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THEORETICAL AND EMPIRICAL MINIMUM DETECTABLE DISPLACEMENTS FOR DEFORMATION NETWORKS

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What is Minimum Detectable Displacement (MDD)?

- sensitivity,
  - Detectability of expected displacements and deformations at the designated network (Even-Tzur, 2006)
- reliability
  - Displaced points that are detected by Conventional deformatin analysis can be validated whether these points are actually displaced or not by means of simulation.
- accuracy assessment
  - Difference between estimation value and simulated value

Objective

- Theoretical Minimum Detectable Displacement (TMDD) depends on the power of test; such as %80, %70.
  - Which one is more realistic?

- To reach an optimal Minumum Detectable Displacement (MDD), Empirical Minimum Detectable Displacement (EMDD) can be obtained as an alternative.
  - Empirical Technique: Using Displacement Ellipse
  - Empirical Technique 2: Step-by-Step Approach

Theoretical Minimum Detectable Displacement (TMDD)

$$\lambda = \frac{\mathbf{d}_\mathbf{k}^\mathbf{T} \mathbf{Q}_\mathbf{dd}^{+} \mathbf{d}_\mathbf{k}}{\sigma_0^2} \quad ; \quad \lambda > \lambda_0$$

If $\lambda > \lambda_0$ it is inferred that the deformation network is sensitive. This comparison process called as sensitivity analysis (Caspary et al., 1983; Niemeier, 1985; Cooper, 1987; Even-Tzur, 2006; Aydin et al., 2004).

$\mathbf{d}_\mathbf{k}$: the expected deformations vector
σ_0^2: a priori variance of unit weight
$\mathbf{Q}_\mathbf{dd}^{+}$: Pseudoinverse of the cofactor matrix

$\mathbf{g} = [\cos t_1 \; \sin t_1 \; \cos t_2 \; \sin t_2 \; \ldots \cos t_p \; \sin t_p]^T$

p: The number of points

In vector **g** the components of the undisplaced points are assumed as “0”

$\mathbf{d_k} = \mathrm{a}\mathbf{g}$

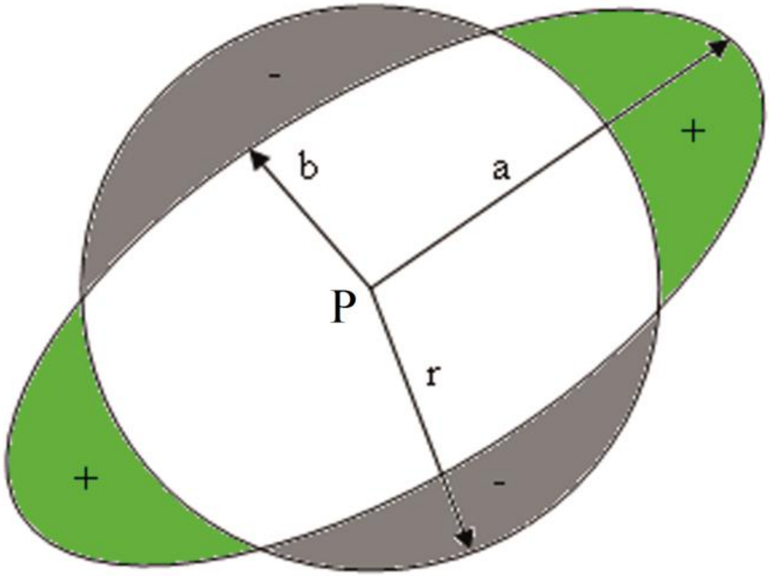
a : The computable scale factor

$\lambda = \frac{\mathrm{a}^2}{\sigma_0^2} \mathbf{g}^{\mathbf{T}} \mathbf{Q}_{\mathbf{dd}}^{+} \mathbf{g} \quad ; \lambda > \lambda_0$

$\frac{\mathrm{a}^2}{\sigma_0^2} \mathbf{g}^{\mathbf{T}} \mathbf{Q}_{\mathbf{dd}}^{+} \mathbf{g} > \lambda_0 \quad \Rightarrow \quad \mathrm{a}_{min} = \sigma_0 \sqrt{\frac{\lambda_0}{\mathbf{g}^{\mathbf{T}} \mathbf{Q}_{\mathbf{dd}}^{+} \mathbf{g}}}$

Empirical MDD: Using Displacement Ellipse

The circle whose area is equal to the area of the expected displacement ellipse could be chosen as given in Fig. 1, so that the total area of the positive part is equal to the total area of the negative part.



Expected displacement ellipse and corresponding displacement circle of Point P (a and b are the semi-major and semi-minor axes of the expected displacement ellipse) (Hekimoglu et al., 2010).

EMDD - Using Displacement Ellipse

The semi axes of error ellise

$$\lambda_1 = \frac{Q_{dxixi}+Q_{dyiyi}+w}{2} \quad , \quad \lambda_2 = \frac{Q_{dxixi}+Q_{dyiyi}-w}{2}$$

- Q_{dxixi} and Q_{dyiyi} = elements of the respective submatrix of the cofactor matrix $\mathbf{Q_{dd}}$, which belongs to the i^th point
- λ_1 and λ_2 = Semi-major and semi-minor axes of the standardized expected displacement ellipse

$$w = \sqrt{(Q_{dxixi} - Q_{dyiyi})^2 + 4Q_{dxiyi}^2}$$

The formulas for the elements of the rescaled expected displacement ellipse

EMDD - Using Displacement Ellipse

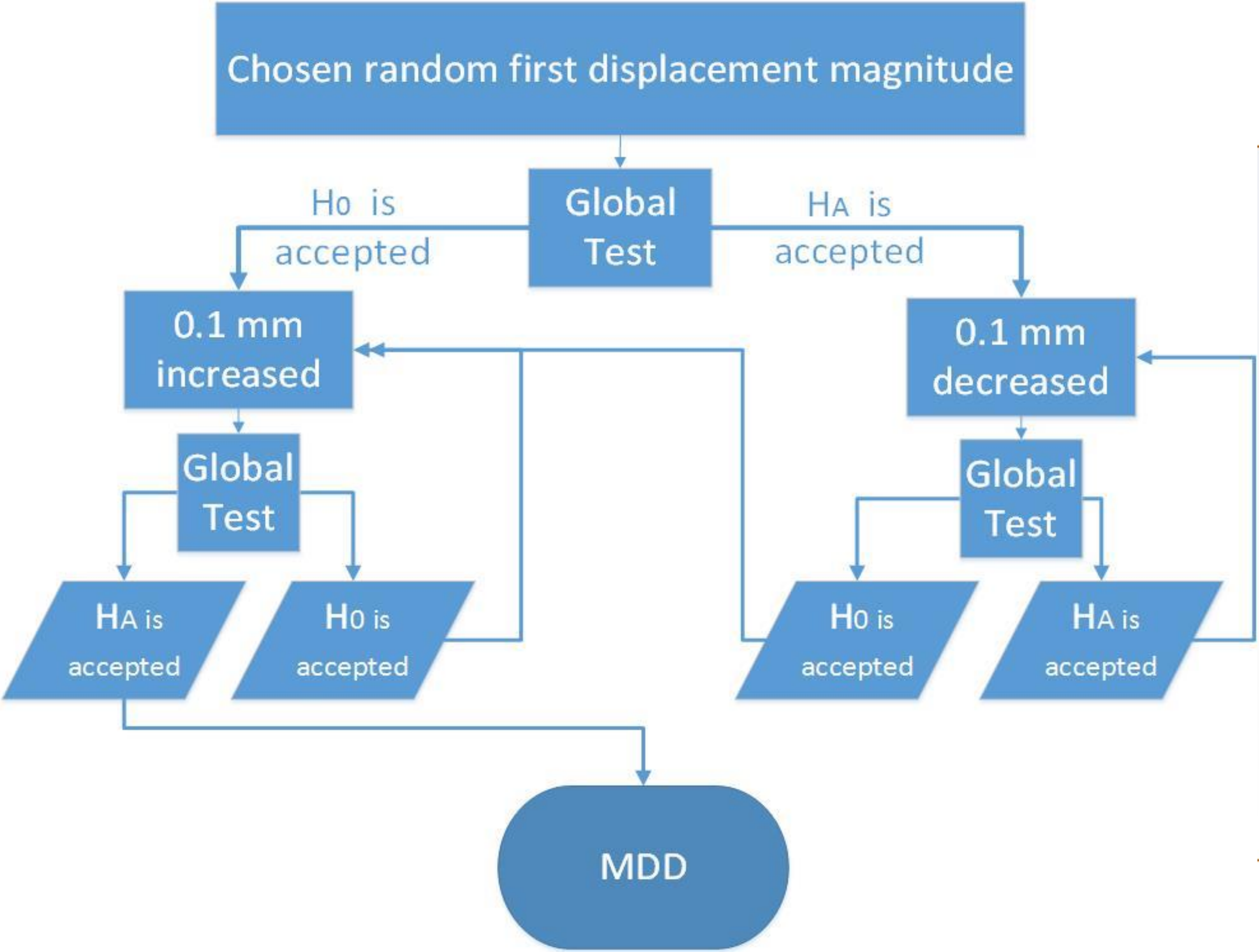
$$a = \sigma_0 \sqrt{\lambda_1 \chi^2_{2,1-\alpha}} \qquad b = \sigma_0 \sqrt{\lambda_2 \chi^2_{2,1-\alpha}}$$

σ_0^2 = "True" variance component

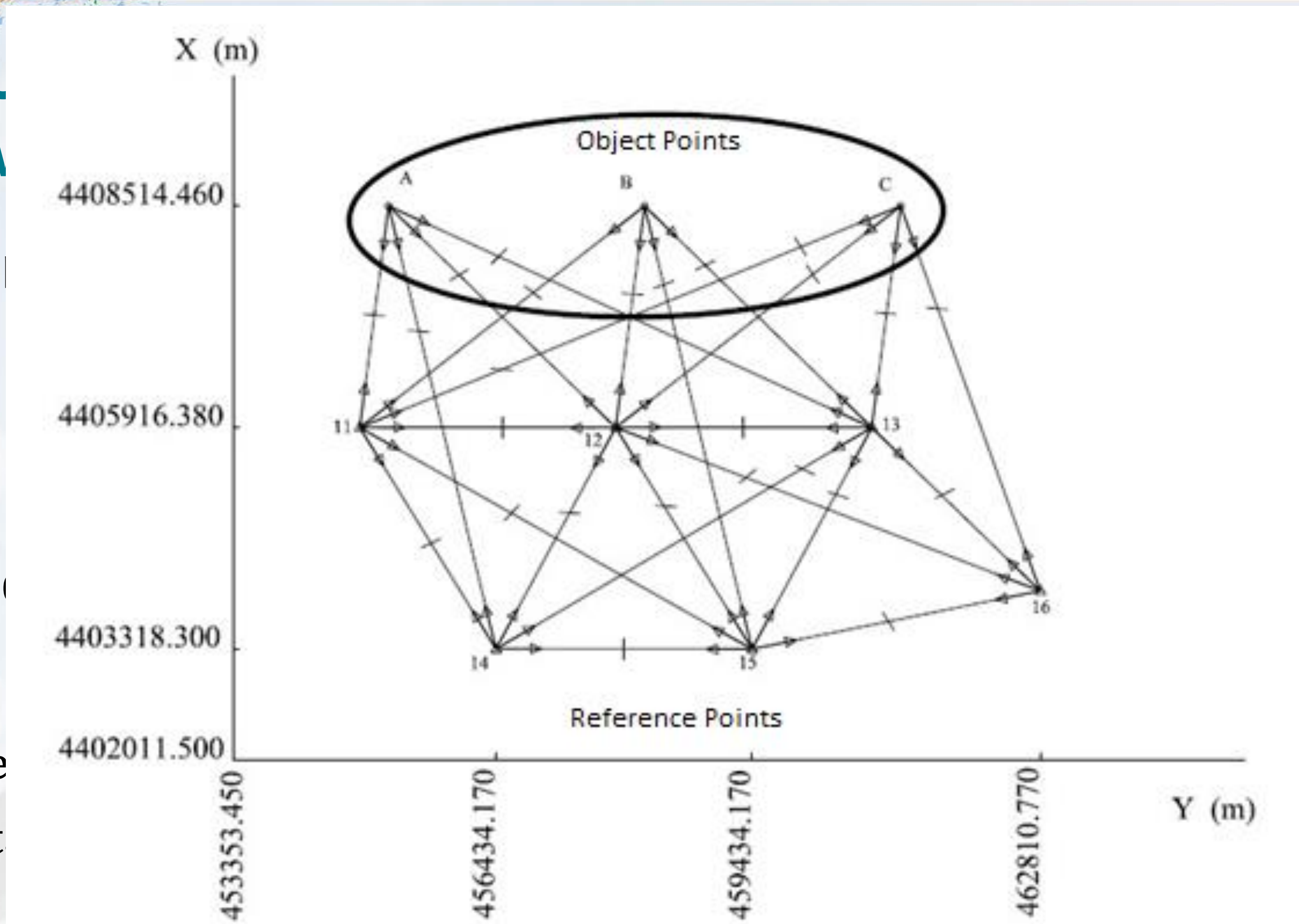
- $\chi^2_{2,\alpha}$ = α- fractile of the χ^2_2-distribution for 2 degrees of freedom
- α = Error probability ; a, b= semi-axes for each points
  $r = \sqrt{ab}$
- r = Radius of the corresponding displacement circle
- α is chosen here as 0.001 so the stochastic effect is reflected almost entirely in the simulated displacement magnitude.

EMDD – Step-by-step Approach

- Second EMDD magnitude obtained by step approach testing.
- This technique estimates the minimum magnitude of displacement for different directions depending on the global congruency test.
- To reach the final step quicker, the first value has chosen randomly which can both detectable and reflect the stochastic model of network.
- Then first value was increased or decreased 0.1 mm to get the minimum displacement magnitude.


Chosen random first displacement magnitude
Global Test
H0 is accepted
HA is accepted
0.1 mm increased
0.1 mm decreased
Global Test
Global Test
HA is accepted
H0 is accepted
H0 is accepted
HA is accepted
MDD

SıMU
Netw



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σ_{dire}

σ_{dist}

$\mathbf{l_1} = \bar{\mathbf{l}} + \mathbf{e_1}$

$\mathbf{l_2} = \bar{\mathbf{l}} + \mathbf{e_2} + \mathbf{A} * \mathbf{z}$

A : Coefficient matrix

z : deformation vector

$\bar{\mathbf{l}}$: uncontaminated measurements

$\mathbf{e_1}$ and $\mathbf{e_2}$: normally distributed random error vectors

random errors differently for each epoch $N(\mu=0,\sigma_d^2)$,

$\mathbf{z} = [z_{1x} \quad z_{1y} \quad z_{2x} \quad z_{2y} \quad \cdots \quad z_{ux} \quad z_{uy}]$; displacements of the points

z_x : The projection of deformation vector **z** to x axes of the deformation,

z_y : The projection of deformation vector **z** to y axes of the deformation

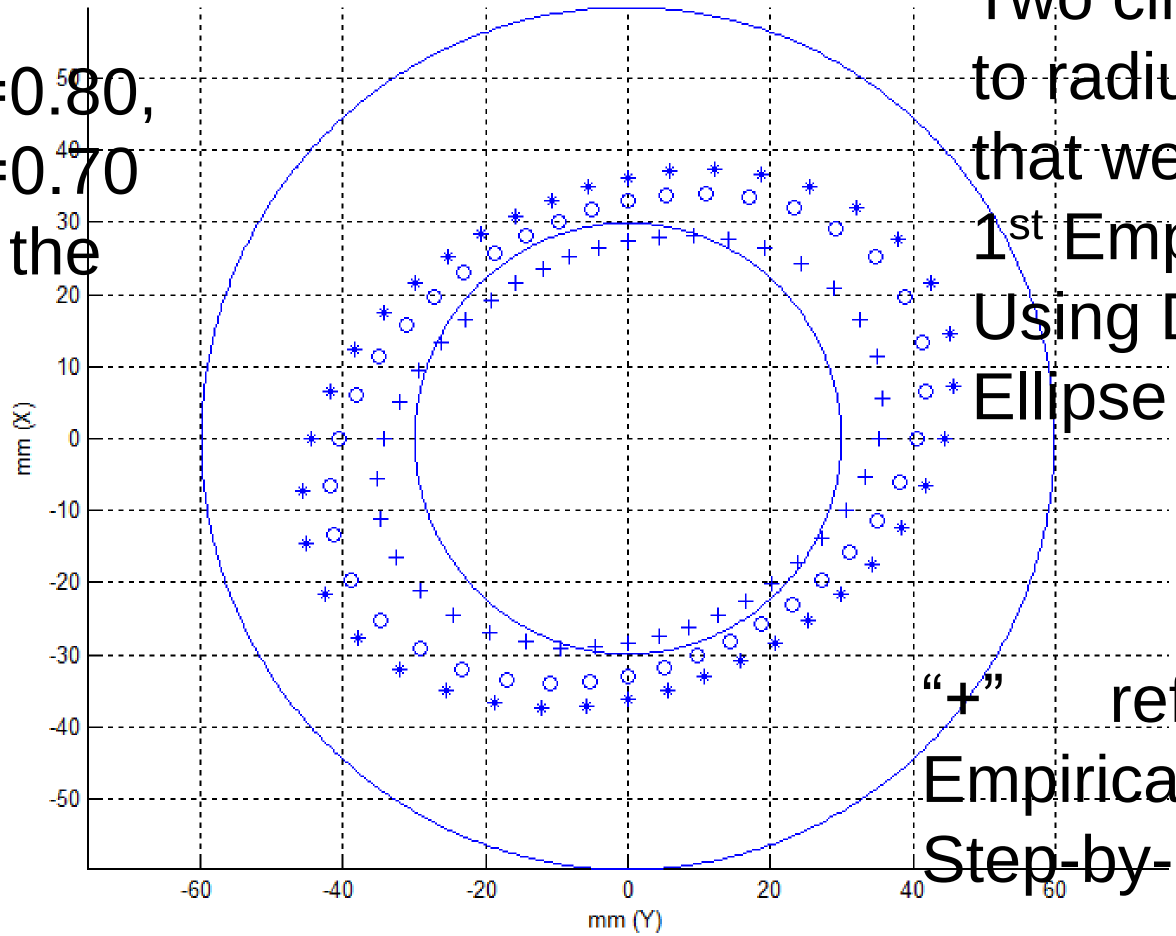
$\mathbf{A} * \mathbf{z}$: The deformation vector for the corresponding measurements

Results for Horizontal Control Network

- The components of displacement circle were obtained for points A, B and C which are assumed as displaced and radius was calculated as r = 29.9 mm.
- MDD were computed at 40 different directions for 3 MDD techniques
- with $\alpha = 0.05$ and $\beta = 0.20$. The displacements magnitudes were obtained for $\alpha = 0.05$ and $\beta = 0.30$ as well

TMDD and EMDD magnitudes for point A

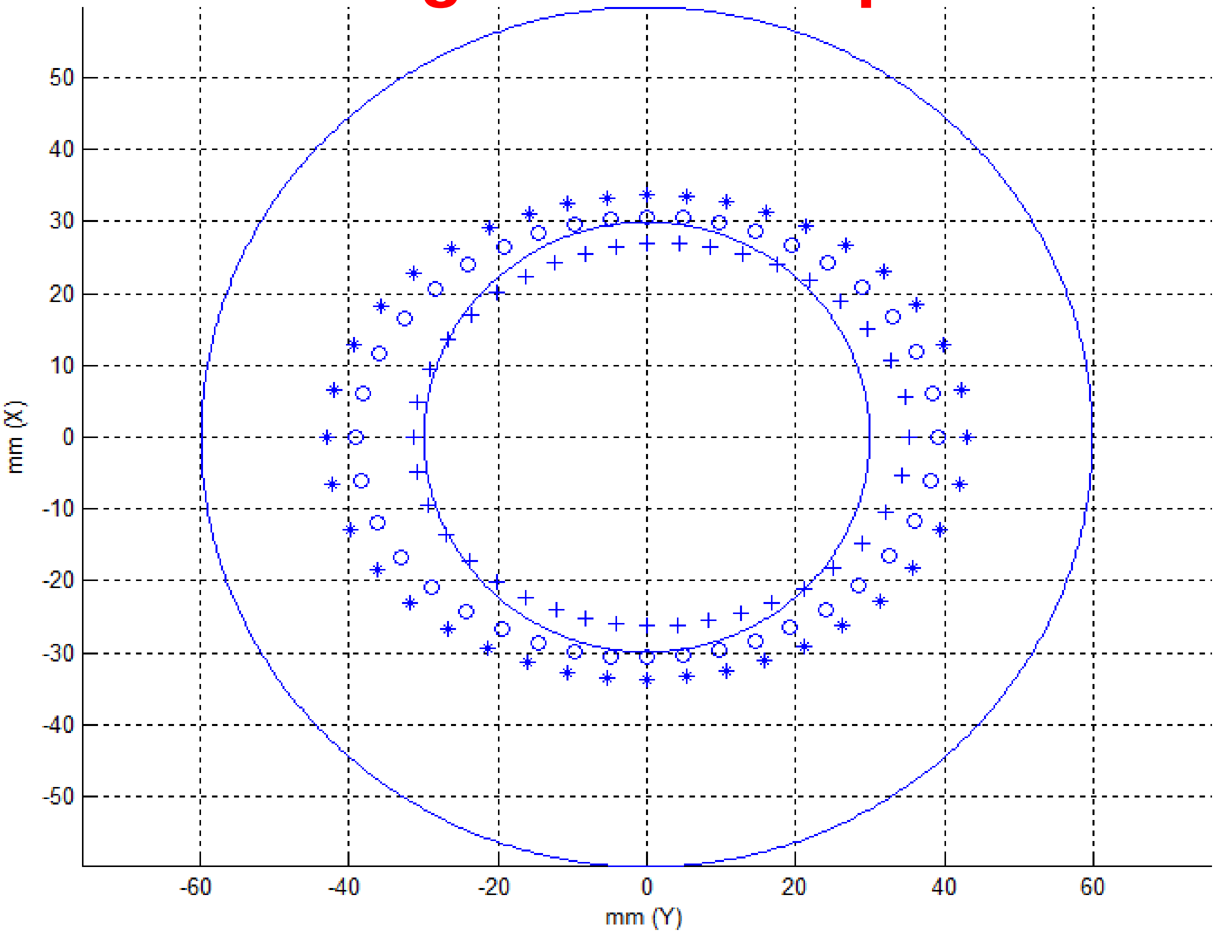
“*” 1-β=0.80,
“o” 1-β=0.70
shows the TMDD



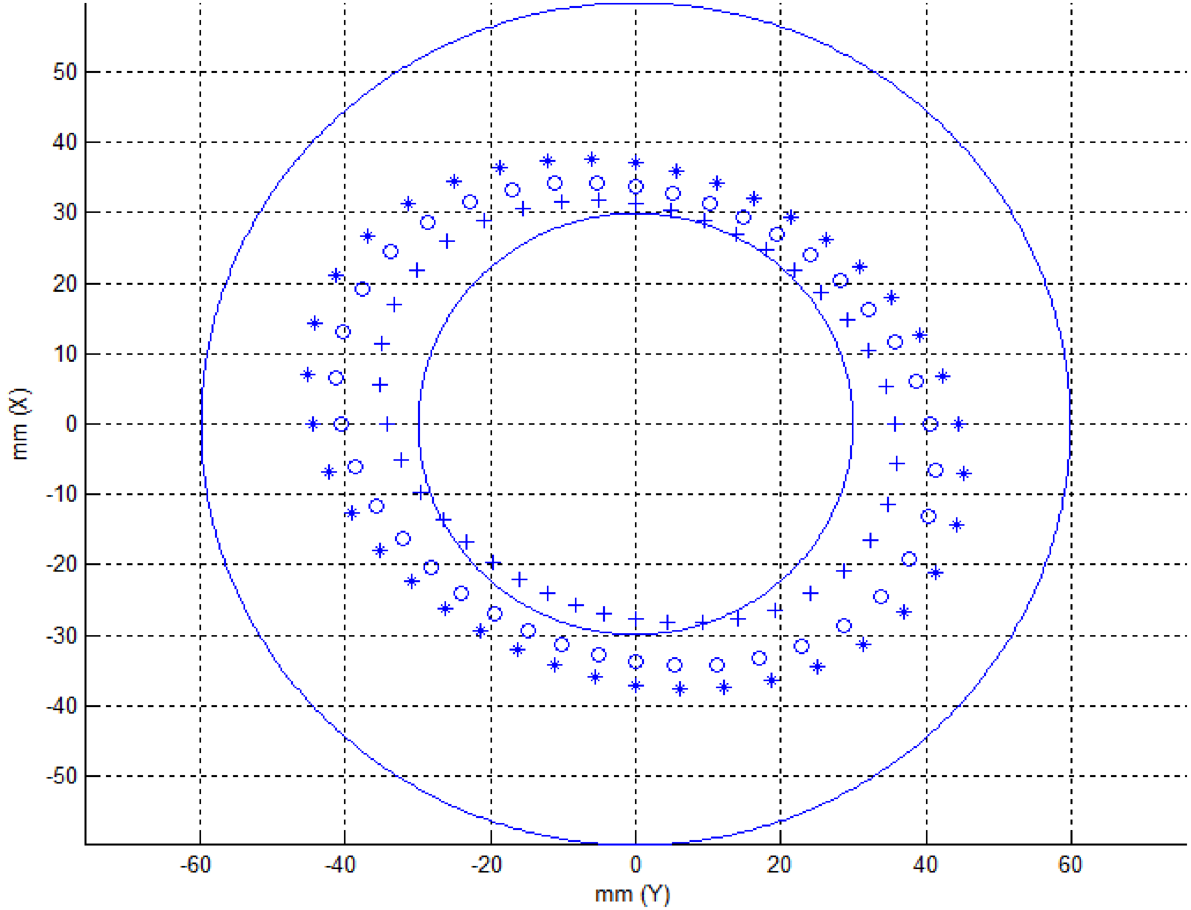
Two circles corresponds to radius (r, 2r) interval that were obtained from 1st Empirical Technique: Using Displacement Ellipse

“+” refers to 2nd Empirical Technique: Step-by-Step Approach

TMDD and EMDD magnitudes for point B



TMDD and EMDD magnitudes for point C

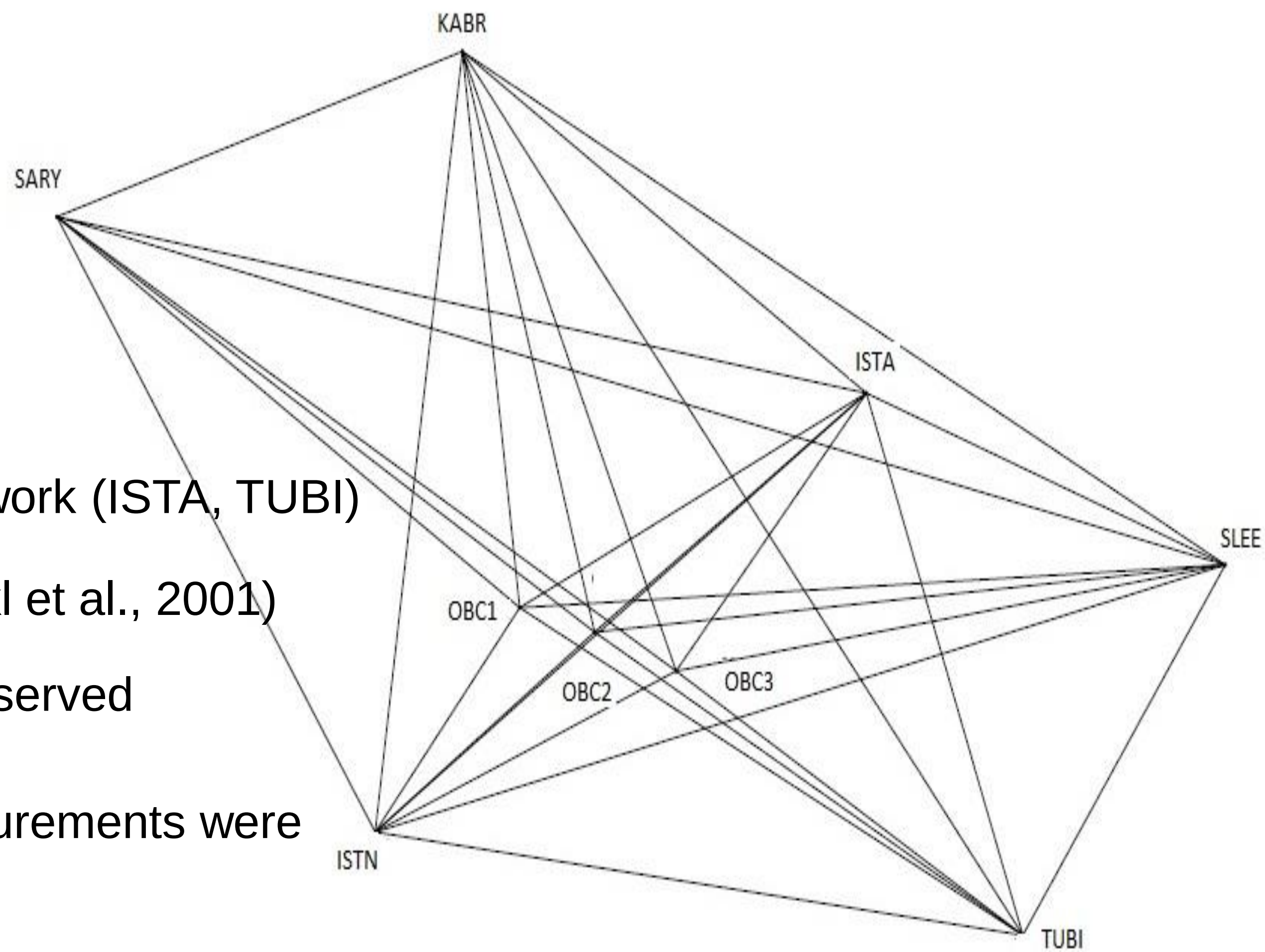


SIMULATED NETWORKS: GPS Network

- 3 points at the Davutpasa Campus of Yildiz Technical University
- 4 points TUSAGA-AKTIF (SARY, KABR, SLEE, ISTN)
- 2 points from IGS network (ISTA, TUBI)

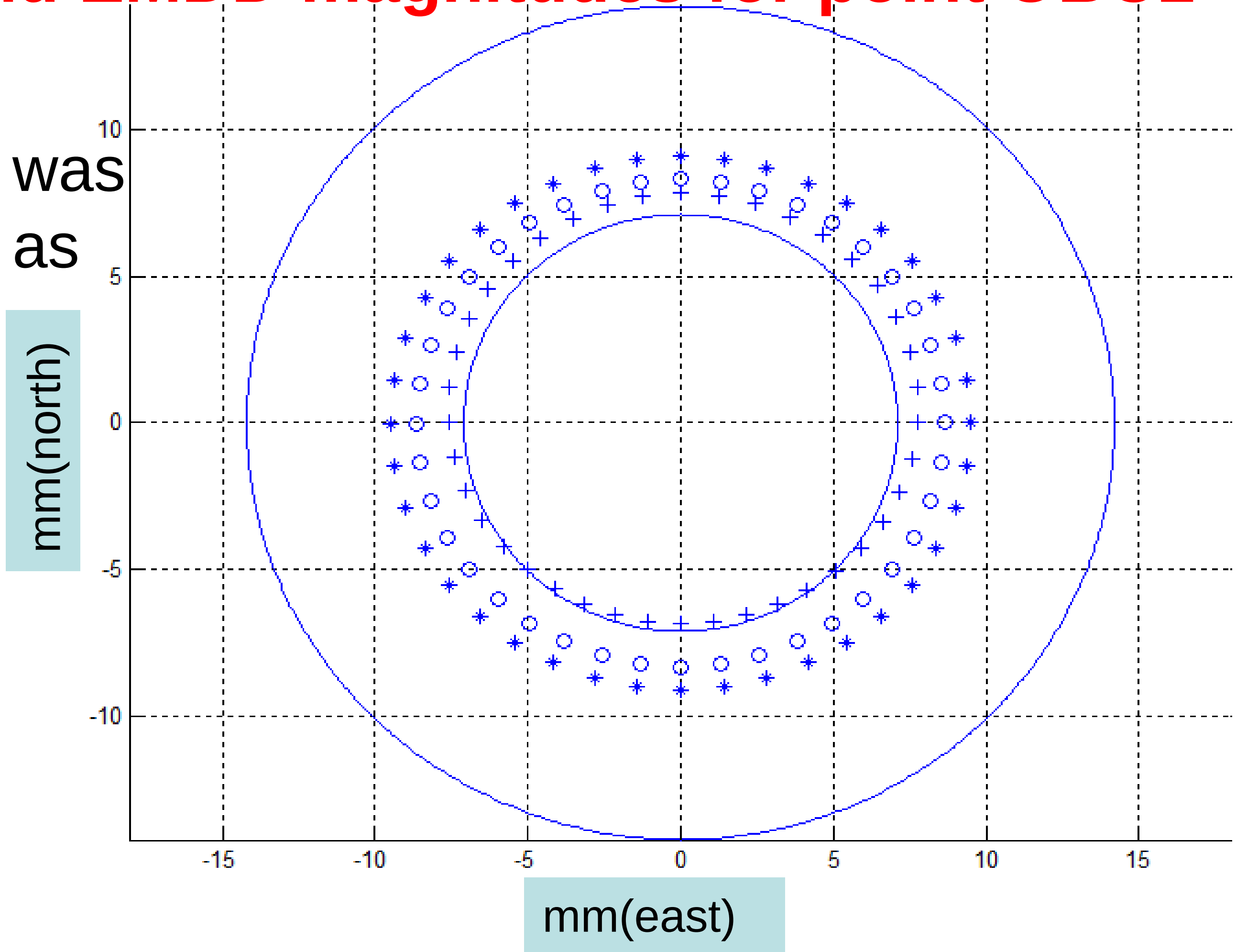
$\sigma_0^2 = 3.59^2 mm^2$ (Eckl et al., 2001)

- 32 baselines were observed
- 7 hours of GPS measurements were carried out

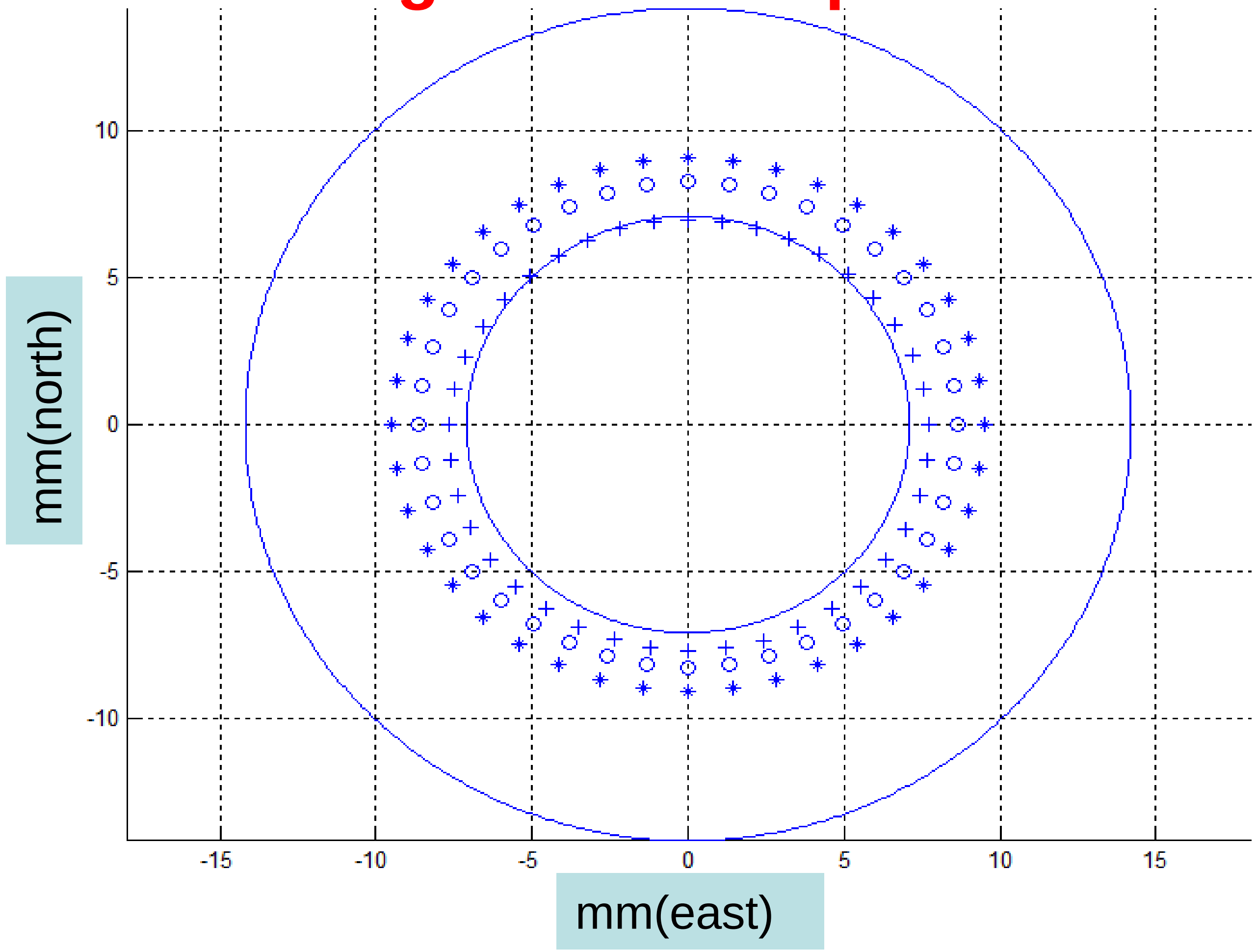


TMDD and EMDD magnitudes for point OBC1

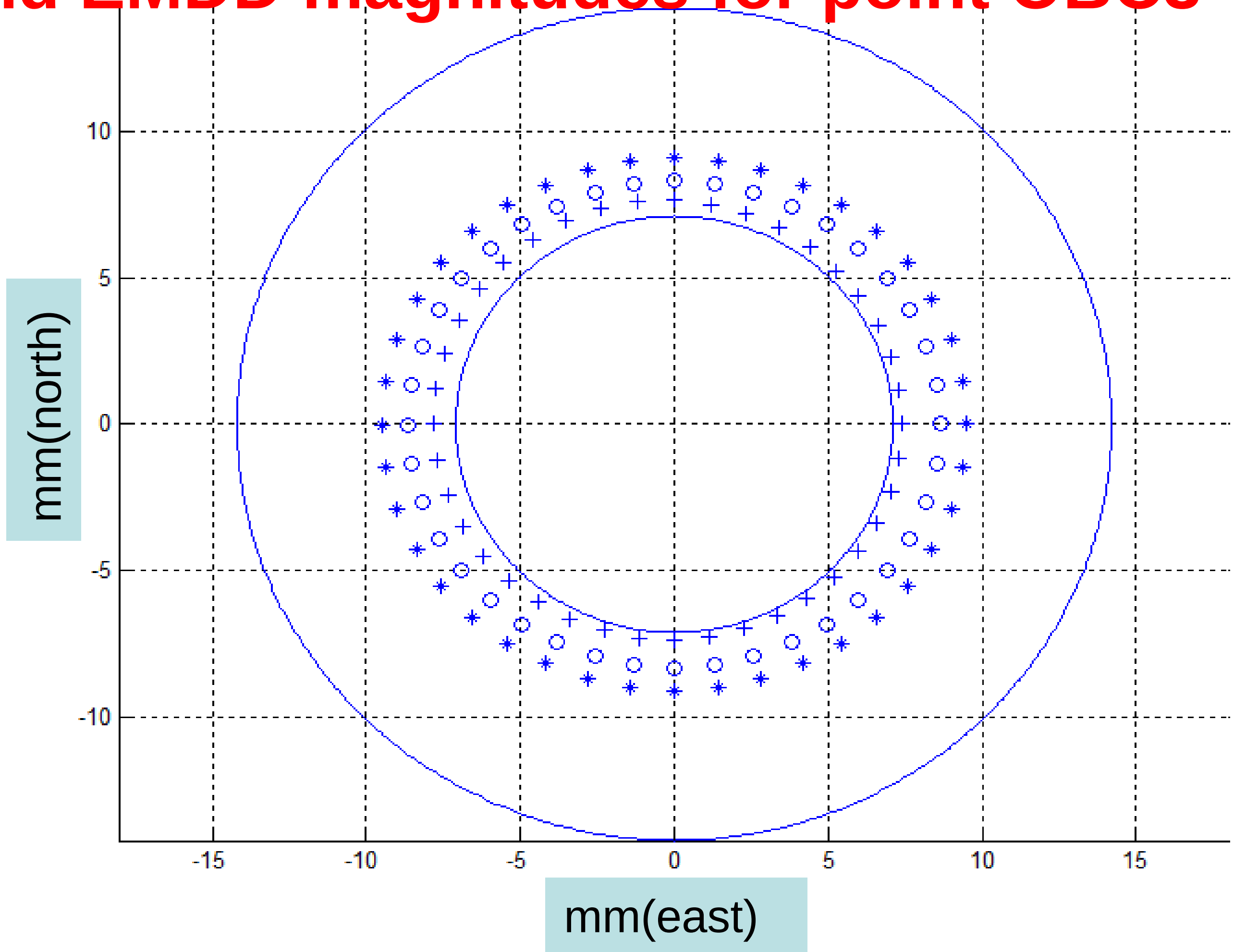
the radius was computed as r=7.1 mm



TMDD and EMDD magnitudes for point OBC2



TMDD and EMDD magnitudes for point OBC3



Conclusions

- The magnitudes of the TMDD techniques are greater than the ones of the EMDD techniques.
- Accordingly, the simulation of the displacement should be based on EMDD techniques for the performance of the deformation analysis method.
- The obtained result values from Empirical MDD more realistic than the Theoretical MDD.
- Two Empirical techniques converges to each other.
  - Step-by-step approach is the slowest amongs these techniques.

Thank you for your attention !