

# Coordinates of Boundary Points in Norway

Helge NYSÆTER, Norway

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## SUMMARY

This paper focuses on today's practice concerning cadastral surveying, and calculation of coordinates of cadastral boundaries, in Norway. Cadastral surveying in Norway is performed mainly by public surveyors, employed by the municipalities. These are obliged to follow relatively strict demands concerning measurements and computations, issued by the mapping authorities. However, these demands may be a bit outdated and not that well adapted to the newest technologies. I discuss the need for an adjustment computation with blunder detection and reliability analysis, when RTK-GNSS is the preferred measuring technique. This is also compared to practice in other European countries. I also discuss whether the surveyor should be given more freedom to choose the optimal method of measuring and calculation of coordinates in each situation. This question is connected to the question of formal qualification demands for the cadastral surveyor. Finally, I discuss the significance of the coordinates of the boundary points in the Norwegian cadastral system. This is also compared with some other countries.

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## 1. DESCRIPTION OF THE NORWEGIAN SYSTEM

### 1.1 Origin

Norway has a kind of German-style cadastre (Mjøs 2016) , in the sense that title is registered. Every land parcel has a unique number, and the parcel, with its owner, is registered in the land register. The geographic extent and the boundaries of the parcels are registered in the cadastre, and shown in the cadastral map. But there are still a lot of parcels that are not registered in the cadastre (Mjøs 2016).

### 1.2 Land register and cadaster

The geographical extent and the boundaries of the parcels are registered by means of coordinates in an official reference system. The coordinates are stored as latitude/longitude in the Euref89 reference system, as specified by the Norwegian mapping authorities. When displayed on a map, these coordinates are projected to the desired UTM zone. Which UTM-zone to be used, is specified for each of the 19 Norwegian counties, but all of southern Norway uses UTM-zone 32. This zone is formally extended to also include the most western parts of Norway. It is also used for the most eastern parts of southern Norway, despite these areas geographically falls within zone 33.

### 1.3 Mandatory registration

To subdivide land and thereby create a new property, a cadastral survey is mandatory in the current Norwegian system. Without a cadastral survey, a parcel number will not be issued, and the new property will not be registered in the land register. Without being registered in the land register, it will not be possible for the property to be object of mortgage. The government guarantees the correctness and completeness of the land register. But the Norwegian system of land registration does not necessarily require, nor guarantee, that the correct borders of the new property are registered in the cadastre (Mjøs 2016). The process of property registration is based on the cadastral law (Matrikkelova 2005) and the cadastral bylaw (Matrikkelforskriften 2009).

### 1.4 Rules for cadastral surveying and mapping

Cadastral surveying and mapping is the responsibility of the Norwegian municipalities, and most often carried out by surveyors who are employees of the municipality. Some municipalities hire private surveyors to do the technical work, but are still responsible for the registration in the cadastre. The land consolidation court is also permitted to do cadastral surveying and mapping in Norway, but the registration in the cadastre is always carried out by the municipality.

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The cadastral law demands that all boundaries are to be marked and measured in the field, and that they are identified by coordinates given in the official reference system (Matrikkellova 2005). Further details are given in the cadastral bylaw where it is stated that along with the coordinates, their quality and the method of measurement must be specified. The bylaw also demands that measuring and calculation must be done according to the specifications given in technical standards issued by the Norwegian mapping authorities (Matrikkelforskriften 2009).

The most relevant standard is “Surveying of cadastral boundaries” (Kartverket 2011). This standard requires cadastral surveying to be performed using controlled measuring equipment. Another requirement is making redundant measurements, so that every measurement is controlled by another measurement. The standard specifies that coordinates are to be calculated by an adjustment computation, which must include both blunder detection and statistical reliability analysis. It further states that the quality requirements for the result are the same for the boundary points and the temporary points utilized in the survey.

Concerning fieldwork when GNSS is applied, this standard refers to another standard by the mapping authorities, “Satellite-based positioning” (Kartverket 2009) and concerning reference system it is referred to the standard “Coordinate-based reference system” (Kartverket 2009). The practical application of the standards, concerning fieldwork and calculation will now be further discussed.

## **2 FIELDWORK AND CALCULATION**

The requirement to use controlled measuring equipment is normally maintained by regularly calibrating the total station, and concerning the GNSS-receiver, to measure the coordinates in a known position before further use. For both simple and complex cases, the required procedures concerning the fieldwork are the same. To illustrate, examples will be given, first of a rather complex case and then a rather simple.

### **2.1 Complex case**

Typically, a case is complex when it is impossible to measure every boundary mark directly using GNSS. In the first example there is a parcel lying between the sea and a very high cliff that partially hangs over the parcel. The conditions for receiving a direct satellite signal are bad, both on the parcel and also nearby it (see figure 1). The boundaries of the parcel are measured by setting up a total station approximately 400 meters away, on the other side of the sound (see figure 2). From that location, there are not many visible fixed points, so the setup points will be measured with GNSS to achieve better redundancy. The final calculation of the boundary’s coordinates will involve more than 20 observations of at least four different kinds, namely horizontal and vertical angles, slope distances and GNSS observations.



Figure 1: Parcel that is impossible to measure by use of GNSS only.

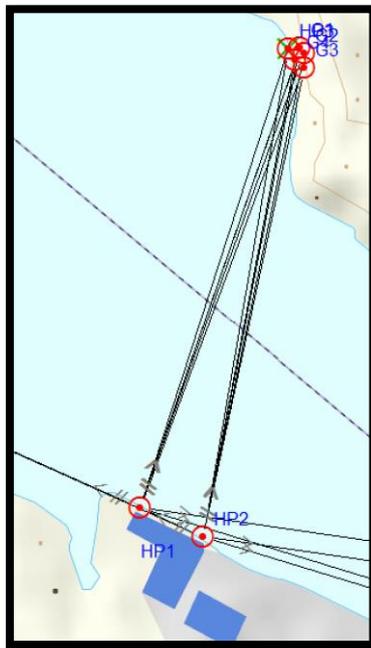


Figure 2: Combination of GNSS and total station from the other side of the sound.

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## 2.2 Simple case

An example of a simple case may be a parcel where every boundary point has good conditions for receiving direct and undisturbed satellite signals, and at the same time good conditions for receiving some kind of differential correction signal. The boundaries of the parcel shown in figure 3 were measured solely with RTK GNSS and CPOS, which is the name of the differential service offered by the Norwegian mapping authorities. The standard named “Satellite-based positioning” (Kartverket 2009) requires each point to be measured two or three times to obtain redundancy. The surveyor can choose between two measurements with an interval of at least 45 minutes, or three measurements with an interval of at least 15 minutes between each of them. These intervals are defined so to secure sufficiently independent measurements.

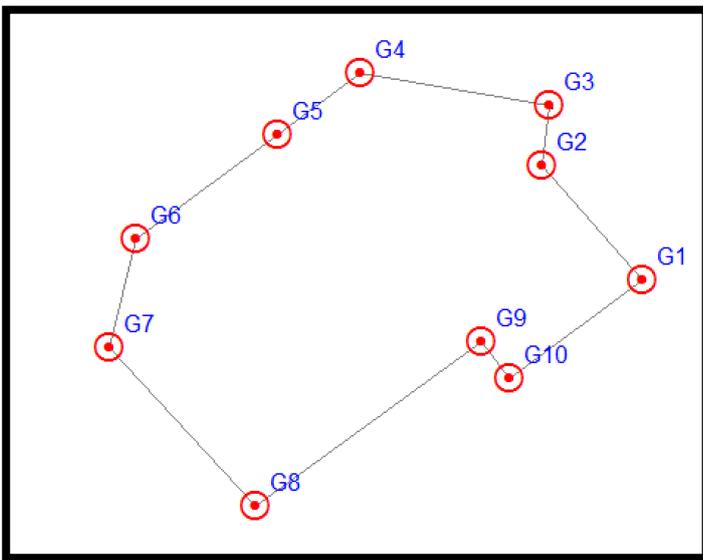


Figure 3: Every boundary point measured with RTK-GNSS

## 2.3 Analyses

In the complex case, it is understandable that both blunder detection and a statistical reliability analysis is required. The commonly used software packages offer a blunder detection procedure that estimates a blunder in every observation and uses a t-test to try if the blunder is significant. The standards demand that every significant blunder must be removed from the observation material before calculation of the final result. If removal of detected blunders results in lack of redundancy, additional measurements has to be undertaken. The reliability analysis consists of two steps. First, internal reliability is calculated. This is an estimation of the size of blunders that still can be present in the observational material, without having been detected in the blunder detection process. The standard requires a significance level of 0.05 or less in this analysis. The second step is calculation of external reliability, or point deformation. Here is calculated the maximum movement (deformation) of the coordinates, caused by the maximum undetected blunders calculated in step

one. The standard demands the maximum deformation to be less than 10 cm for every boundary point and for every temporary point that is part of the final adjustment computation (Nysæter and Leiknes 2014). When performing these analyses on our “complex case” (figures 1 and 2), we find all deformations to be less than 6 cm. This means that the boundary is mapped according to the demands of the standard, even though the satellite observations may have been affected by multipath, and the total station measurements may have been disturbed by varying refractive conditions over the sea surface.

For the “simple case”, the requirements concerning the procedures of calculation are exactly the same. The coordinates must be calculated by adjustment, and the standard explicitly calls for a *unified adjustment* (Kartverket 2011). The meaning of this is not defined in the standard, and it is not self-evident. To calculate two or more points in a common weighted least-squares adjustment computation will produce exactly the same coordinates as if the points were calculated one-by-one, when the points don’t have any observations in common (Nysæter and Leiknes 2014). So the question is if one can really speak of a unified adjustment when the different boundary points are measured only with RTK-GNSS. The Norwegian software vendors seem to think that a unified adjustment is possible, because adjusting the points together is the only logical way of using their software. If this “simple case” should be calculated one point at the time in the software “Gemini Oppmåling” (Powel 2016), it would result in 10 adjustment reports, 10 blunder detection reports and 10 reliability analysis reports. This would add up to more than 60 pages of report for mapping this single parcel.

A so-called unified adjustment will produce, as mentioned, exactly the same coordinates, but the computed accuracy of the coordinates will be different. This is because the unified adjustment produces one single matrix of residuals for the whole computation. The residuals are used to calculate one common weighted sum of squared residuals, which in turn is used to calculate the accuracy of the coordinates. In this way, residuals in one of the points will affect the calculated accuracy in every other point, even if the points have no observations in common. Any justification for this method of calculating accuracy is hard to imagine, but it is still the most common practice because of the way the software packages work.

### **3. RTK GNSS**

Today’s cadastral surveyor prefer using Real Time Kinematic GNSS (Global Navigation Satellite System) whenever possible, because of the efficiency of this method and equipment. When the Norwegian technical standards for cadastral surveying were developed, other measuring methods dominated.

#### **3.1 CPOS and similar services**

Cadastral surveyors use GNSS-receivers that are capable of measuring with centimetre-accuracy in real time. To achieve this, requires reception of data from a differential network. In the process of

calculating coordinates for the desired point, a three-dimensional vector is first calculated, either from a base station in the differential network or from a virtual reference station calculated by the network service. As these vectors themselves are calculated by least squares adjustment, the accuracy of the vector components are calculated in the same operation. The accuracy is given in form of variance, and co-variance between the vector components is also calculated. The output file from the GNSS receiver after one observation could look like figure 4.

```

00 Output from Trimble Survey Controller job
01 Parcel_38_5 26102015 002 $111
00 VRS base: 60.38591°, 5.15303°, 80.864m
05 G1 6700584.839 288000.599 33.087
47 0.00006429 0.00002664 0.00015849 0.00000709 0.00004454 0.00002087
46 26102015 10:35:04 16 1.45 0.000 11 94
00 WA Fixed Prec:0.006/0.011 PDOP:1.5 Pos:11 SVs:16 UTC:10:35,26/10-2015

```

Figure 4: Output from GNSS receiver

In the line starting with 05, the name of the point and the observed UTM-coordinates are given. The line starting with 47 contains the variances and the co-variances of the vector, given in the geocentric coordinate system. The last line, starting with 00, contains the quality information that is displayed on screen when measuring. In the example, both GPS and GLONASS-satellites are utilised, and the total number of satellites is 12. When the Galileo satellite system is declared operative, even more satellites will be available.

### 3.2 Computation of coordinates

As previously stated, the adjustment computation must include all the boundary points in the project, and also temporary points that have been used for setup, aiming, or distance measurements. A unified adjustment computation with blunder detection and reliability analysis is required. This is a reasonable approach when mapping parcels like the one in the complex example, but not necessarily in the simple case. When practicing according to the standard, the GNSS point observations are treated in the same way as static GNSS-vectors, using calculation methods developed for obtaining sub-centimetre accuracy in a static survey. When mapping boundary points within 10 cm accuracy, it is difficult to see any need for taking co-variances between vector components into account. The actual effect of the covariance-values on the calculated result is documented in the following section. Even in Austria, where coordinates on boundaries are considered legally binding, co-variances are not part of the computation<sup>1</sup>. It is also difficult to see that the accuracy provided on screen, in real-time, is not sufficient. The example listed in figure 4 shows a horizontal accuracy of 0.6 cm, which is a typical accuracy displayed for RTK-GNSS when measuring and taking average for about 10 seconds. If another observation is made after the required time interval of 45 minutes, the surveyor can easily check if it agrees within reason with the first observation. This can be done manually by comparing the numbers, or automatically by the

<sup>1</sup> Based on own experience when joining “Mayr Vermessung” in a cadastral survey in Tyrol in 2014.

GNSS-fieldbook. If the result is considered ok, the two sets of coordinates could be averaged, with or without weighting based on the displayed accuracy. Alternatively, one could just state that the second measurement confirms the first one, which then is kept. This last method is the approach of the “Kadaster” in the Netherlands<sup>2</sup>.

### 3.3 Effect of covariance values

The effect of taking covariance values into account when calculating adjusted coordinates, was tested in a bachelor thesis at Western Norway university of Applied Sciences in 2016 (Sellevoll, Gjuvsland et al. 2016). The analysis shows only minor effects both on the adjusted coordinates and on their calculated accuracy. Based on the data from the bachelor thesis, I have made some further calculations and some modified analyses, which confirm the results from the bachelor thesis. The observations were made at two different sites. The first is an ideal site for making GNSS observations, on top of a tall building. The second site is a more ordinary site, with a tall building on the south side and some trees on the north side, but still a site where it is easy to obtain a fix-solution. All observations can be considered free of centering errors. Observations are made every minute for one hour. When calculating the adjusted coordinates, these observations are combined three by three, with 15 minutes between each, according to the standard procedures for cadastral surveying in Norway. By this we obtain 15 sets of adjusted coordinates, and these are adjusted both with and without taking covariance values into account. Not taking covariance values into account is done by setting the values to zero. By this, every set consists of three observations that are sufficiently independent from each other, according to the standard given by the mapping authorities (Kartverket 2009). It can of course be argued that the 15 sets within one hour are not independent from each other, so the bachelor group therefore made more than twenty observation sessions on different days, on different times of the day, and on two different sites.

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<sup>2</sup> Based on own experience when visiting the Dutch «Kadaster» in 2016.

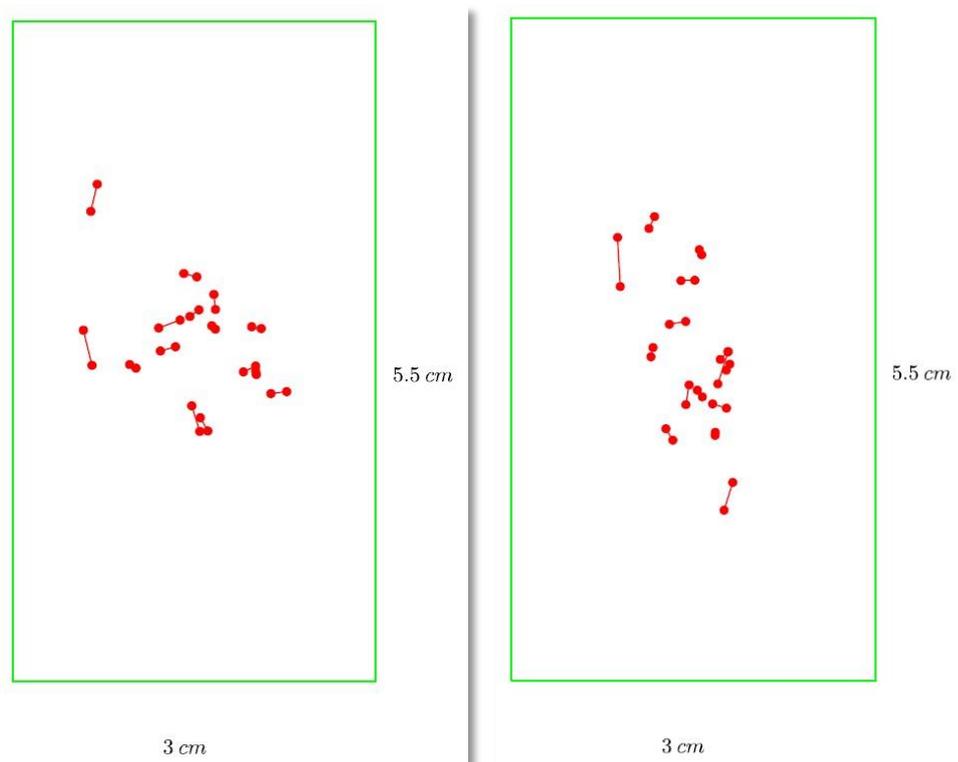


Figure 5: Changes of adjusted coordinates because of covariance values, shown on authorized boundary mark seen from above.

Figure 5 shows the 15 sets of adjusted coordinates for both observation sites. To the left are the results for the “ideal site” and to the right are the results for the “ordinary site”. In the diagrams, two and two dots are connected by lines. The connected dots are the same adjustment made with and without covariance values. The maximum effect of taking the covariance values into account is 7.2 mm for the ideal site and 4.9 mm for the ordinary site. The maximum effect on the calculated standard deviation of the coordinates is 5 mm for both sites. The rectangular frame that is drawn around the results shows the dimension (30 mm x 55 mm) of an official Norwegian boundary mark, seen from above.

The calculations are made by own programming in Scilab (Scilab\_Enterprises 2017), based on standard textbooks (Ghilani 2010, Leick, Rapoport et al. 2015). The results have also been checked by using “Gemini Oppmåling” (Powel 2016), which is a commonly used commercial Norwegian surveying software.

The effect of covariance values is normally less than the effect of tilting the antenna pole while measuring, and tilting of the boundary mark itself when it is mounted in the ground. It can be argued that using covariance values will affect whether observations will be classified as blunders or not, and thus influence the adjusted coordinates by several centimetres.

However, a careful surveyor will easily notice if the discrepancy between his observations indicates one of them containing a blunder. Then he will just make one extra measurement.

#### **4. THE SURVEYOR**

The cadastral surveyor in Norway is appointed by the municipality. The municipality may appoint whoever they consider able to maintain the work. There are no formal demands for any specific education nor any specific skills, nor any authorization of cadastral surveyors. This is probably part of the explanation for the rigid and detailed specifications of measuring routines and calculation routines issued by the mapping authorities. If the cadastral surveyor is without sufficient education and knowledge, one cannot expect him to be able to choose the optimal procedures for each different case. The solution is to always measure and calculate all that is needed in the worst cases. All the “thinking” is left for the computer software.

#### **5. DOCUMENTATION AND SIGNIFICANCE OF COORDINATES**

The calculated coordinates of the boundary points are registered in the cadastre (Norw.: “matrikkelen”). The cadastre is administered by the mapping authorities, and the registration is done by the municipality. Together with the coordinates, their accuracy, in form of standard deviation, is also registered. The way of calculating the standard deviation, however, is not specified, nor is it self-evident. This has led to some confusion and differences in practice (Nysæter and Leiknes 2014). The landowners will receive a document that contains the registered coordinates shown both in a list, and on a map. The registered accuracy of the coordinates, and the distances between them are also shown. For the landowners, the documentation is of limited value as a tool for relocating boundary points. The landowner does not possess any equipment for measuring coordinates within centimetres and the boundary points are never measured relative to any building or other terrain detail. A landowner may require the municipality to point out a property boundary. The registered coordinates will then serve as a tool for the cadastral surveyor. However, if two landowners disagree about the boundary, the coordinates will only serve as one of many possible proofs. If the conflict is taken to court, experience shows that the coordinates often will be of minor importance (Nysæter and Leiknes 2014). It seems like a paradox that Norway puts more effort into measuring and calculating accurate coordinates than any other European country, even Austria, when the legal significance of the coordinates is both undefined and unclear. In Austria, as mentioned earlier, the coordinates are considered binding since 1969 (Abart 2011).

#### **6. CONCLUSION**

As more and more boundary points are measured solely with RTK-GNSS, the procedures given in the technical standards need to be revised. Today’s cadastral surveyors may need strict procedures to follow, because some of them are without relevant education. Nevertheless, detailed procedures that are to be sufficient for every situation inevitably get complicated. And complicated procedures

can easily be misunderstood, especially by surveyors without formal qualifications. While the short-term solution seems to be updated procedures for measuring and calculation, the long-term solution must be cadastral surveyors who are both able, and free, to choose what they consider the optimal procedures in each case.

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## **BIOGRAPHICAL NOTES**

Helge Nysæter is assistant professor at Western Norway University of Applied Sciences since 2006, where he has been teaching land surveying, mathematics, cadastral systems and Geographical Information Systems. He graduated in mapping technology from the Agricultural University of Norway in 1994. He has also studied history and pedagogy at university level. Current research interests are cadastral surveying and cadastral mapping.

## **CONTACTS**

Assistant professor Helge Nysæter  
Western Norway University of Applied Sciences  
Postbox 7030  
5020 Bergen  
NORWAY  
Tel. +4755587660  
Email: [helge.nyseter@hvl.no](mailto:helge.nyseter@hvl.no)  
Web site: [www.hvl.no](http://www.hvl.no)