Analysis of GPS Coordinates Time Series by Kalman Filter

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Key words: Times series, Kalman filter, identity model, kinematic model, deformation analysis.

SUMMARY

In this paper we try to process time series of position coordinates using Kalman filter and Kalman smoother and to predict the position coordinates in intervals that contain erroneous data. Coordinates are originally obtained by relative GPS in kinematic way. The Kalman filter which can be implemented in real time, utilizes for computing the current estimate the precedent measurements and current measurement together with their variances. In turn, the Kalman smoother, also known as the RTS (Rauch-Tung-Striebel) smoother gives better estimates than Kalman filter since it exploits all data, i.e., after the total measurements have been done, then, it can be used only in post-processing.

Due to the unknown or badly known internal geophysical effects, a purely dynamic model of deformation that involves the actual causative forces is then very difficult to establish with sufficient accuracy. Consequently, in this study, two descriptive models of deformation are chosen to play the role of two different dynamic systems: first, the identity model whose state vector contains position, second, the kinematic model whose state vector contains position and velocity. After results analysis, we conclude that the identity model is more suitable one than the other, because it describes better the behavior of motion of the receiver antenna.

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1. INTRODUCTION

Until now, Kalman filter still an appropriate tool for analyzing time series of position when the deformations are modeled as a linear dynamic system. Kalman filter gives the best estimate. By definition this algorithm uses uncertain dynamic system that describes the dynamic behavior of the moving object and the measurement model. Both models should be disturbed by zero mean white noise. If the noise is white and Gaussian the obtained estimate is qualified as *optimal*. Two dynamic systems are chosen to describe the same dynamic of the deformation: first, the identity model when position follows a Random Walk Process (RWP) and second, the kinematic model when velocity follows a RWP.

The present paper is organized by sections. Section two, briefly presents the rules that allow the operation of discretizing i.e. the passage from continuous-time to discrete-time dynamic system. Section three develops the identity and kinematic model as two dynamic systems that will be implemented and the corresponding measurement models in discrete-time. Section five gives a review of the algorithms of the different tasks that the Kalman filter is capable to achieve and discusses about their properties. These tasks are: filtering, prediction and smoothing. In section five, we describe the experiment we made to create the GPS coordinates of the moving receiver. Section six gives the results and analysis of time series processed by the precedent algorithms.

2. DISCRETIZING STATE SPACE MODEL

This section gives the rules that allow passage from linear dynamic system in continuous-time to a linear dynamic system in discrete-time. In our study, the implementation of Kalman filter is started from the knowledge of a continuous-time state space model

$$\dot{X} = F(t)X(t) + G(t)w(t) \quad (1)$$

$$\tilde{Z} = H(t)X(t) + \varepsilon(t) \quad (2)$$

The equation (1) and (2) are called the dynamic and measurement system respectively.

In discrete-time, the equivalent state space model are written as

$$X_k = \Phi_{k-1}X_{k-1} + \Gamma_{k-1}w_{k-1} \quad (3)$$

$$\tilde{Z}_k = H_k X_k + \varepsilon_k \quad (4)$$

Where, X_k : State vector at time t_k containing n parameters to estimate. Φ_{k-1} : Transition matrix. Γ_{k-1} : Coupling matrix. w_{k-1}, ε_k : Process noise and measurement noise respectively.

$\tilde{Z}_k$: Vector of m measurements. H_k: Observation matrix.

The transition matrix Φ_{k-1} can be obtained according to the expression [Demailly 2006]

$$\Phi_{k-1} = e^{F.\Delta t_k} \quad (6)$$

$\Delta t_k = t_k - t_{k-1}$: The sample time.

The following two integrals help to compute the coupling matrix Γ_{k-1} and the process covariance matrix Qd_{k-1} in discrete-time

$$\Gamma_{k-1} = \int_{\tau=0}^{\tau=\Delta t_k} e^{F(\Delta t_k - \tau)} G(\tau)\, d\tau \quad (7)$$

$$Qd_{k-1} = cov(\Gamma_{k-1} w_{k-1}) = \int_{t_{k-1}}^{t_k} e^{F(\Delta t_k - \tau)}\, G(\tau) Q_w(\tau) G^T(\tau) e^{F^T(\Delta t_k - \tau)} d\tau \quad (8)$$

Q_w in equation (8) denotes the diagonal matrix with the PSD (Power Spectral Densities) of process noises in continuous-time [Farrell 2008], [Grewal et al. 2001].

The dynamic system is considered observable if the observability matrix $\mathcal{O}$ expressed as

$$\mathcal{O}^T = [H^T \quad \Phi^T H^T \quad \cdots \quad (\Phi^T)^{(n-1)} H^T] \quad (9)$$

has rank n [Grewal et al. 2001].

3. DESCRIPTIVE MODELS OF DEFORMATION

- **Identity model**

In this case, the generic coordinate ξ follows a RWP. The velocity is driven by zero mean, white process noise w, satisfying the equation: $\dot{\xi} = w(t)$. This scalar expression leads to the following linear continuous-time system

$$\dot{X} = w \quad (10)$$

and in discrete-time, equation (10) becomes

$$X_k = I_3\, X_{k-1} + I_3\, \Delta t\, w_{k-1} \quad (11)$$

Where, $X_k = [E \;\; N \;\; U\;]_k^T$: State contains displacements according to East, North, Up axis. $w_{k-1} = [w_E \;\; w_N \;\; w_U]_{k-1}^T$: Noise vector in East, North, up direction. I_3 : Identity matrix (3×3).

We can use in computing the covariance matrix Qd the following expression

$$Qd = cov(\Gamma w) = Q_w . \Delta t = \begin{bmatrix} \sigma_{\overline{w}_E}^2 & 0 & 0 \\ 0 & \sigma_{\overline{w}_N}^2 & 0 \\ 0 & 0 & \sigma_{\overline{w}_U}^2 \end{bmatrix} \quad (12)$$

with the vector noise $\overline{w} \equiv \Gamma w$.

In our case, we don't have Q_w, however the matrix Qd can be directly computed according to the rule stated in [Kuhlmann 2003] : "system noise should be of the same magnitude as the deformation between two epochs". During the observation session (see section 5), the sample time was fixed to 1 s, the antenna has been displaced horizontally along the line of displacement (as depicted in the Figure 2) every half hour in steps of 10 cm, the position change took about 10 s in each displacement. Regarding the Figure 2, the standard deviations can be expressed as: $\sigma_{\overline{w}_E} = 1 sin\alpha$ (cm), $\sigma_{\overline{w}_N} = 1\; cos\alpha$ (cm), $\sigma_{\overline{w}_U} = 0$ (cm), with: α =62°.

The measurement system can be modeled as

$$\tilde{Z}_k = I_3.X_k + \varepsilon_k \quad (13)$$

This equation matches the equation (4).

The state X_k is observable for all epochs because the observability matrix O given by equation (9) is of full rank.

- **Kinematic model**

In practice, the velocity undergoes at least slight changes. This can be modeled by a continuous-time zero-mean white noise $w(t)$ [Bar-Shalom et al. 2001]

$$\ddot{\xi} = w(t) \quad (14)$$

Where: $\ddot{\xi}$ is the second derivative of position (or acceleration).

The state space representation in continuous-time of equation (14) is given by

$$\begin{cases} \dot{\xi} = v \\ \dot{v} = w(t) \end{cases} \quad (15)$$

In discrete-time, equation (15) becomes for three-dimensional deformations

$$X_k = \begin{bmatrix} I_3 & I_3\Delta t \\ 0_3 & I_3 \end{bmatrix} X_{k-1} + \begin{bmatrix} I_3\,\Delta t^2/2 \\ I_3\Delta t \end{bmatrix} w_{k-1} \quad (16)$$

Where in equation (16): $X_k = [E \;\; N \;\; U \;\; v_E \;\; v_N \;\; v_U]_k^T$, $w_{k-1} = [w_E \;\; w_N \;\; w_U]_{k-1}^T$.

v_E, v_N, v_U : velocities in East, North, Up direction. 0_3 : Null matrix of size (3×3).

Equation (16) is of the form of equation (3).

$$Qd = \begin{bmatrix} Q_w\,\Delta t^3/3 & Q_w\,\Delta t^2/2 \\ Q_w\,\Delta t^2/2 & Q_w\Delta t \end{bmatrix} \quad (17)$$

with:

$$Q_w\Delta t = \begin{bmatrix} \sigma_{\overline{w}_E}^2 & 0 & 0 \\ 0 & \sigma_{\overline{w}_N}^2 & 0 \\ 0 & 0 & \sigma_{\overline{w}_U}^2 \end{bmatrix} \quad (18)$$

The process noise intensity is chosen according to [Bar-Shalom et al. 2001] in which it is stated that: “The changes in the velocity (Δv_i) over a sampling time Δt are of the order of $\sqrt{[Q_w]_{ii}\Delta t}$. A nearly constant velocity model is obtained by the choice of a small intensity in the following sense: The changes in the velocity have to be small compared to the actual velocity”. Then for our experiment, it can be written

$\sigma_{\overline{w}_E} = \Delta v_E$= 0.3 $sin\alpha$ (cm/s), $\sigma_{\overline{w}_N} = \Delta v_N$= 0.3 $cos\alpha$ (cm/s), $\sigma_{\overline{w}_U} = \Delta v_U$=$10^{-8}$ (cm/s).

After that the matrix $Q_w\Delta t$ has been constructed then we can easily determine the process covariance matrix Qd by substituting $Q_w\Delta t$ in the blocs of Qd expressed by equation (17).

The measurement model is described by

$$\tilde{Z}_k = [I_3 \quad 0_{3,3}]X_k + \varepsilon_k \quad (19)$$

$$R_k = cov(\varepsilon_k) = \begin{bmatrix} \sigma^2_{\varepsilon_E} & 0 & 0 \\ 0 & \sigma^2_{\varepsilon_N} & 0 \\ 0 & 0 & \sigma^2_{\varepsilon_U} \end{bmatrix}_k \quad (20)$$

R_k : Measurement covariance matrix. Although the assumption of white measurement noise (ε_k) is not fulfilled (since in reality ε_k is correlated with time due to slow change of multipath and signal propagation at neighboring epochs in case of GPS [Kuhlmann 2003]), for simplification we consider here that the noise ε_k is zero-mean and white.

Because, in practice, the values of correlations between observations are not available within the data, we assume them nulls. Therefore, the covariance matrix R_k is diagonal. Similarly to that of identity model, the state parameters are observable since the observability matrix $\mathcal{O}$ is of full column rank.

4. FILTERING, PREDICTION AND SMOOTHING

Kalman filter is a state estimator with a minimum mean-squared error [Ribeiro 2004]. It is possible to using Kalman filter either in filtering, prediction or in smoothing. In this section we give the corresponding algorithms.

- **Filtering:** The classical Kalman filter was first established by Rudolph E. Kalman in his influential paper [Kalman 1960]. The filtered estimate of X_k only takes into account the past information relative to X_k [Pieter 2008]. The Kalman filter is applicable in real time. The algorithm is divided in two steps: prediction step and measurement update step.

- *Initial conditions:* $\hat{X}_{0/0}, P_0$

- *Prediction:*

$$\hat{X}_{k/k-1} = \Phi \hat{X}_{k-1/k-1} \quad (21a)$$

$$P_{k/k-1} = \Phi P_{k-1/k-1} \Phi^{\mathrm{T}} + Qd \quad (21b)$$

- *Measurement update:*

$$\begin{gathered} K_k = P_{k/k-1} H^T [H P_{k/k-1} H^T + R_k]^{-1} \\ \hat{X}_{k/k} = \hat{X}_{k/k-1} + K_k [\tilde{Z}_k - H \hat{X}_{k/k-1}] \\ P_{k/k} = [I - K_k H] P_{k/k-1} \end{gathered} \quad (22)$$

Where $\hat{X}_{0/0}, P_{0/0}$ *:* State vector at time $t = 0$ and its corresponding initial covariance matrix.
$\hat{X}_{k/k-1}$, $P_{k/k-1}$: Predicted estimate and its covariance matrix. K_k : Kalman gain,
$\hat{X}_{k/k}, P_{k/k}$: Updated estimate and its covariance matrix.

- **Prediction**: In general, when a measurement is missing or qualified as faulty or erroneous the corresponding update measurement step in the precedent algorithm must not occur [Farrell 2008]. In our case, erroneous data are present in some epochs then it is possible to predict states at such epochs (using the time propagation step only).

- **Smoothing:** The smoother is called RTS since its implementation was derived by H. Rauch, K. Tung and C. Striebel in 1965 [Grewal 2001]. By incorporating the past and future observations relative to X_k, we can obtain a more refined state estimate [Abeel 2008]. For this effect, smoothing can be done only in post-processing. The smoothed state $\hat{X}_{k/N}$ (N is the total number of epochs) and its covariance matrix $P_{k/N}$ are calculated with the following equations [Bar-Shalom 2001], [Hartikainen 2007]:

$$\begin{aligned} \hat{X}_{k+1/k} &= \Phi \hat{X}_{k/k} \\ P_{k+1/k} &= \Phi P_{k/k} \Phi^{\mathrm{T}} + Qd \\ C_k &= P_k \Phi^T [P_{k+1/k}]^{-1} \qquad (23) \\ \hat{X}_{k/N} &= \hat{X}_{k/k} + C_k [\hat{X}_{k+1/N} - \hat{X}_{k+1/k}] \\ P_{k/N} &= P_{k/k} + C_k [P_{k+1/N} - P_{k+1/k}] C_k^T \end{aligned}$$

with C_k : The smoothed gain on time step k.

The difference between Kalman filter and Kalman smoother is that the recursion in the filter moves forward and in smoother backward. In smoother the recursion starts from the last time N with $\hat{X}_{N/N}$ and $P_{N/N}$ obtained by the filter.

5. DATA AQUISITION

On 13th May 2014 at 11:00, an experiment of GPS observations in kinematic mode was conducted, on the roof of the Space Techniques Centre (CTS) building, (Figure 1). A short baseline of 45 m was observed by two receivers of type ZMAX with a sample time Δt =1 s and the measurement duration of more than 9 h. The receiver of the station T01 placed on the ground of the roof is moved manually in the horizontal plan by a distance of 10 cm every half hour as shown in the Figure 2. Moving the receiver took about 10 s. The time series obtained after processing by commercial software (WINPRISM) consist of local geodetic coordinates of the station T01.

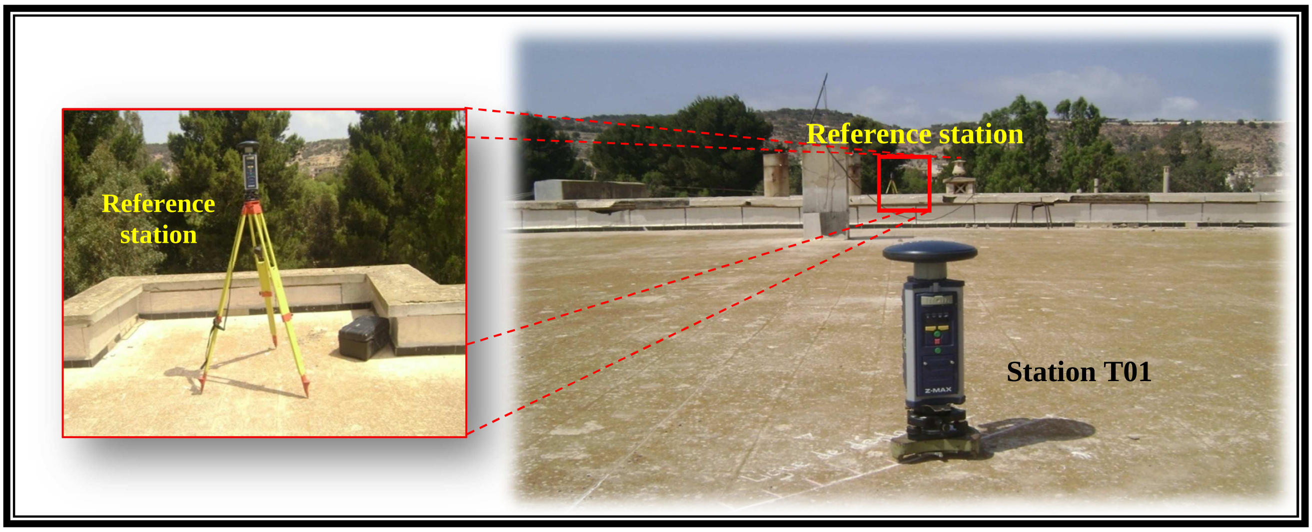

Fig 1: GPS observations performed on the roof of the CTS Centre building.

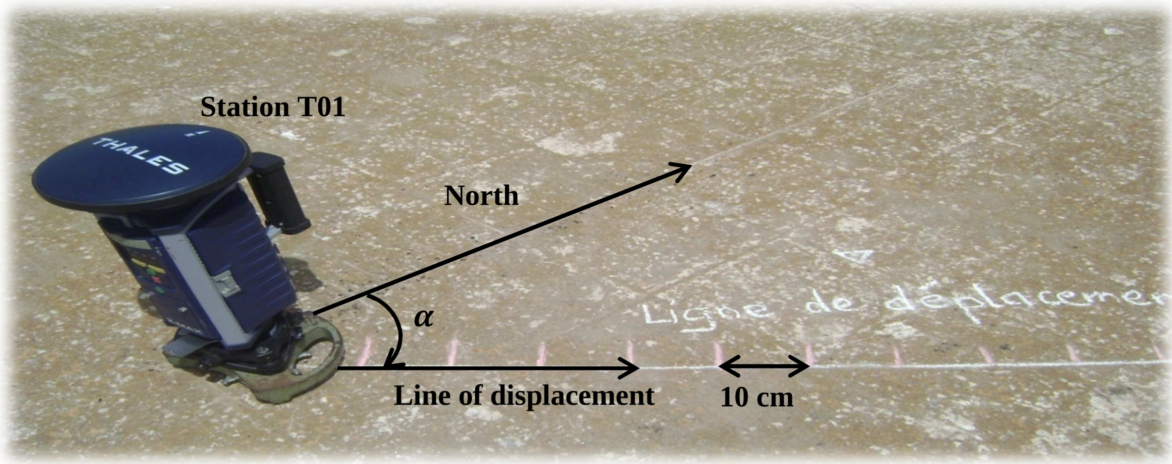

Fig 2: Displacement line of the receiver.

6. RESULTS AND ANALYSIS

The Figure 3 illustrates the time series of the position residuals of station T01 according to local components (E, N, U). We can see that the planimetric components (E, N) increase gradually every 30 mn while the vertical component remains relatively fixed in time as it happened in the experiment. In the same figure, some outliers (inside ellipses) occur in different times. On the figure, the outliers surrounded by black ellipse can reach up to 15 cm and are probably due to the initialization phase. The outliers of green ellipses attain 33 cm and are possibly caused by multipath since the corresponding value of PDOP in the Figure 4 is about 4 indicating a relatively good geometry while for red ellipse the anomalies are due to a bad geometry (PDOP>6).

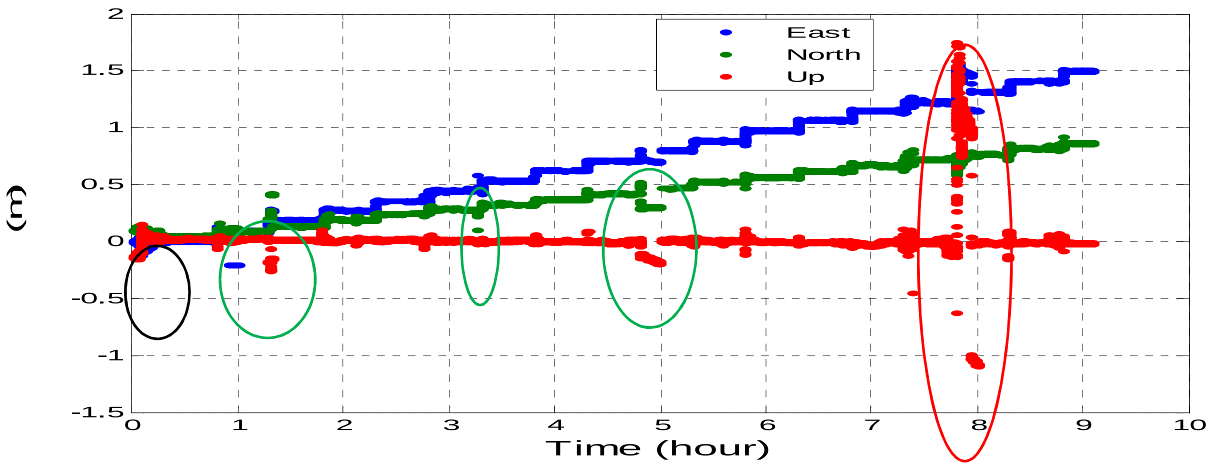

Fig 3: Time series of the coordinates according to East, North and Up.

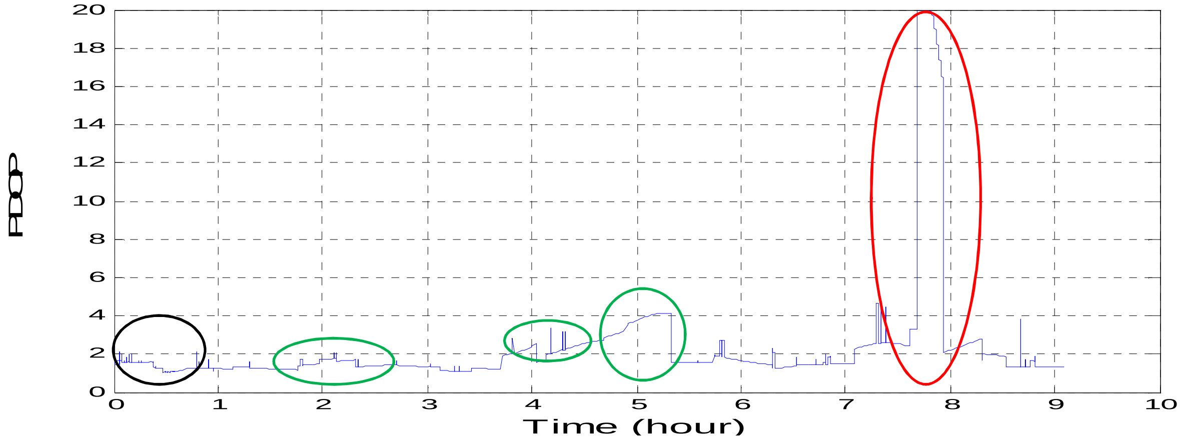

Fig 4: PDOP of the observations.

Firstly, the time lag: (4.5 to 6 h) is divided into three parts: (4.5 to 4.8 h), (4.8 to 5 h) and (5 to 6 h). In the second part of 12 mn, only the predicted estimates were computed instead of filtered and smoothed estimates since the outliers possibly caused by multipath occur inside this portion (See Figures 5 and 6). Inversely, in the first and third part we account only for filtered and smoothed estimates.

We can see that the predicted coordinates in the Figure 5a, 5b, 5c relative to the identity model remain constant over time. On the other hand, in the Figure 6a, 6b, 6c the predicted coordinates by kinematic model follow a linear form. This is because in case of prediction we assume that the dynamic system in continuous-time is not noisy: $\dot{\xi}$ =0 and $\ddot{\xi}$ =0 and thus the generic coordinate after integration becomes ξ =constant for the first model and $\xi = c_1 t + c_2$ (equation of line) for the second model. The predicted coordinate can have large difference (38 cm in Figure 6b) from the true (simulated) deformation as the time of absence of measurements becomes significant (12 mn).

Because the anomalies occurred in the observed times series the Gaussian distribution of the observations is not fulfilled, thus the estimates obtained by the Kalman filter and Kalman smoother can be qualified as *best* and not *optimal*.

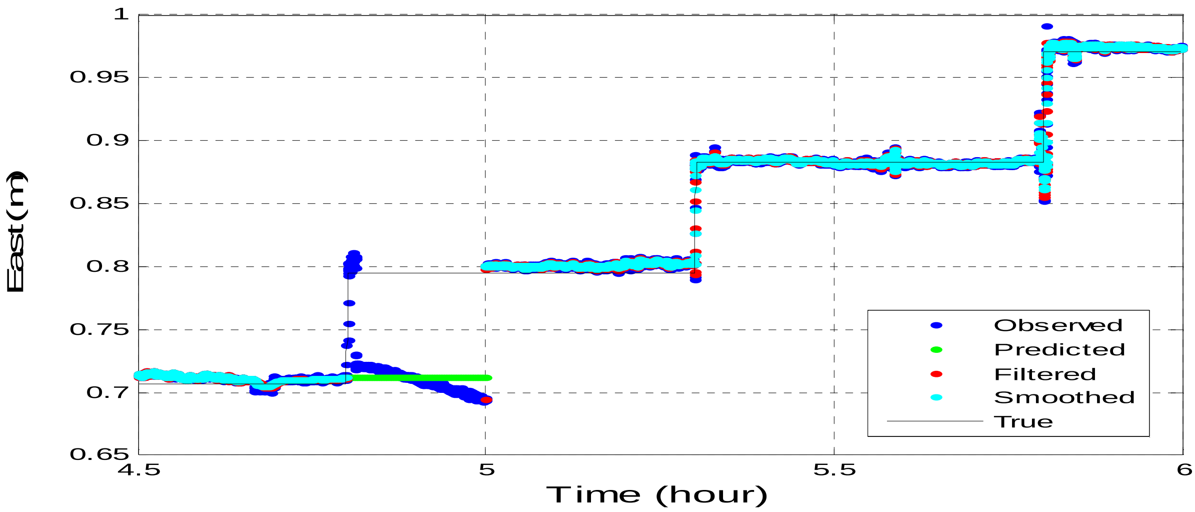

Fig 5a: Observed and processed East coordinates by identity model (Time lag: 4.5 to 6 h).

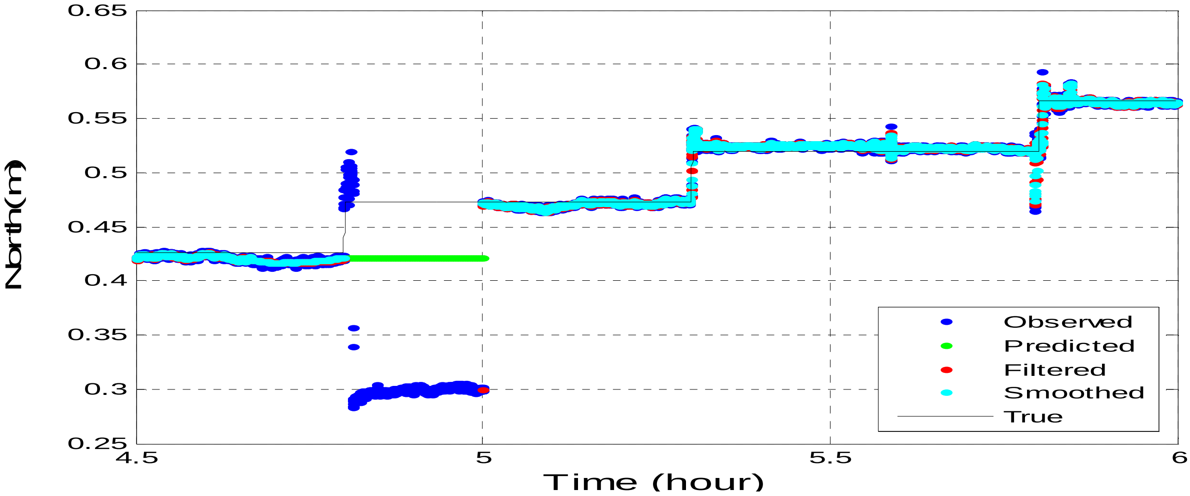

Fig 5b: Observed and processed North coordinates by identity model (Time lag: 4.5 to 6 h).

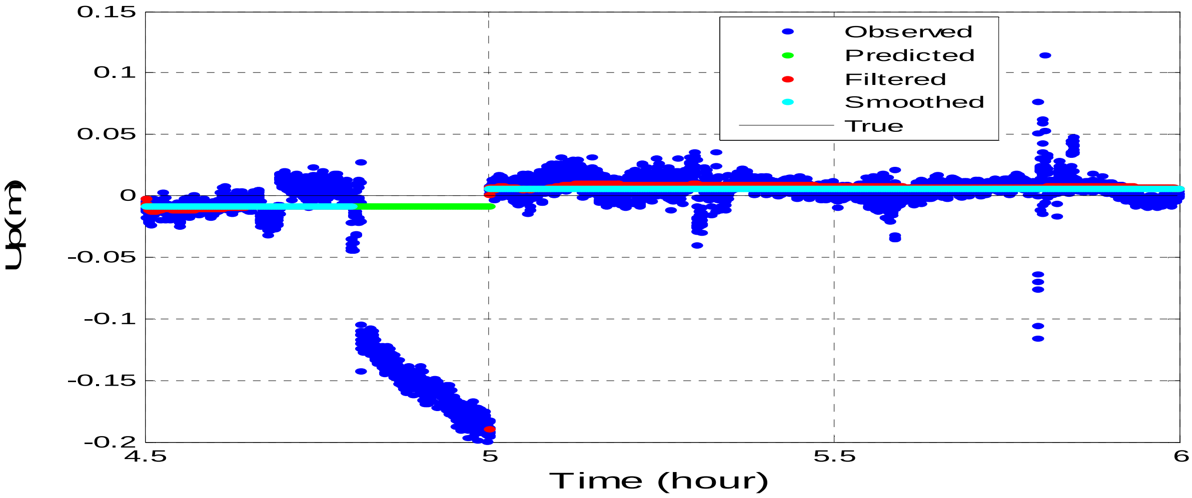

Fig 5c: Observed and processed Up coordinates by identity model (Time lag: 4.5 to 6 h).

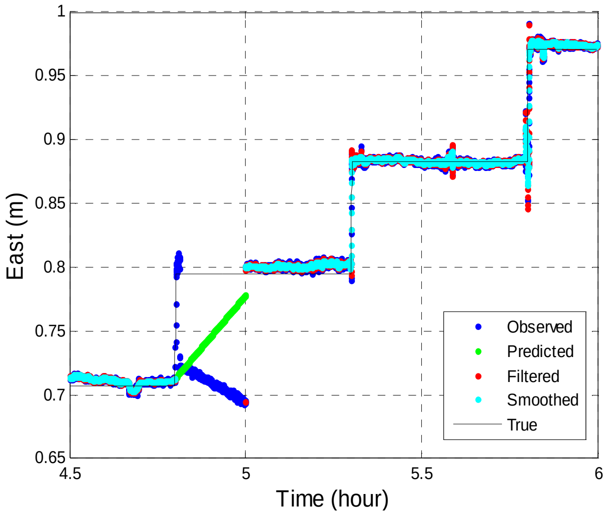

Fig 6a: Processed East coordinate using Kinematic model (Time lag: 4.5 to 6 h).

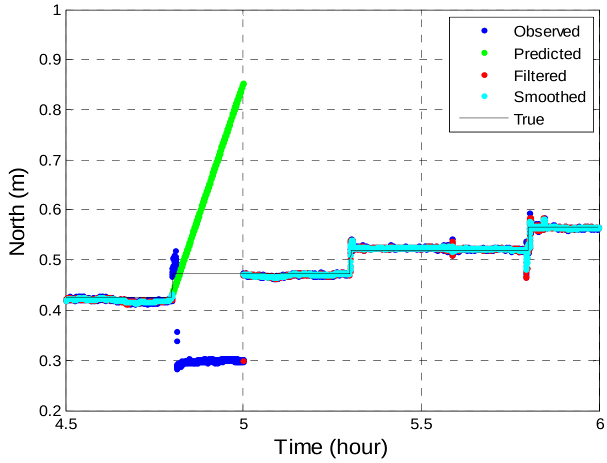

Fig 6b: Processed North coordinates using Kinematic model (Time lag: 4.5 to 6 h).

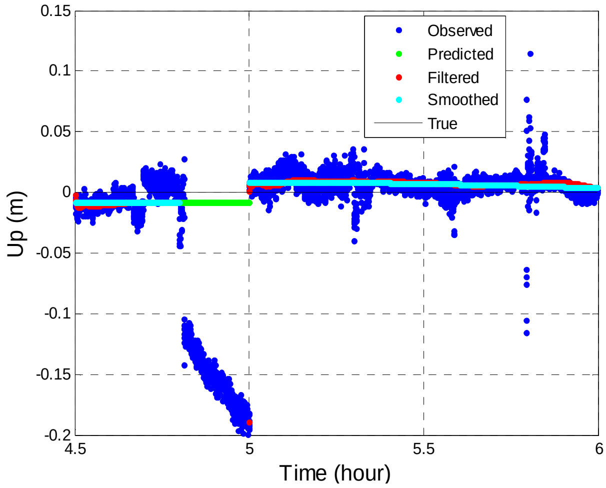

Fig 6c: Processed Up coordinate using Kinematic model (Time lag: 4.5 to 6 h).

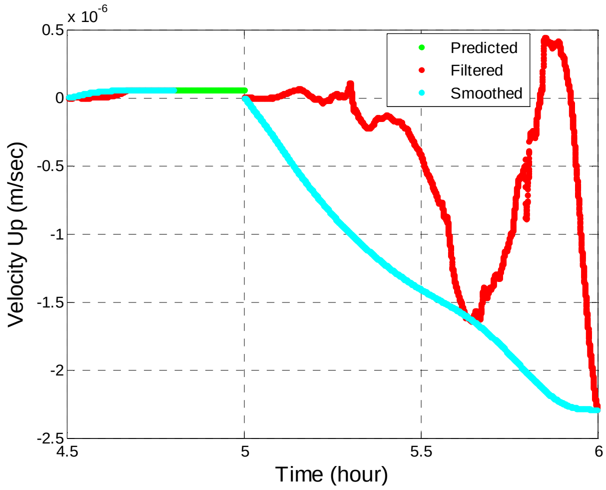

Fig 6f: Processed Up velocity using Kinematic model (Time lag: 4.5 to 6).

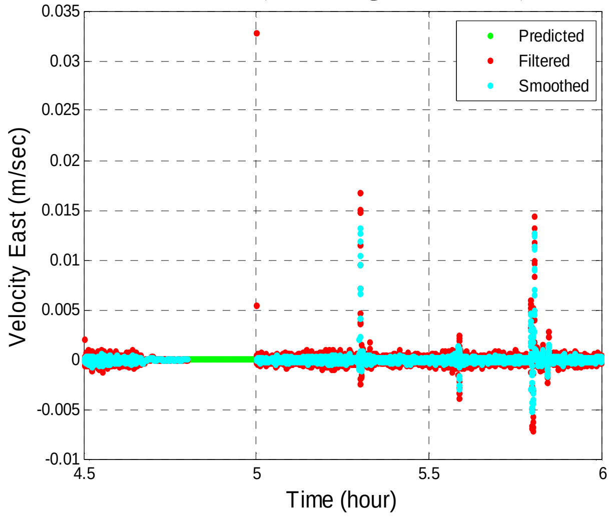

Fig 6d: Processed East velocity using Kinematic model (Time lag: 4.5 to 6 h).

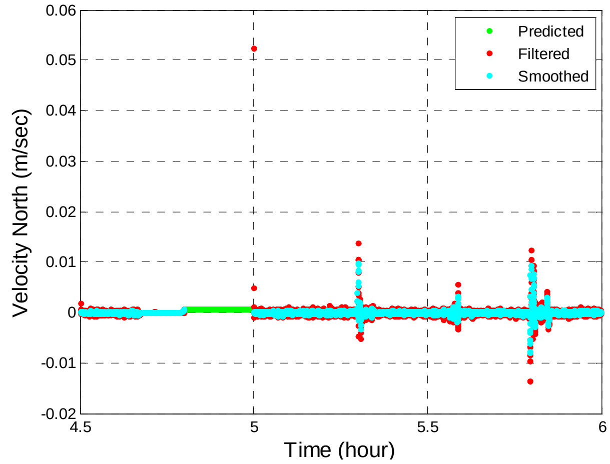

Fig 6e: Processed North velocity using Kinematic model (Time lag: 4.5 to 6 h).

Average of standard deviations	Observed	Identity model				Kinematic model			
		Filtered	Imp (%)	Smoothed	Imp (%)	Filtered	Imp (%)	Smoothed	Imp (%)
$\bar{\sigma}_E$ (mm)	30.2	10.45	65.4	8.31	72.5	14.08	53.4	7.80	74.2
$\bar{\sigma}_N$ (mm)	31.6	6.73	78.7	5.29	83.2	11.39	74	6.14	80.6
$\bar{\sigma}_U$ (mm)	74.8	0.65	99.1	0.28	99.6	0.73	99	0.34	99.5

Table 1: Averages of standard deviations of observed and processed times series.

Imp in the Table above means improvement of the processed (filtered or smoothed) times series relative to the observed time series.The statistics shown in the Table take into account only the results of the first and third part (second part contains predicted time series).

There are some remarks in the Table to exhibit:

First, it is clear from this Table that the filtered and smoothed times series are of better quality in term of standard deviation than that of the observed time series. The improvement of accuracy varies from 65.4 to 83.2% for identity model and from 53.4 to 80.6% for kinematic model in the horizontal plan and about 99% for the Up-component processed time series.

Second, as it has been expected, smoothing gives more accurate coordinates than the filtered coordinates for both models. This is caused probably by the absence of acceleration during the motion of the antenna receiver, furthermore, the antenna receiver remains fixed during 30 mn before each displacement.

Third, the identity model seems to perform better than the kinematic model. Effectively, for North and Up component, the corresponding averages of standards deviations of identity model are less than those of kinematic model except for the smoothed East coordinate for kinematic model where the improvement (74.2%) is greater than that of smoothed East coordinate for identity model (72.5%).

Fourth, the standard deviation average for the Up component is improved to a great extent (99% to 99.6%) that is because the Up component is invariant over time and therefore it is well predicted by the two dynamic systems.

Fifth, although the observed East component (30.2 mm) is slightly more accurate than the observed North component (31.6 mm), the processed East component is less accurate than the processed North one. This is due to the azimuth angle α =62° which makes the most displacement of the receiver happen on the East axis and thus the noise (See section 3 and how the process noise magnitude is chosen).

Sixth, in the Figure 5a and 5b at t=5.8 h, filtered estimates appear affected -at some degree- by erronous observations, i.e. filtering cannot neutralize their effects, on the other hand smoothing is less sensitive to these anomalies.

7. CONCLUSION

Throw the present paper, the kalman filter-based coordinate time series analysis was evoqued. The experiment concerned the obseravtion of short baseline by kinematic positioning of two GPS recveivers with 1s sample time and during 9 hours. Two descriptive models -identity model and kinematic model- are adopted to decsribe the motion behaviour of the receiver antenna. The Kalman filter was used to analyse the position time series, exploiting both dynamic systems. Three tasks of Kalman filter have been implemented: filtering, prediction and smoothing.

As relevent results of our application:

- The identity model seems to be the more adaquete model for describing the motion of the receiver than the kinematic one.

- The prediction precision decreases as the time of measurement lack increases.

- Compared to the filtering, smoothing provides more accurate solution of about (72.5%, 83.2%, 99.6%) and (74.2%, 80.6%, 99.5%), acoording to local coordinates (E, N, U), for identity and kinematic models, respectively.

- Filtering is more sensitive to the presence of outliers than smoothing.

As perspectives,

- The correlations between the three observed time series should be investigated and taken into account in the construction of the measurement covariance matrix.

- The GPS coordinate time series are often correlated with time, so the white noise assumption of the noise is not justified, consequently a shaping filter should be appended to the dynamic system to reduce this correlation.

- Establishing a mechanism that enable -at a certain level- detection of anomalies caused by bad geometry or some error sources inherent in relative GPS observations.

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BIOGRAPHICAL NOTES

Bachir GOURINE received an Engineer Diploma in *Geodesy* in 1994 at the National Centre of Spatial Techniques (CNTS, Arzew/Algeria) and obtained his Magister degree in *Space Techniques and Applications* at the Centre of Spatial Techniques (CTS), in 2004. During 2004, he is a permanent researcher at the Space Geodesy Division of CTS. On January 2011, he is PhD graduated in *Telecommunication*, Faculty of Electrical Genius, Department of Electronics - University Sciences and Technology Mohamed Boudiaf (USTO, Oran/Algeria).
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