

# Developing Raster Datasets for Length Differences between Topography, Geoid, Ellipsoid and Map Projection for Macedonian Territory

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**Keywords:** Raster, Ellipsoid, Geoid, Map Projection, Macedonia

## SUMMARY

Procedure of reducing lengths from topography surface up to map projection, through geoid and ellipsoid undulations and approximations, by using digital technology, give us an advantages for more deeply and faster computes-analyses in this field. The only thing which is necessary for performing this kind of analyses is owning of quality data.

Till now, all analyses within the coordinate systems have been carried out separately, between the different approximations, i.e. reduction from topography surface to geoid, than undulation from geoid to earth ellipsoid, and in the end projection from earth ellipsoid to map projection. Within previous scientific analyses, only length differences between the earth ellipsoid and any map projection can be recognized, without taking into account both another approximations from topography surface to earth ellipsoid jointly.

In this paper, results of some analyses of real length differences between topography surface and map projection will be shown. Analyses have been realized in the state territory of Macedonia, by using some most appropriate map projections for its territory, WGS84, SRTM DEM and EGM08, based on grid with 1km spatial resolution in total with 25635 points. From generated data, as outputs were compiled maps with iso-lines of the length differences between topography and map projection, statistical results from basic statistical analyses, and some conclusions.

From obtained results within very complex and deeply analyzes of partial and progressive length reductions between some surfaces, it is so clear that during establishing of the state coordinate systems were not taken in to account all length reductions beginning from the topography up to map projection. This conclusion can be verified by computed linear deformation values, which in most of cases have largely avoided from expected values.

Extreme length differences in all cases doesn't reached same values with opposite prefix, mean linear differences in all cases didn't have values nearby zero, and dispersion of length differences didn't reach the criteria of uniform dispersion which was resulted with to large differences between areas with negative and positive linear deformations.

Modern software gives us opportunity to accelerate such complex analyses, by using complex mathematical models and large data. In the past, nobody could imagine performing analyses with over than 25000 points, by defining their altitude and geoid heights, as well calculation their length reductions between four surfaces.

Conclusions from research represent a gateway for further analyses in this field, which are planned to be done next years!

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## 1. INTRODUCTION

Horizontal lengths between points in topography surface, before mapping and calculating coordinates of edges, should pass at least three approximations between three surfaces. During the approximation process, lengths changes their values based on many factors, such as altitude, geoid height, earth ellipsoid, and map projection parameters. Changing the length value for many times before its mapping, should be known in details in order to have whole control on mathematical elements of geospatial data and qualitative spatial coordinate system. In a figure 1, scheme of length approximations from mean altitude between two points in the topography surface up to the map projection is shown.

First approximation is reduction of horizontal length from topography in to geoid surface (sea level), based on the mean altitude value of line edges. Second approximation is reduction of length from the geoid surface in to the earth ellipsoid surface, known as geoid undulation, based on the geoid height between the earth ellipsoid and local geoid surface. Third and final approximation is length projecting from earth ellipsoid in to the flat area of map projection.

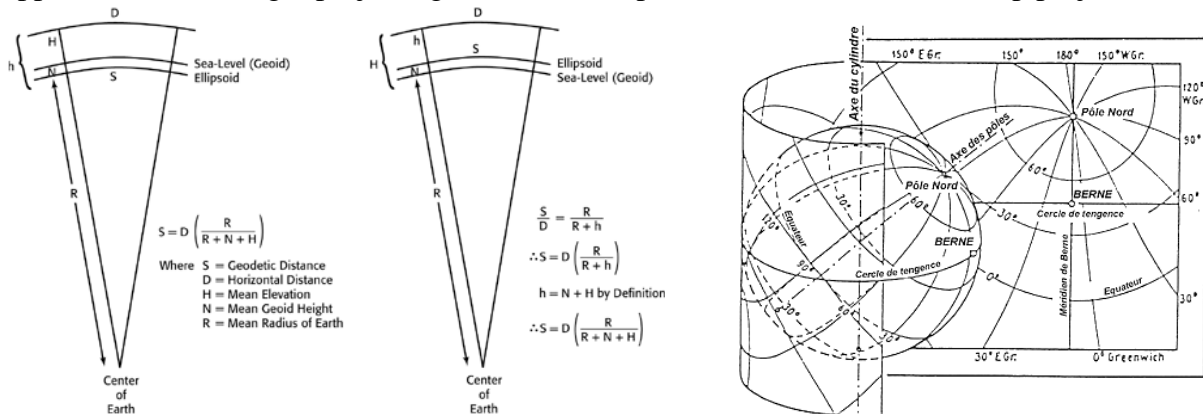


Figure 1. Scheme of lengths approximation from topography up to map projection

Within the research methodologies which have been done in past period for defining most appropriate state coordinate system through defining map projection, only separate length differences between two surfaces have been analyzed. During the literature review, any joint analyses of length differences between topography and map projection (state coordinate system) was not found! Even any national dataset with the length differences was not found during our research analyses!!!

Length differences can be calculated as absolute values, and they are not errors! Differences are known based on parameters of surfaces (topography, geoid, ellipsoid and map projection), and the accuracy of their calculation depends on the accuracy of used surfaces.

Developing dataset for length differences with high accuracy between topography and state coordinate system for country areas is very important data for much type of field measurements and works, geodetic and engineering works, high accuracy geodetic measurements, combination of GPS and total station measurements, transformation of local networks (micro triangulations, etc.) from free oriented in to state coordinate system, etc. One of main objectives of this research is developing and providing national dataset of length differences in 1km for many cases and between many surfaces for the country area of Macedonia.

## **2. SOURCE DATA & TEST MODEL DEFINING**

Due to current conditions within Macedonian geo market and copyright rules on geospatial data defined on a law for real estate cadaster, there are no possibilities for utilization official data of our National Mapping Organization (NMO) free of charge for scientific research projects, except the Macedonian Global Map dataset which is free and open for noncommercial uses via Global Map's web site. Because of this situation, beside Macedonian global map dataset (figure 1), our research analyses have been conducted by using global, open and free access datasets: ASTER GDEM and EGM08.

Based on border line obtained from Macedonian Global Map dataset (figure 2), with the aim of creating test model to be used for our research, point vector grid with 1km distance between points has been developed for Macedonian territory, in total with 25635 points which covers entire area of Macedonia (figure 3). Given the fact that differences between two or more surfaces (between topography and geoid, between geoid and ellipsoid, etc.) are expressed in 1km, during the test model creation 1km distance between test model points have been used. By using QGIS software, from ASTER GDEM (figure 4) and EGM08 (figure 5), has been extracted orthogonal/geographic coordinates, altitudes and geoid heights for all 25635 points of test model.

Referering to spatial position of 25635 points of test model in 1km spatial resolution, and EGM08 data, geoid heights for all points have been calculated for the entire area of Macedonia with precision of 4 decimals after meter (0.1mm), and 1mm RMS of interpolation of geoid heights in points of test model. From the calculations, maximum value of geoid heights within EGM08 within Macedonian territory is 46.396m, while the minimum is 41.1318m. Based on calculated geoid heights, map of geoid heights of Macedonia was compiled (figure 5) as GIS raster dataset with 1km spatial resolution (Idrizi B., 2013).



Figure 2. Map of Macedonia, based on Global Map VI vector data (Idrizi B, 2006)

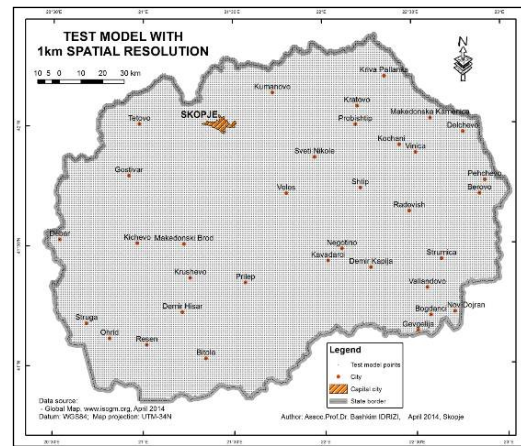


Figure 3. Test model with 1km spatial resolution; 25635 points

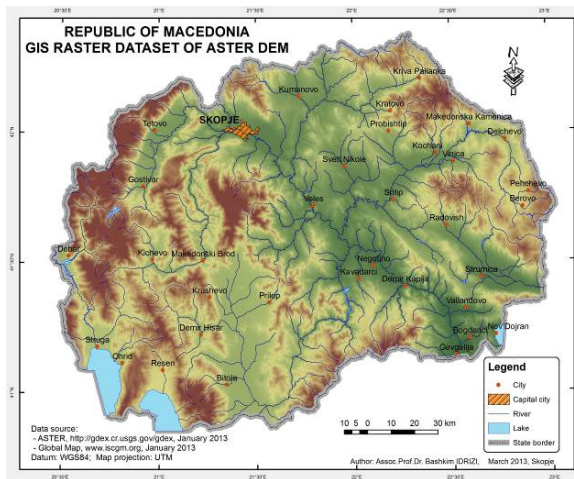


Figure 4. Relief of Macedonia, based on ASTER Global DEM (Idrizi B, 2013)

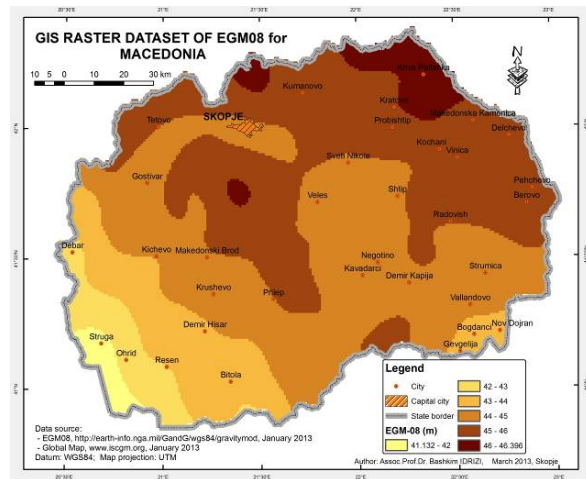


Figure 5. GIS raster dataset with 1km spatial resolution of EGM08 for Macedonian territory (Idrizi B, 2013)

Assessment and comparison of length reductions between many surfaces have been realized through calculation of mean linear deformations of 1km lengths, dispersion of deformations (from-to), as well areas with positive, negative or without deformations. Through calculated values, the quality of final length difference between topography and map projection can be evaluated. Theoretically, calculated mean linear deformation should approach to zero, minimum and maximum value of differences within the dispersion should approximately have same values, and areas with positive and negative differences should approach same values!

### 3. CALCULATION LENGTH DIFFERENCES

Calculation process of the length differences within research have been conducted in three phases:

- Calculation of 1km length differences between topography and geoid, as well geoid and earth ellipsoid;
- Calculation of 1km length differences between ellipsoid and some characteristic map projections for the Macedonian territory; and
- Calculation progressive differences of 1km topography horizontal length from topography up to geoid, ellipsoid and map projections.

#### 3.1. Calculation of 1km length differences between topography and geoid, as well geoid and earth ellipsoid

From the ASTER global DEM, EGM08 and geometric parameters of WGS84 earth ellipsoid, as well by using mathematical models for calculation length reduction from topography to sea level (geoid) and geoid undulation, partial length differences of 1km lengths between three reference surfaces (ASTER, EGM08 and WGS 84) for all 25635 test model points have been calculated. Calculations have been realized in Microsoft Excel by using domestic mathematical models for this type of computation from ellipsoidal geodesy, physical geodesy and topography, while the cartographic representation of calculated results have been realized in QGIS software.

At the beginning, length difference of 1km horizontal length in topography surface between the ASTER and EGM08 was computed. Due to very large difference between extreme values of altitudes for Macedonian territory (over than 2700m), and having in consideration that whole state territory is above the sea level, reduction values have negative prefix with largest length difference about -41cm/km. Minimal calculated reduction value is -0.38cm/km, maximal is -41.43cm/km, while mean reduction is -13.03cm/km (table 1). From calculated reductions for 25635 test model points, vector grid and raster dataset with 1km spatial resolution have been developed (figure 6).

Based on geoid heights (figure 5, Idrizi B. 2013), geoid undulation calculations between EGM08 and WGS84 surfaces have been conducted as second research step. Range between extreme values of geoid heights in Macedonian territory is about 5m, which means that undulation value is too short with just about 7mm/km. Because the EGM08 surface on Macedonian territory is above the WGS84 earth ellipsoid, all length differences between EGM08 and WGS84 have minus values, and entire national area reducing its surface after transition from geoid in to earth ellipsoid surface. Extreme values of length differences in 1km length are in range from -0.65 to -0.73cm/km, while the mean difference is -0.7cm/km (table 1). From computed values of 1km length differences between EGM08 and WGS84 surfaces, vector grid and raster dataset with 1km spatial resolution have been developed, as well visualized in complied map in figure 7.

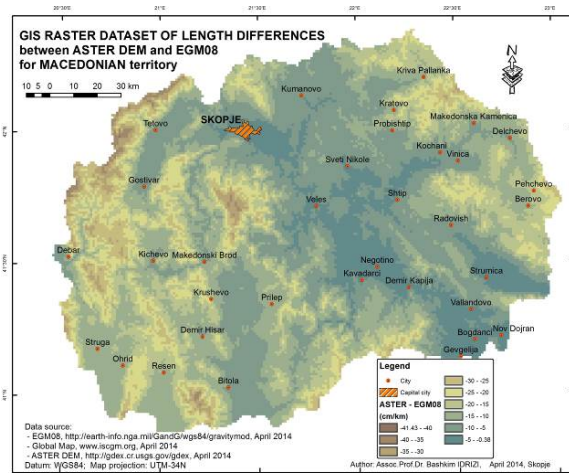


Figure 6. Length differences of 1km length between ASTER and EGM08

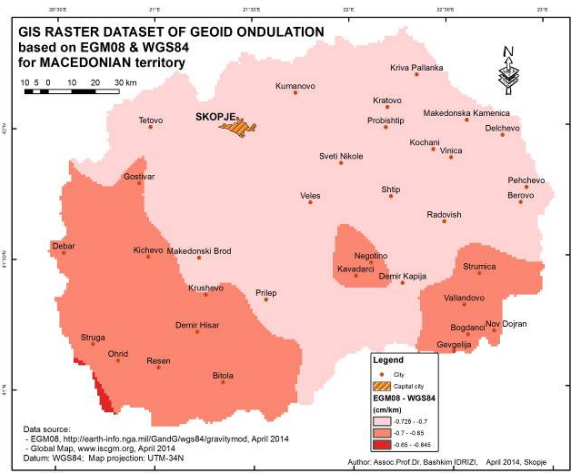


Figure 7. Length differences of 1km length between EGM08 and WGS84

Table 1. Partial length differences of 1km lengths between six reference surfaces

SURFACES	ASTER DEM – EGM08	EGM08 – WGS84	WGS84 – UTM34N	WGS84 – Gauss-Kruger (21°E, 0.9999)	WGS84 – Gauss-Kruger (21°45'E, 0.99993)
⊖	-13.03cm/km	-0.7cm/km	-32.28cm/km	-2.27cm/km	-3.44cm/km
Dispersion of deformations	-0.38 to -41.43cm/km	-0.65 to -0.73cm/km	-4.87 to -40cm/km	-10 to 25.16cm/km	-7 to 7.25cm/km
d positive	-	-	-	31.11%	18.99%
d negative	100%	100%	100%	68.39%	81%
d without deformation	-	-	-	0.5%	0.01%

### 3.2. Calculation of 1km length differences between ellipsoid and some characteristic map projections for the Macedonian territory

Third and final approximation is projecting points from the earth ellipsoidal in to map projections flat surface. Research for third approximation has been realized based on some variants of Gauss-Kruger projection, related to previous researches for quality analyses of official Macedonian map projection (Idrizi B., et all, 2005) and selecting the best map projection for Macedonian territory (Ribarovski R., et all, 1998), as well projection with international importance. Based on three mentioned criteria's, third approximation was realized by using map projections listed below, while their parameters are given in table 2:

- UTM 34N
- Macedonian state map projection and
- Macedonian state map projection with displaced central meridian and changed scale factor.

From geometric parameters of WGS84 earth ellipsoid, utilized map projections parameters, as well by using domestic mathematical models for projecting lengths from earth ellipsoid to map projection from mathematical cartography, partial length differences of 1km lengths between WGS 84 and three map projections for all 25635 test model points have been calculated.

### ***UTM 34N***

In first case the UTM map projection for Macedonian territory, as map projection with international importance and unified model for projection zones has been used. Macedonia is located in 34<sup>th</sup> projecting zone of Northern Hemisphere (UTM 34N). During projecting of 1km lengths from the earth ellipsoid in to map projections flat surface, due to central meridian of 34<sup>th</sup> zone crosses Macedonian area, and distance between the central meridian and eastern coordinate as extreme point is about 170km, whole national area of Macedonia reduces its lengths and areas with minus values, which means that whole area is getting smaller after projecting from WGS84 in to UTM 34N.

Extreme linear deformation values are in a range from -40 to -4.87cm/km, where the mean linear deformation is -32.28cm/km (table 1). From calculated values for 25635 test model points, grid and raster datasets with 1km spatial resolution have been developed, as well map with linear deformations has been compiled (figure 8).

### ***Macedonian State Map Projection***

Macedonian state map projection is defined with parameters given in table 2, based on Law for Real Estate Cadastre of Macedonia and Regulation for basic geodetic works in Macedonia. This map projection was evaluated by 25635 test model pints, by referring to WGS84 earth ellipsoid. From computed values, extreme linear deformations are -10cm/km along the central meridian and 25.16cm/km in eastern point of test model, while mean linear deformation is -2.27cm/km (table 1). Since the deformation values have both, positive and negative prefixes, compared with above approximations, in this case area with positive length differences is recognized, i.e. 31.1% of area has positive linear deformations, despite 68.4% with negative and 0.5% without deformations. From calculated deformation values for all test model points, grid and raster datasets with 1km spatial resolution were developed, as well map was compiled (figure 9).

### ***Macedonian State Map Projection With Displaced Central Meridian And Changed Scale Factor***

Within conducted research on year 1998 by group of authors (Ribarovski R., 1998), current Gauss-Kruger projection has been analyzed by adapting to national shape and dimensions with displaced central meridian and changed scale factor. In referred research, one of analyzed options was Gauss-Kruger projection with central meridian 21°45'E and scale factor along the central meridian 0.99993. This projection aimed to reach same level of absolute linear deformations in extreme points (eastern and western points) and along central meridian

( $\pm 7\text{cm/km}$ ), as well to have uniform dispersion of linear deformations among entire national area. Its parameters are shown in table 2.

This map projection was utilized for our research as options to be analyzed for computing the length differences between earth ellipsoid WGS84 and map projection flat area. For 25635 test model points, extreme linear deformations, mean linear deformation and areas with positive and negative deformation were calculated. In table 1, is shown that linear deformation are in range from  $-7$  to  $7.25\text{cm/km}$  with mean value of  $-3.44\text{cm/km}$ , while 81% of territory has minus value despite 19% with positive deformation. From calculated values, it is so clear that linear deformations don't have uniform dispersion in whole national area. As in previous cases, raster and vector grid datasets, as well map were compiled (figure 10).

Table 2. Parameters of utilized map projections for research

UTM – zone 34N	Gauss-Kruger (21°E, 0.9999)	Gauss-Kruger (21°45'E, 0.99993)
Central meridian: 21°	Central meridian: 21°	Central meridian: 21°45'
Origin of latitude: Equator	Origin of latitude: Equator	Origin of latitude: Equator
Scale factor: 0.9996	Scale factor: 0.9999	Scale factor: 0.99993
False easting: 500000m	False easting: 500000m	False easting: 500000m
False northing: 0m	False northing: 0m	False northing: 0m

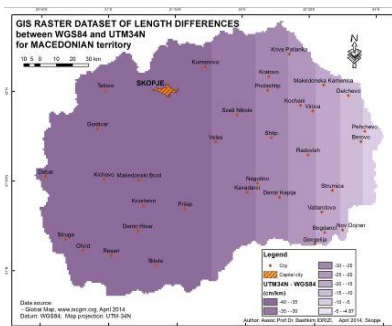


Figure 8. Length differences of 1km length between WGS84 and UTM34N

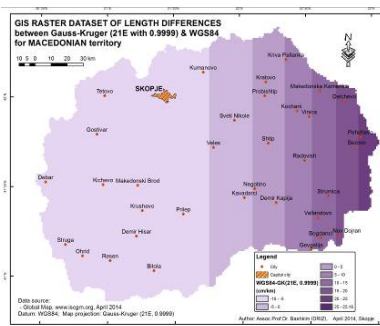


Figure 9. Length differences of 1km length between WGS84 and Gauss-Kruger (21°E, 0.9999)

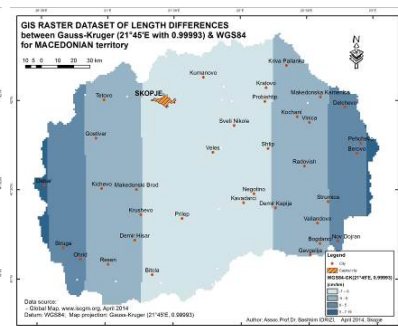


Figure 10. Length differences of 1km length between WGS84 and Gauss-Kruger (21°45' E, 0.99993)

### 3.3. Calculation progressive differences of 1km topography horizontal length from topography up to geoid, ellipsoid and map projections

Third and final phase of length difference analyses was conducted by computing progressive length differences from topography surface up to other surfaces, i.e. to sea level (geoid), earth ellipsoid and map projection. Calculated values in this phase represent difference of 1km horizontal length in topography surface up to three mentioned surfaces. From 25635 test model points, in table 3 are given range of length differences, mean values and areas in percentage with negative, positive and without length differences.



Second column of third table has same values with those in table 1. It represents the 1km horizontal length differences between topography and sea level (geoid), i.e. between ASTER DEM and EGM08.

In third column of table 3, are shown results from calculated differences between topography and earth ellipsoid, i.e. between ASTER and WGS84, based on 1km horizontal length in topography surface. From obtained results, approximation of all Macedonian area is with negative value, with range of differences from -1.07 to -42.11cm/km and mean value - 13.74cm/km. Vector and raster grid datasets with 1km spatial resolution have been developed from 25635 test model points. Results are shown in compiled map, shown in figure 11.

Final calculation represented in last three columns (columns 4, 5 and 6) of table 3, belongs to length differences between 1km horizontal lengths in topography and projected lengths in map projections flat surface, length that previously passed successive approximations through geoid and earth ellipsoid.

Within first case of utilization of UTM 34N, length differences have negative values among entire national area with range from -10.55 up to -81.55cm/km and mean value of - 46.01cm/km. From 25635 test model points, vector and raster grid datasets were developed, as well map was compiled (figure 12).

In a case of Macedonian national Gauss-Kruger map projection, situation is relatively better in the aspect of extreme and mean values, as well dispersion of differences. Horizontal lengths of 1km from topography surface are reducing their lengths with mean value of - 16.01cm/km, in range from -51.56 to 19.48cm/km. From all researched cases, this is first time in which considerable area of Macedonians territory has positive values of length differences between topography and map projection. In fifth column of table 3, can be recognized that 88.6% of territory is in minus area, while 11.3% is in positive and 0.1% in area without length differences. From calculated values for 25635 test model points, grid and raster datasets with 1km spatial resolution have been developed, as well map of length differences was compiled (figure 13).

In a last analyzed case of length differences of 1km horizontal topography length between topography and Gauss-Kruger projection with central meridian 21°45'E and scale factor 0.99993 (7cm/km negative linear deformation along central meridian), a small insignificant area of 0.1% belongs to area with positive differences, despite all area which is in negative zone except 0.01% without differences. Extreme length difference values are in range between -44.46 and 2.01cm/km, with the mean value of -17.17cm/km. From calculated values for 25635 test model points, grid and raster datasets have been developed, as well map of length differences has been compiled (figure 14) also.

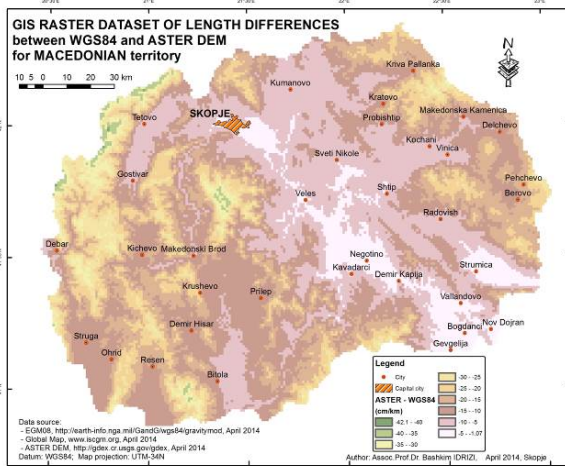


Figure 11. Length differences of 1km length between ASTER and WGS84

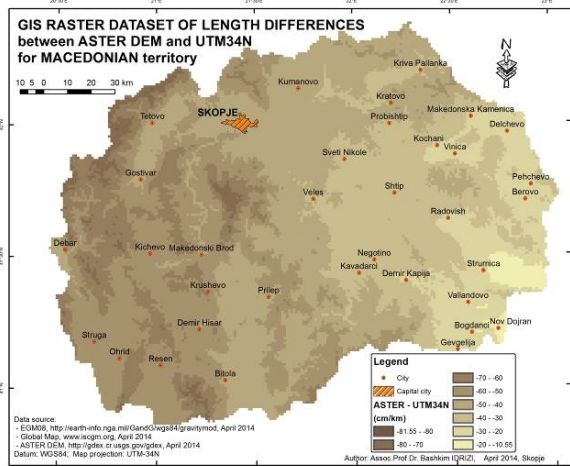


Figure 12. Length differences of 1km length between ASTER and UTM 34N

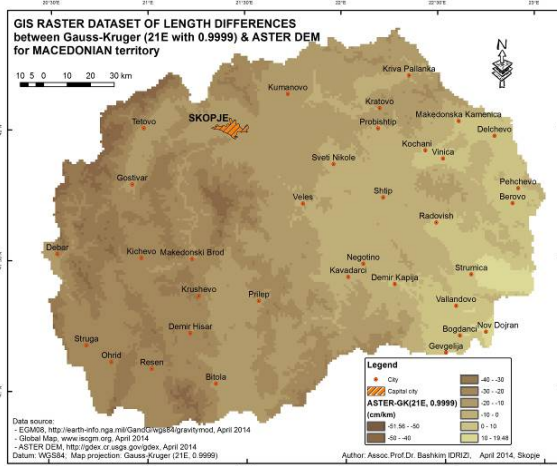


Figure 13. Length differences of 1km length between ASTER and Gauss-Kruger (21°E, 0.9999)

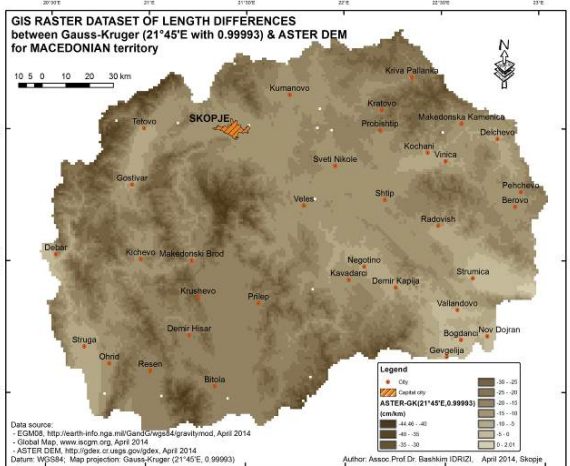


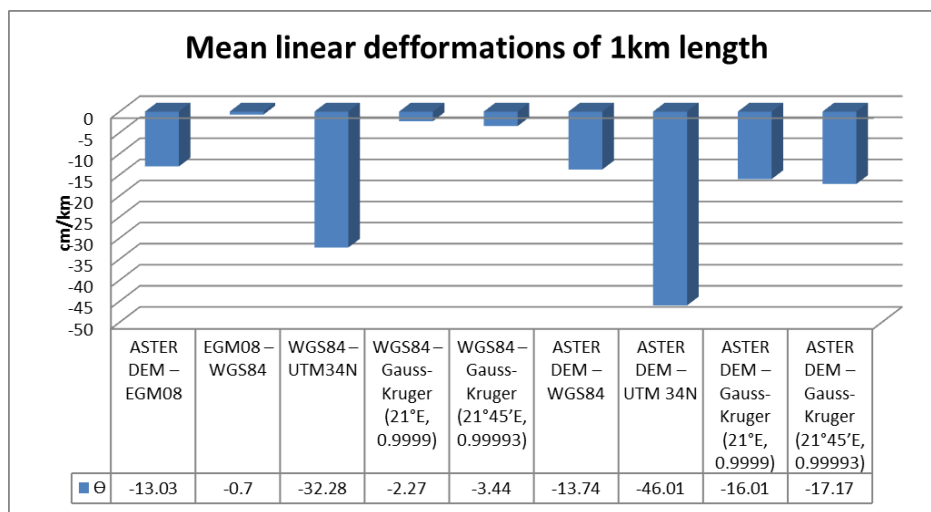
Figure 14. Length differences of 1km length between ASTER and Gauss-Kruger (21°45'E, 0.99993)

Table 3. Progressive length differences of 1km lengths between ASTER and five reference surfaces

SURFACES	ASTER DEM – EGM08	ASTER DEM – WGS84	ASTER DEM – UTM 34N	ASTER DEM – Gauss-Kruger (21°E, 0.9999)	ASTER DEM – Gauss-Kruger (21°45'E, 0.99993)
$\ominus$	-13.03cm/km	-13.74cm/km	-46.01cm/km	-16.01cm/km	-17.17cm/km
<i>Dispersion of deformations</i>	-0.38 to -41.43cm/km	-1.07 to 42.11cm/km	-10.55 to -81.55cm/km	-51.56 to 19.48cm/km	-44.46 to 2.01cm/km
<i>d positive</i>	-	-	-	11.32%	0.09%
<i>d negative</i>	100%	100%	100%	88.62%	99.9%
<i>d without deformation</i>	-	-	-	0.06%	0.01%

#### 4. CONCLUSIONS

From obtained results within very complex and deeply analyzes of partial and progressive length reductions between some surfaces, it is so clear that during establishing of the state coordinate systems were not taken in to account all length reductions beginning from the topography up to map projection. This conclusion can be verified by computed linear deformation values, which in most of cases have largely avoided from expected values. Extreme length differences in all cases doesn't reached same values with opposite prefix, mean linear differences in all cases didn't have values nearby zero, and dispersion of length differences didn't reach the criteria of uniform dispersion which was resulted with to large differences between areas with negative and positive linear deformations. Just as an example, in next chart are shown mean values of 1km length deformations between researched surfaces, where it is so clearly seen that in all cases the value of mean linear deformations have negative value.



Having information for length differences is of a big importance for developing database of length differences between approximated surfaces. This database in national level has to be developed, and it has to be open and free for usage by geo community.

Contemporary technic and technology enables us realizing the calculation methodology of length differences with such a big number of test model points. In the past, nobody could imagine performing analyses with over than 25000 points, by defining their altitude and geoid heights, as well calculation their length reductions between four surfaces. Modern software gives us opportunity to accelerate such complex analyses, by using complex mathematical models and large data. In our research case, any special software was not used for calculations, but in Microsoft Excel has been defined mathematical models and performed calculations.

Beside the practical application of computed data within our analyses, which will be open and free for use, have to be mentioned fact that this research will open doors for our further researches in this area, mainly analyses for defining of state coordinate systems by including

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of successive length reductions from topography up to map projections surface through geoid and earth ellipsoid, as one of preconditions to reach the basic theoretical criteria's!

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## BIOGRAPHICAL NOTES



**Bashkim IDRIZI**, was born on 14.07.1974 in Skopje, Macedonia. He graduated in geodesy department of the Polytechnic University of Tirana-Albania in 1999year. In 2004, hot the degree of master of sciences (MSc) in Ss.Cyril and Methodius University-Skopje. In 2005 he had a specialization for Global Mapping in Geographical-Survey Institute (GSI) of Japan in Tsukuba-Japan. On year 2007, he held the degree of Doctor of sciences (PhD) in Geodesy department of Ss.Cyril and Methodius University–Skopje. He worked in State Authority for Geodetic Works from May 1999 until January 2008. During those period, in 2004 he appointed for head of cartography department, i.e. geodetic works. From October 2003 up to January 2008, he worked as an outsourcing lecturer in State University of Tetova. From February 2008, he works as a cartography and GIS Professor at the State University of Tetova–Tetova. He continuo with working as outsourcing lecturer in geodesy department of the University of Prishtina-Kosova. He is the author of three cartography university books, and more than 90 papers published and presented in national and international scientific conferences related to geodesy, cartography, GIS and remote sensing. From March 2010, he is appointed as president of Geo-SEE Institute (South-European Research Institute on Geo Sciences). From November 2011 to February 2014, he was first President of Pan-national Association of Albanian Surveyors “ Karl Gega”.

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