Comparison of Normal-orthometric Height and Helmert orthometric Height in Korea

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Key words: Normal-Orthometric height, Helmert orthometric height, Leveling

SUMMARY

The normal-orthometric height has been published and used in Korea. The normal-orthometric height adds orthometric correction value, which is derived from ellipsoid, to the leveling data. Since the effect of gravity was not considered from actual gravity data, normal-orthometric height would be different from the orthometric height. Especially at the mountain or steep area where equipotential surface changes drastically, the difference between the two heights is significantly large.
In the past in Korea, gravity surveying had not been done with leveling, normal-orthometric height has been replaced the orthometric height. However, many gravity data were obtained when KNGeoid (Korea National Geoid model) was developed. Therefore, it is possible to consider the effect of gravity from actual gravity data. In this study, with leveling and gravity data, the comparison of normal-orthometric height and orthometric height was conducted. Orthometric height was calculated as Helmert orthometric height, which is using prey reduction. As a result, the difference between normal-orthometric height and Helmert orthometric height was distributed uniformly over the flat land and was larger in the mountainous area.

SUMMARY (optional summary in one other language in addition to English, e.g. your own language)

현재까지 대한민국에서 사용되고 있는 정규정표고는 직접수준측량 된 높이 차이에 정규중력으로부터 계산된 보정량을 적용한 높이이다. 이는 실제 중력값이 아닌 타원체로부터 계산된 정규중력을 사용하기 때문에 실제 연직선과는 다를 수 있다. 특히 산악지나 급경사지에서는 등 포텐셜면의 변화가 급격해 지기 때문에, 정규정표고와 실제 연직선 사이의 차이값이 커지게 될 수 있다. 이러한 현상은 산악지가 많은 대한민국에서 높이의 일관성을 떨어뜨릴 수 있다.
과거 대한민국에서는 직접수준측량과 함께 중력측량이 수행되지 않아 정표고대신 정규정표고를 사용해 왔으나, KNGeoid 구축 당시 획득된 중력자료와 수준망 조정 시 획득된 직접수준측량 자료를 활용하여 실제 중력의 영향을 고려 할 수 있게 되었다. 따라서 본 연구에서는, 획득된 중력자료와 직접수준측량 자료를 활용하여 정표고를 계산하고 기존의 정규정표고와의 비교 분석을 실시하였다.
그 결과 두 표고의 차이는 평지에서는 일정한 반면에, 산악지로 갈 수록 그 차이가 더 커지는 것을 알 수 있었다.

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1. INTRODUCTION

Normal-orthometric height, which is calculated from spirit leveling data and normal gravity, has been used in Korea. It might be different from the length of the plumb line at the mountain or steep area because the correction is not from real gravity data but from reference ellipsoid. In Korea**,** since the mountain area constitutes more than 70% of their whole area, this may cause significant height error (Lee et al., 2010). It is, therefore, necessary to consider the orthometric height, which is calculated from the leveling and real gravity data (Choi and Lee, 1997). Since many gravity data had been obtained when built KNGeoid (NGII, 2014), it is possible to calculate the orthometric height. In this study, comparison of normal-orthometric height and orthometric height is conducted with leveling and gravity data. In order to calculate the orthometric height, approximation formula from prey reduction was used, and this height is Helmert orthometric height.

2. METHODS

The principle of spirit leveling is measuring the height difference between two points with level and staff, but it has no physical meaning when performed solely. Since the equipotential surface is not always parallel due to the irregular distribution of material inside the earth (Meyer et al., 2006), the spirit leveling has path-dependent.

2.1 Normal-orthometric height

Normal-orthometric, which adds an orthometric correction to the measured height difference, is being used in Korea. This correction is derived from normal gravity that is calculated on the basis of the reference ellipsoid (Filmer et al., 2010). The process to calculate the normal-orthometric height shows as follows:

1. Obtain height difference through leveling.
2. Calculated the orthometric correction value, which is from the normal gravity.
3. Add the correction value to each bench marks.
4. Adjusting network to each leveling routes.
5. Calculate the normal-orthometric height.

2.2 Helmert orthometric height

In reality, the leveling loop closure generally cannot be zero because equipotential surfaces are not always parallel. When conducting the leveling in the wide area or highland, the size of misclosure grows larger. Therefore, the length of physical plumb line and sum of height differences are not same. To overcome this problem, gravity surveying should be conducted with leveling to calculate the geopotential number. Geopotential number is calculated by equation (1):

$$\int_{0}^{A} \boldsymbol{g}\, \boldsymbol{dn} = \boldsymbol{W_0} - \boldsymbol{W_A} = \mathbf{C} \quad (1)$$

where $\boldsymbol{g}$ is the gravity value, $\boldsymbol{dn}$ is height difference, $\boldsymbol{W_0}$ is potential at the geoid, $\boldsymbol{W_A}$ is potential at the surface and C is geopotential number.
Since this geopotential number C is independent on the path of integration, one great advantage of geopotential number is that it can provide a unique height value. Various heights are calculated from this geopotential number (e.g., orthometric, normal and dynamic height).

Orthometric height, which is derived from geopotential number, means the length of the plumb line from geoid to earth's surface. In order to calculate this height, Geopotential number is divided by mean gravity value along the plumb line (equation (2)).

$$\boldsymbol{H} = \frac{\boldsymbol{C}}{\overline{\boldsymbol{g}}} \quad (2)$$

where H is the orthometric height, C is Geopotential number, $\bar{g}$ is the mean gravity value calculated by integrating the gravity along the plumb line ($\overline{\boldsymbol{g}} = \frac{\mathbf{1}}{\boldsymbol{H}}\int_{\mathbf{0}}^{\boldsymbol{H}} \boldsymbol{g}\, \boldsymbol{dH}$).
However, since $\bar{g}$ locate in the underground, it could not be computed exactly (Jekeli, 2000). Instead, in practical computation, calculate $\bar{g}$ from gravity value at the earth's surface through prey reduction, and it is defined by equation (3):

$$\overline{\boldsymbol{g}}_{\boldsymbol{P}}^{\boldsymbol{Prey}} = \boldsymbol{g_P} + (\mathbf{0.0424}\boldsymbol{mgal/m})\boldsymbol{H} \quad (3)$$

where $\bar{g}_P^{Prey}$ is the mean value of gravity using prey reduction and g_P is observed gravity value at the earth's surface.

When C is divided by $\bar{g}_P^{Prey}$, it is called Helmert orthometric (Hofmann-Wellenhof and Moritz, 2006). The process to calculate the Helmert orthometric height shows as follows:

1. Obtain height difference and gravity data through surveying.
2. Calculate the geopotential number.
3. Adjusting network to make unique geopotential number at the bench mark.
4. Calculate the mean value of the gravity along the plumb line using prey reduction.
5. Calculate the Helmert orthometric height at each bench marks.

In this study, there are two different Helmert orthometric height depend on gravity data. Firstly, using actual gravity data, calculate Helmert-orthometric height. Secondly, using gravity value from the map of the gravity anomaly, calculate Helmert-orthometric height (anomaly). Helmert-orthometric height (anomaly) was calculated in order to look into whether the map of the currently constructed gravity anomaly map is sufficient for the area where gravity surveying were not conducted. The gravity anomaly data was extracted from gridded gravity anomaly of bench marks which is used in ths study.

3. STUDY AREA AND RESULT

3.1 Study area

Since there are various topographical characteristics, The third, fourth, and fifth vertical circuit were selected as the study area among the Korean vertical circuits. This study area consists of 12 route and 356 leveling data. In addition, to look into the effect of terrain, mountain area and flatland are chosen in the study area. Standard deviation of leveling data for each route was calculated that make up the three circuits. The largest standard deviation area, the 19th route was chosen as mountain area. The smallest standard deviation area, the 11th route was chosen as flatland. The study area is shown in **Figure 1.** Leveling data was obtained when adjusting the vertical network at 2006 (NGII, 2006). Gravity on the each bench marks that are published by NGII (National Geographic Information Institute) is used in this study.

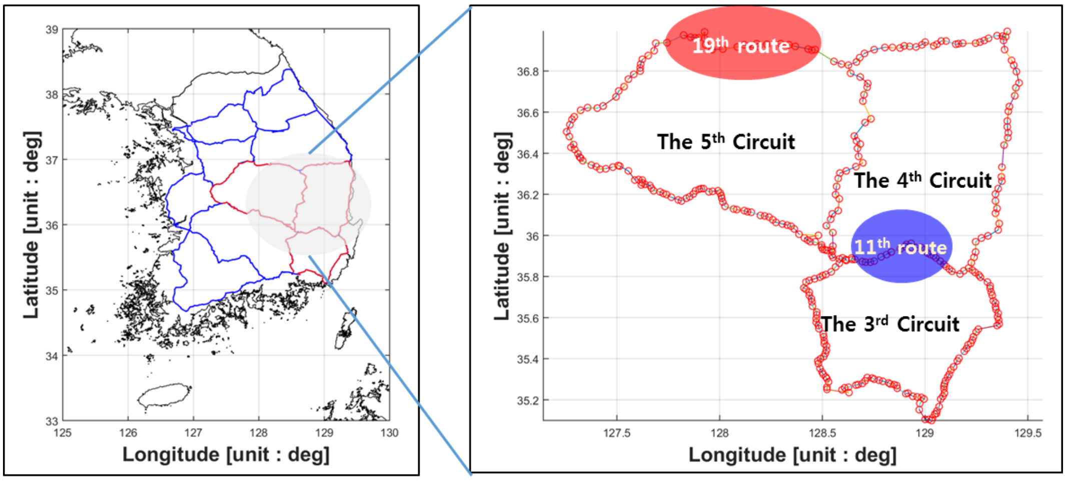


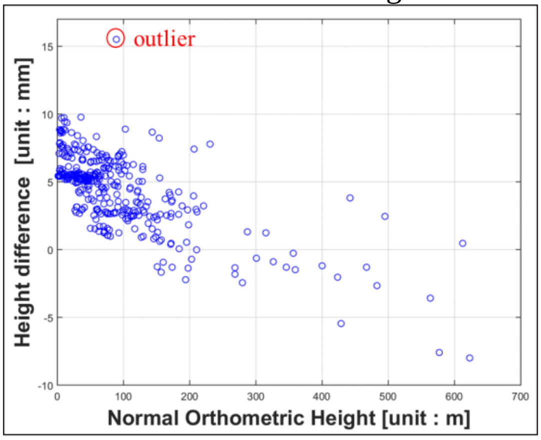
Figure 1. Korea vertical network and study area

3.2 Result

<u>3.2.1 Comparison of Normal-orthometric height and Helmert orthometric height</u>

Since normal-orthometric height has been used in Korea, normal-orthometric height is considered as reference height when comparing the normal-orthometric height and orthometric height.

Figure 2 shows the height difference between normal-orthometric height and Helmert orthometric height. The result shows that there is bias about 4mm and the maximum difference is 9.76mm between two heights except one outlier. It is considered that there is a negative correlation between normal-orthometric height and difference of two heights. When normal-orthometric height is increased, the difference between two heights becomes larger. These effects are greatly affected by gravity. At the highland, the variation of equipotential surfaces became greater. As a result, Helmert orthometric height is larger than the normal-orthometric height at the mountain area.

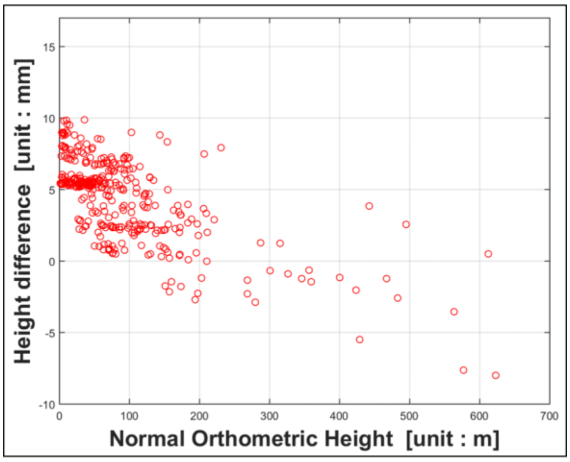


Unit : mm

Range	Mean	STD	RMSE
-8.01 ~ 9.76	4.37	2.73	5.15

Figure 2. Comparison of Normal-orthometric height and Helmert orthometric height

The result of the difference between normal-orthometric height and Helmert orthometric height (anomaly) is shown in **Figure 3**. The maximum difference between two heights is 9.88mm and bias is 4.39mm. Normal-orthometric height and Helmert orthometric height (anomaly) also show a negative correlation. It indicates that the conversion of gravity anomaly to gravity value was highly precise. Because gravity anomaly maps contain the bench marks used in this study.



Unit : mm

Range	Mean	STD	RMSE
-8.00 ~ 9.88	4.39	2.90	5.26

Figure 3. Comparison of Normal-orthometric height and Helmert orthometric height (anomaly)

3.2.2 Analysis effect of terrain

In the case of the 11th route, plain area, the difference between normal-orthometric height and Helmert orthometric height was distributed uniformly, which is shown in **Figure 4**. On the other hand, in the mountain area, 19th route, it was found that the difference between two heights became bigger when normal-orthometric height gets higher as shown in **Figure 5**.

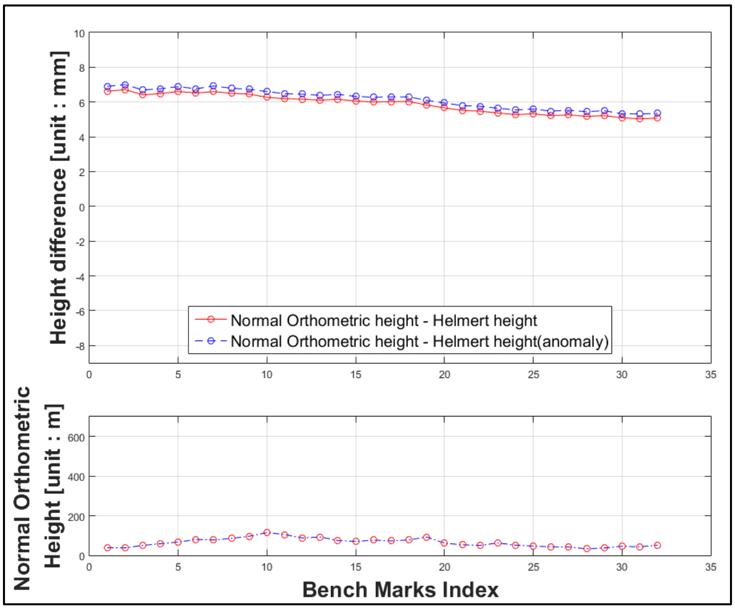


Figure 4. Height comaprison at the flatland

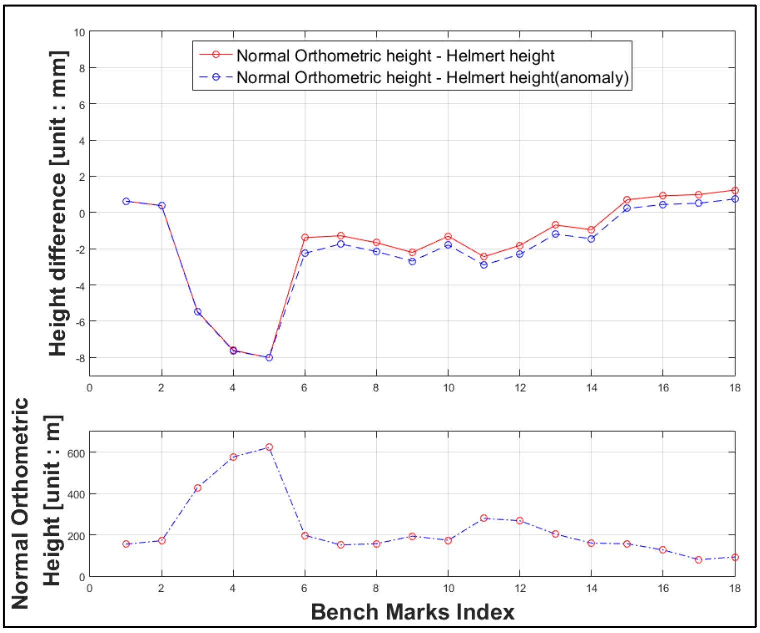


Figure 5. Height comparison at the mountain area

4. CONCLUSION

Since equipotential surface changes significantly at the mountain or steep area, difference between normal-orthometric height and Helmert orthometric height depends on the elevation. At the highland, plumb line and normal-orthometric height vary considerably. The maximum difference between two heights was 9.76mm in this study area.

Helmert orthometric height and Helmert orthometric height (anomaly) are relatively similar. Since gravity anomaly map includes the bench marks which are used in this study, conversion was conducted precisely.

In near future, the height system of Korea is changed to Orthometric height. In this case, denser gravity data would be necessary to secure the accuracy of the height especially in mountainous areas.

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