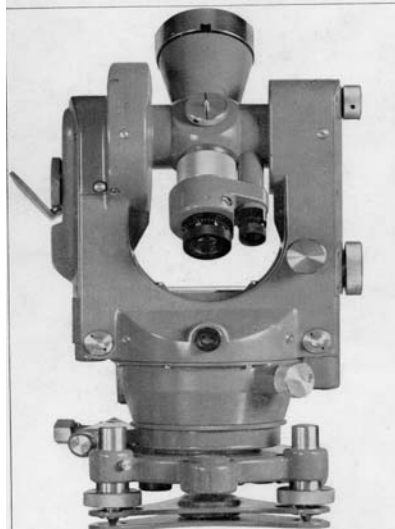


Fig. 7 Circle Reading with the Wild T2 Universal Theodolite (Horizontal or Vertical Circle)

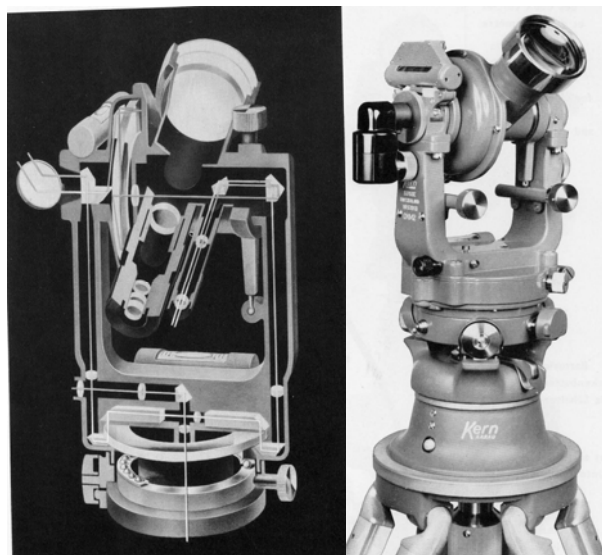
When Heinrich Wild commenced working in Switzerland in 1921, in the town of Heerbrugg, he began by designing the now famous one-second reading Theodolite the Wild T2. This instrument, because of its proven accuracy and robustness became the workhorse for many surveyors throughout the world.



Wild T2 Universal Theodolite

Other designs such as the optical scale reading Theodolite T1 as well as self-reducing tachometers and photogrammetric plotters followed the T2.

In 1932 Heinrich Wild decided to leave the company in Heerbrugg to set himself up as an independent designer, however he continued to provide designs for his old company until 1935 when he entered into an exclusive agreement with the Kern Company in Aarau, to whom he licensed his designs for a double circle reading optical micrometer Theodolite, reading directly to 1 second and estimating to $1/10^{\text{th}}$ called the DKM2, the light weight scale reading instrument called the DK1 and the micrometer version DKM2



Kern DKM2 Theodolite, Circa 1950

These were the first of many optical reading instruments to be produced by the Kern Company to Wild's revolutionary designs.

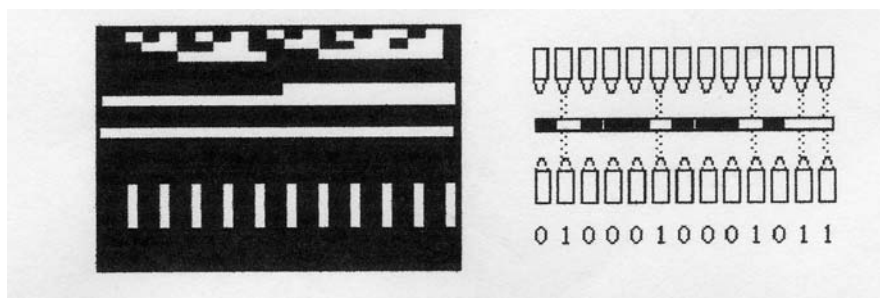
Other manufacturers of surveyors' instruments, finding the growing demand for the glass circle Theodolite designed by Heinrich Wild, entered into licences to enable them to produce their own designs based on the Wild patents. One of the first to do this was the British company Cooke Troughton & Simms, who in 1926, with encouragement for the UK government, designed and built the 1-second reading Tavistock Theodolite.



CT&S Geodetic Tavistock Theodolite

7. CODED CIRCLES

Today the leading companies in the forefront of surveying instrument design and manufacture are focussing on evermore-sophisticated electronic recording instruments, which read the angles using binary coded circles. Otto Fennel of Kassel produced the first, rather bulky electronic Theodolite in Germany. These recorded the encoded readings on film, which when developed had to be passed through a reader, similar to the early Kine Theodolites produced by Askania Werk and used for aircraft and missile tracking. Later but equally cumbersome instruments were produced like the Zeiss REGALTA Electronic Tacheometer, which recorded distance as well as the angle readings via a punched paper tape.



Binary Coded Circle System

The Wild T2000 the Kern E1 & E2 and the Zeiss Elta were among the first electronic reading Theodolite to become popular due to the fact that they were not much larger or heavier than a standard analogue instrument, although they did require rather heavy power supply batteries.

As electronics have become smaller thereby requiring much less power consumption, it has been possible to integrate the power supply within the body of the instrument. Also the new generation of electronic Theodolite now incorporates as standard a high degree of computing power, enabling the user to make on the spot calculations and to store site design information that speeds the setting out process.



Trimble 3603 Total Station

The emerging developments included automatic target tracking and point "cloud" data capture speeding up the surveying operation and processing of the data.

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BIOGRAPHICAL NOTE

David A. Wallis Hon.RICS. FBCS. FRSPSoc. FCIM.

He has now retired from the commercial world of surveying instrumentation, having been the founder of Survey & General Instrument Co. Ltd the general agent for the Kern Company of Aarau for the United Kingdom.

He has held many offices in learned societies in the fields of surveying and cartography, including President of the British Photogrammetric Society.

He is a Past Master of the Worshipful Company of Scientific Instrument Makers of the City of London also a Liveryman of the Spectacle Makers Company.

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