Analysis of DORIS Stations Coordinates Time Series by the Singular Spectrum Analysis (SSA)

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Key words: Time series, SSA, Denoising, Periodogram, DORIS

SUMMARY

The aim of this paper consists in introducing the Singular Spectrum Analysis method (SSA) into the analysis of the time series of co-ordinates of space geodesy stations, in order to collect maximum information on the signals and their noises, which allowed apprehending the time variability of stations motion.

The application of SSA method on time series of a set of residual coordinates of one DORIS (Doppler Orbitography and Radio-positioning Integrated from Satellite) station enabled us to better arise the trend and the periodicity of the studied series. However, for the separation between the noise and the true signal (denoising), this method presents difficulties at the level of the determination of the eigenvalues number corresponding to the true signal to take in consideration.

The results obtained show that the studied series are affected by an important noise and that the increase of number of reconstructed components (or eigenvalues numbers) involves the augmentation in the standard deviation of the series denoised compared to original ones.

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1. INTRODUCTION

The time series occupy a significant place in all the fields of the observation or the data-gathering. In the field of geodesy, collected measurements result from various techniques of ground observations (angles, distances) or space (VLBI, SLR/LLR, GPS and DORIS). These measurements are pretreated and affected of a date of measurement. Thus one forms the time series.

The analysis of a time series consists, in general, to recognize to him three types of components [Chatfield, 89], [Dégerine, 04] : a trend (long-term evolution of the series), a seasonal component and a random variation (noise). It is then useful to separate these components, and this for two reasons :
- First is to detect the general growth or decrease of the phenomenon observed. It is also interesting to highlight the possible presence of a periodic variations.
- Second is to remove the phenomenon of its trend and its periodic variations, in order to observe the random phenomenon more easily.

The technique of analysis of this type of time series of stations co-ordinates of space geodesy, presented in this article, rests on method SSA (Singular Spectrum Analysis). The goal is to obtain diagnoses on the signals and the noise which they contain by making the distinction between the "signal noise" and the "true signal" which allows to determine and to quantify the systematic signals such as trend and periodicity.

2. SINGULAR SPECTRUM ANALYSIS (SSA)

SSA method allows to extract the significant components from a time series (trend, seasonal component and noise). The method is based on calculating the eigenvalues and the eigenvectors of the lag-covariance matrix C formed from the time series $\{X_i\ i=1...,N\}$ and the reconstruction of this time series from the principal eigenvectors [Ghil and al., 02], [Vautard and al., 92] and [Broomhead and al., 86] :
:

$$C = \frac{1}{N'} D^t D \text{ with } D = \begin{pmatrix} X_1 & X_2 & \cdots & X_M \\ X_2 & X_3 & \cdots & X_{M+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N-M+1} & X_{N-M+2} & \cdots & X_N \end{pmatrix}$$

M is the lag-covariance and N is the length of the time series.

Representing the eigenvalues by diagram in descending order, one immediately identifies the isolated and the closed eigenvalues. Thus the theory of the SSA allows to conclude that:

- the signal has a trend if the diagram contains an isolated eigenvalues,
- the signal is periodic if there are two closed eigenvalues, and that
- the small eigenvalues constitute the noise of the signal.

3. APPLICATION

3.1 Data Description

The data used during the processing are constituted of sets of residual coordinates (dN: North component, dE: East component and dH: Vertical component) of the station DORIS (SYPB) (table1) provided by the IGN/JPL analysis centre. These series are expressed in the local geodetic reference frame after removal of the ITRF2000 model of positions and velocities (plate tectonics movement) [Altamimi and al., 2002].

Station acronym	Site	Country	Data span (years)	Period (years)	Observations number
SYPB	Syowa	Antarctica	1999.2-2003.6	4.4	225

Table 1 : Data of the DORIS station studied

3.2 Results and Analysis

The SSA application requires a priori the choice of the Lag-covariance *M* which depends on the periodicity of the signal. This periodicity is estimated by the means of the power spectral density (periodogram), given by the equation 1. The most significant peaks of the power spectral density (PSD) explain the existence of the dominant cycles in the series.

$$S_x(f) = \frac{1}{\mathrm{N}}\left|TFD\left\{X(n)\right\}\right|^2 = \frac{1}{\mathrm{N}}\left|\sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{X(n)e}^{-2i\pi f\mathrm{n}}\right|^2 \qquad (1)$$

N is the length of the series, *TFD* is the discrete Fourier transform of the series $\{X_{n}$, n=1...N} and *f* is the frequency of the signal : $f \in$ {n/N ; n=1...N}.
The PSD graphs of the series according to the Northern component (dN) (figure 1) and the vertical component (dH) (figure 3), shows one peak much more significant of 52 weeks (one year); what explain that the two series have a dominant cyclical component. However, the PSD of the component (dE) (figure 2), revealed a peak at 15 weeks and a peak at 52 weeks.

The PSD of the three studied series (dN, dE and dH) shows that there is a more or less significant systematic peak at one year.

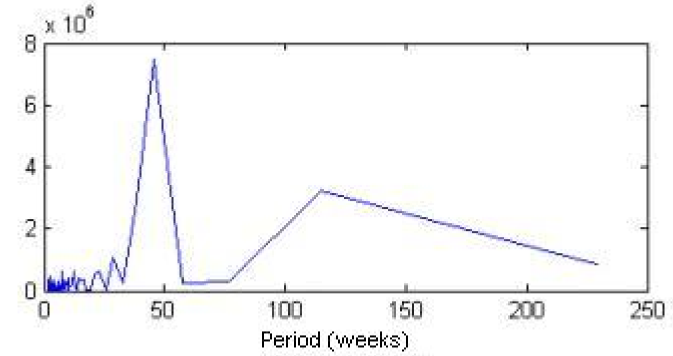

Figure 1 : Periodogram of the series (dN)

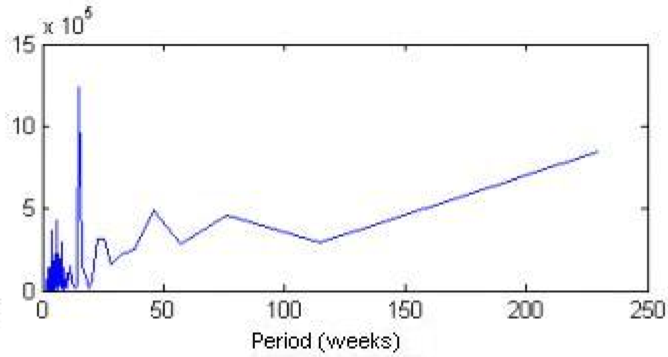

Figure 2 : Periodogram of the series (dE)

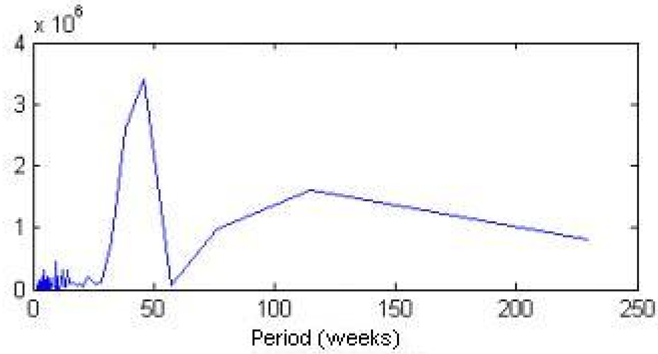

Figure 3 : Periodogram of the series (dH)

For a lag-covariance *M*=52 weeks , the diagram of the eigenvalues (figure 4 and 5) shows two close eigenvalues which are clearly detached from the 50 remaining ones, which indicates a dominant periodic component with a partial variance of 41% for the series (dN) and of 46% for the series (dH). The third eigenvalue which is well separated from the others indicates the signature of a trend with a partial variance of 7% for the two series (dN) and (dH). The remaining eigenvalues correspond to a significant noise and are characterized by much lower values forming a flat floor or a soft slope.

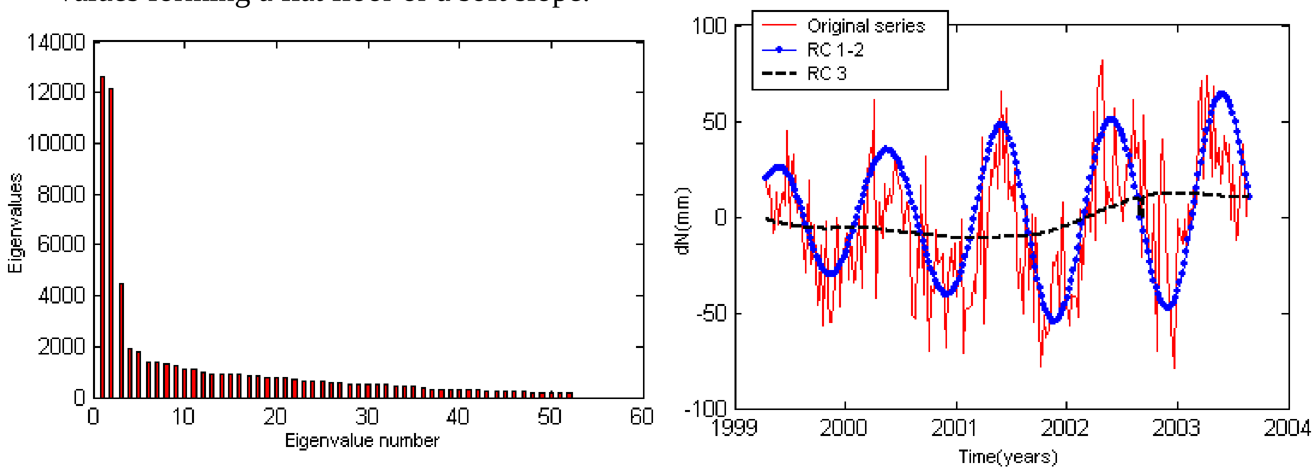

Figure 4 : Diagram of the 52 eigenvalues and superposition of the series (dN) with the principal components

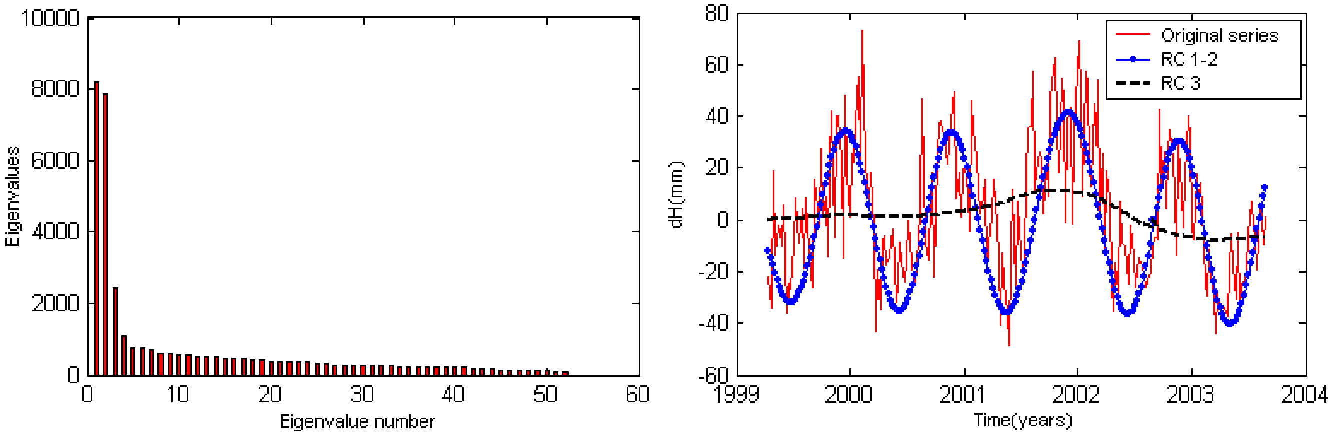

Figure 5 : Diagram of the 52 eigenvalues and superposition of the series (dH) with the principal components

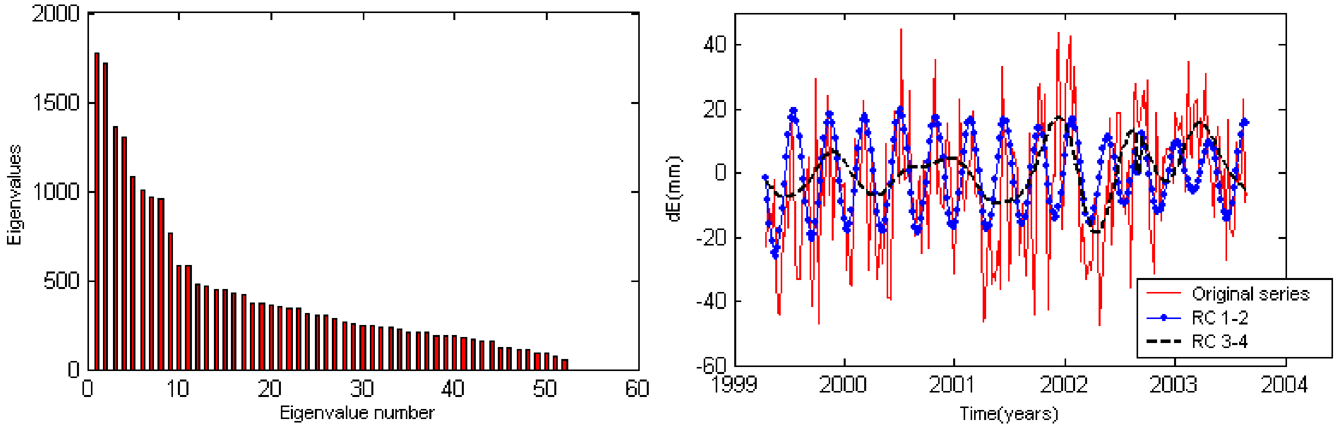

Figure 6 : Diagram of the 52 eigenvalues and superposition of the series (dE) with the principal components

For a lag-covariance *M*=52 weeks, it results from the diagram (figure 6) two pairs of close eigenvalues (four first) which are low detached from the other remaining values, which indicate the existence of two periodic components. The first reconstructed component RC 1-2 based on the first both EOFs explains 16% of the signal while the second RC 3-4 explains only 12%. It arise from the first component a quasi periodic signal and for the second component, there is no periodic signal.

The SSA enabled, also, to determine the noise affecting the series by projecting the original series on the non principal directions (low eigenvalues), or to extract from the initial series, the series reconstructed on the basis of principal component. However, for the separation between noise and true signal (denoising), this method presents difficulties at the level of the determination of the number of eigenvalues (corresponding to the true signal) to take into account (table2).

Series	Reconstructed Components	Variance partial	Standard deviation
dN	RC 1- 2	41	33
	RC 1-2 et RC 3	48	35
	RC 1-2, RC 3 et RC 4-5	54	37
dE	RC 1- 2	16	12
	RC 1-2 et RC 3-4	27	15
	RC 1-2, RC 3-41et RC 5-6	37	17
	RC 1-2, RC 3-4, RC 5-6 et RC 7-8	45	20
	RC 1-2, RC 3-4, RC 5-6, RC 7-8 et RC 9	49	21
	RC 1-2, RC 3-4, RC 5-6, RC 7-8, RC 9 et RC 10-11	54	22
dH	RC 1- 2	46	25
	RC 1-2 et RC 3	53	27
	RC 1-2, RC 3 et RC 4	56	27
	RC 1-2, RC 3, RC 4 et RC 5,6	60	28

Table 2 : Variances partial and standard deviation of the reconstructed components

The following figures (7, 8 and 9) represent the original series (dN, dE and dH) and denoised series on the basis of sum of the reconstructed components, respectively, (RC 1-2, RC 3, RC 4-5), (RC 1-2, RC 3-4, RC 5-6, RC 7-8, RC 9, RC 10,11) and (RC 1-2, RC 3, RC 4, RC 5-6) of which theirs variances partial are listed in table 2. They are about 54% for the series (dN and dH) and of 60% for the series (dE).

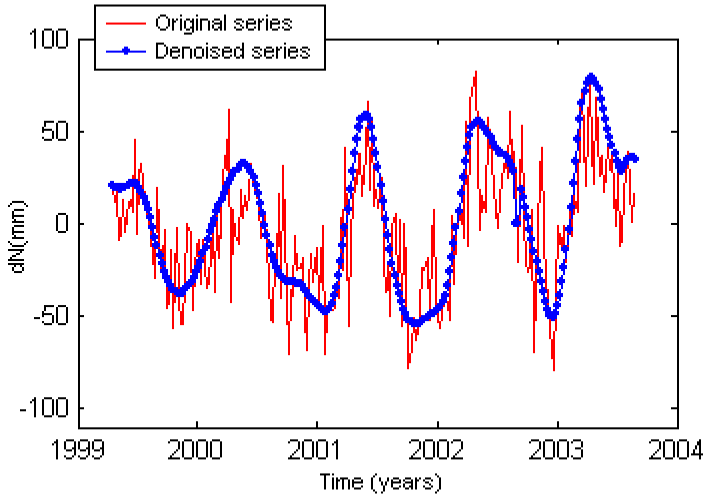

Figure 7 : Component (dN) original and denoised

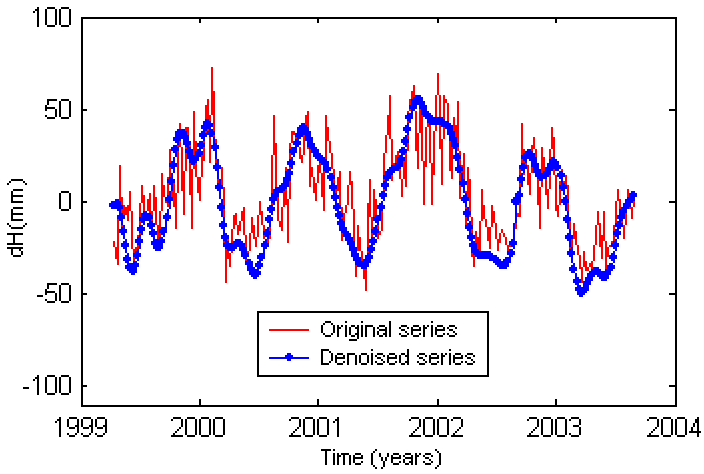

Figure 8 : Component (dH) original and denoised

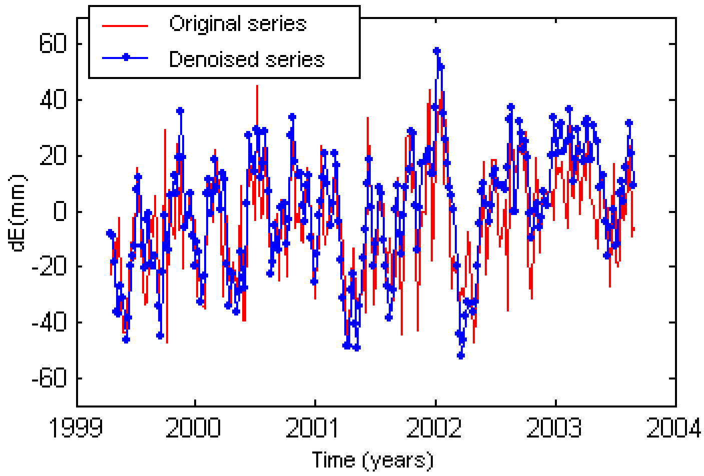

Figure 9 : Component (dE) original and denoised

The table 3 give the statistics of the components (dN, dE and dH) before and after the reconstruction of the signal . We remark that the standard deviation (STD) of the initial series was augmented after the reconstruction which remains dependent on the number of the reconstructed components representing the true signal. Indeed, for example the standard deviation of the reconstructed series (dE) is 12mm (table 2) on the basis of the reconsstructed component RC 1-2 and 22mm on the basis of reconsstructed components (RC 1-2, RC 3-4, RC 5-6, RC 7-8, RC 9 and RC 10,11) to compare with the 20mm of the initial series.

Series		Minimum	Maximum	Mean	STD
(dN)	Original series	-79	82	-4	34
	Denoised series	-54	79	1	37
(dE)	Original series	-47	45	-4	20
	Denoised series	-53	57	0	22
(dH)	Original series	-48	73	4	26
	Denoised series	-49	56	-1	28

Table 3 : Statistics of denoised components (dN, dE and dH) [mm]

4. CONCLUSION

The statistical study of the geodetic time series is a field of research which is currently in a phase of development of analysis methods adapted to the criteria of this new geodesy at four dimensions. The work presented through this paper is a contribution to these methodological developments.

The application of method SSA to the weekly series of sets of residual coordinates of the station DORIS (SYPB), permits to us to better highlight the trend and the periodic components of these series. Indeed, we have clearly identify and quantify a periodic signal at one year and a light trend for the two series (dN) and (dH). Nevertheless, to extract the noise from the original signal, this method presents difficulties at the level of the determination of the number of eigenvalues to take into account to represent the true signal. The results obtained showed that these series are affected by a significant noise and that the increase of principal reconstructed components (true signal) involved the augmentation in the standard deviation of the series denoised compared to original series.

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BIOGRAPHICAL NOTES

Khelifa Sofiane is a researcher in Division of Geodesy of the Center of Spatial Techniques (CTS) since 2000, holder of the diploma of Magister in Space Techniques and applications and PhD student in the Electric Genius faculty at (USTO, Oran, Algeria). His activity of research is related to the analysis of the signals of the time series of stations co-ordinates of space geodesy (SLR, GPS and Doris) with an aim of studying the stability of the stations of space geodesy by the determination of two parameters : the level and the type of the noise of these series.

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