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THE COMPARISON OF THE ADJUSTMENT METHODS IN GEOID DETERMINATION METHOD

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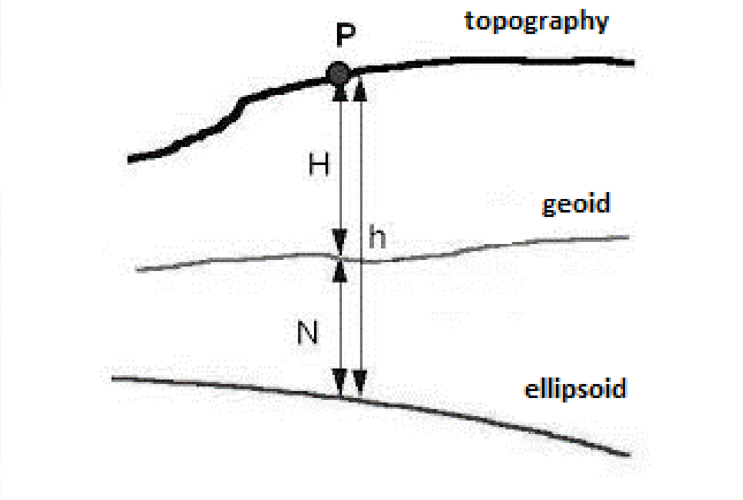
THE COMPARISON OF THE ADJUSTMENT METHODS IN GEOID DETERMINATION METHOD

INTRODUCTION

- The geoid determination is the most important problem for scientist interested in the earth. There are a lot of areas interested in geoid like geodesy, geophysics, geography etc.

- In the geodesy the measurements on the physical earth, but the calculation of measurements is done on the reference surface. Thus, the difference between the reference ellipsoid was called geoid undulation. The geoid determination methods had been developed to obtained geoid undulation values.



INTERPOLATION METHODS

- **POLYNOMIAL INTERPOLATION**
- **MULTI-QUADRATIC INTERPOLATION**

POLYNOMIAL INTERPOLATION

- The polynomial geoid determination technique is based on the determination of polynomial surface. This method is mostly common because of understandability and easy solvability. There are a lot of application in our country. Some of these applications is realized the surface determination, the others are investigated the sensitivity.

- The surface used while determining geoid is generally expressed in high degree polynomials with two variables.
- The orthogonal polynomials can be represented are as follows;

$$N_{(x,y)} = \sum_{i=0}^{m} \sum_{\substack{j=k-i \\ i=0}}^{k} a_{ij} x^{i} y^{j}$$

- The polynomial degree should be chosen and the polynomial equation should be formed for this degree. Polynomial equation can be written for 3rd order polynomial are as follows;

$$N = a_{00} + a_{10}X + a_{01}Y + a_{20}X^2 + a_{11}XY + a_{02}Y^2 + a_{30}X^3 + a_{21}X^2Y + a_{12}XY^2 + a_{03}Y^3$$

- Generally in this problem the point coordinates are taken as measurements. The measurements can be include some outliers. In this case it is prefer that the number of measurements are selected bigger than numbers of unknowns and the adjustment solution is realized for determining the coefficients. The solution of this problem is realized according to the adjustment method and the unknown coefficients are obtained. Then, the geoid undulation of new point can be calculated using obtained polynomial .

MULTI-QUADRATIC INTERPOLATION

- In this method, the purpose is to define the research using only one function. The first stage of the multiquadratic interpolation method is the calculation of ΔN_i of reference points.

- The second stage is the calculation of the unknown coefficients of the polynomial according to adjustment method.
- ΔN_i is obtained as follows;

$$\Delta N_i = N_i - N(x_i, y_i)$$

$$\Delta N_i = N_i - N(x_i, y_i) = N_i - N_{TREND}$$

- The residual value of the undulation in interpolation point is as follows;

$$\Delta N_1 = C_1 a_{11} + C_2 a_{12} + \ldots\ldots C_m a_{1m}$$
$$\Delta N_2 = C_1 a_{21} + C_2 a_{22} + \ldots\ldots C_m a_{2m}$$
$$\downarrow$$
$$\Delta N_m = C_1 a_{m1} + C_2 a_{m2} + \ldots\ldots C_m a_{mm}$$

$$\Delta N = \begin{bmatrix} \Delta N_1 \\ \Delta N_2 \\ \downarrow \\ \Delta N_m \end{bmatrix} \qquad A = \begin{vmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1m} \\ a_{21} & a_{22} & \ldots\ldots & a_{2m} \\ \downarrow & \downarrow & \ldots\ldots & \downarrow \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mm} \end{vmatrix} \qquad C = \begin{bmatrix} C_1 \\ C_2 \\ \downarrow \\ C_m \end{bmatrix} \qquad c = A^{-1}.\Delta N$$

- N_0 undulation value calculated by the equation;

$$N_0 = N(x_0, y_0) + \sum_{i=1}^{n} c_i \left[(x_i - x)^2 + (y_i - y)^2 \right]^{1/2}$$

ADJUSTMENT PROCEDURE

- In applied sciences, measurements are made more than the number of unknowns in order to increase the accuracy and precision obtained from measurements and the results of measurements.
- If the number of measurements is higher than the number of unknowns , there is the more solution of the problem. In such as system, adjustment is made to obtain the only significant result.

- Objective function is chosen according to the minimum number of measurement residual.
- Least Square Method solves with objective function;

$$[\mathrm{Pvv}] = \min$$

- Least Absolute Value Method with;

$$\|\mathrm{pv}\| = [P|v|] = \min.$$

The Least Square Method

- The least squares method (LS) explained by Carl Friedrich Gauss and than Legendre.
- Unknown parameters calculated with the following equation in this method.

$$\underline{X} = \left(\underline{A}^T \underline{Q}_{\ell\ell}^{-1} \underline{A}\right)^{-1} \underline{A}^T \underline{Q}_{\ell\ell}^{-1} \underline{\ell}$$

- Root mean square error (RMSE);

$$m_0 = \pm\sqrt{\frac{\underline{V}^T \underline{P}\underline{V}}{f}}$$

The Least Absolute Value Method

- LAV method is a method given by Laplace in 1789 which is used to solve many different problems.
- In this operation, direct solution is not possible except special cases. The solution can be found as trial and error or linear programming problem. LAV method includes unknown parameters such as X and V.

- New unknowns are as follows for linear programming;

$$X = X^{+} - X^{-}; \qquad X^{+}, X^{-} \geq 0,$$
$$V = V^{+} - V^{-}; \qquad V^{+}, V^{-} \geq 0$$

$$\begin{bmatrix} A & -A & -I & I \end{bmatrix} \begin{bmatrix} X^{+} \\ X^{-} \\ V^{+} \\ V^{-} \end{bmatrix} = [\ell],$$

$$f = b^{T} X = \left[P \left| V \right|\right] = P^{T} V = p^{T} \left[V^{+} \quad V^{-}\right] = \min .$$

NUMERICAL APPLICATION

- In this application, the coordinates of 20 points in Samsun region and their ellipsoidal and orthometric heights were used.
- The unknown parameters were found for these points according to both polynomial and multiquadratic method according to LS method and LAV method.
- Comparisons were made between four methods.

- The finding unknown parameters due to four different methods

X	Polynomial		Multi-quadratic	
	LS	LAV	LS	LAV
a_{00}	28.17204963	28.16889046	226.17233035	218.06967963
a_{10}	-0.03924029	-0.03776712	-0.00003924	-0.00003777
a_{01}	-0.03436085	-0.03185382	-0.00003436	-0.00003185

- The equation of polynomial and multiquadratic interpolation are given follow;
- Polynomial Interpolation (LS)

$$N = 28.1720 - 0.0392\,X - 0.0344\,Y$$

- Polynomial Interpolation (LAV)

$$N = 28.1689 - 0.0378\,X - 0.0319\,Y$$

- Multiquadratic Interpolation (LS)

$$N = 226.1723 - 0.000039\,X - 0.0000344\,Y$$

- Multiquadratic Interpolation (LAV)

$$N = 218.0697 - 0.0000378\,X - 0.0000319\,Y$$

RESULTS AND DISCUSSION

- After finding these equation, selected new 10 points.

Point	East	North	Orthometric Height(H)	Ellipsoidal Height(h)	N=h-H
409	542850.957	4567840.372	1.895	30.236	28.341
413	542336.182	4569055.295	2.593	30.798	28.205
466	546205.568	4564605.175	8.893	37.141	28.248
473	548671.970	4563911.807	10.183	38.296	28.113
472	548221.835	4563915.699	12.231	40.362	28.131
408	542777.297	4567328.650	2.188	30.521	28.333
471	547780.410	4563813.093	16.606	44.748	28.142
469	547051.761	4563884.710	13.185	41.368	28.183
399	545088.541	4564666.379	10.689	39.010	28.321
291	541756.883	4568794.285	1.314	29.639	28.325

- The correction value according to Polynomial Interpolation

Point Number	Geoid Undulation ( ) (m)	Calculated Geoid Undulation( ) (m) (LS)	V (m)	Calculated Geoid Undulation( ) (m) (LAV)	V (m)
409	28.341	28.175	0.166	28.168	0.173
413	28.205	28.145	0.060	28.139	0.066
466	28.248	28.186	0.062	28.184	0.064
473	28.113	28.129	-0.016	28.131	-0.018
472	28.131	28.144	-0.013	28.145	-0.014
408	28.333	28.197	0.136	28.190	0.143
471	28.142	28.163	-0.021	28.163	-0.021
469	28.183	28.185	-0.002	28.184	-0.001
399	28.321	28.222	0.099	28.217	0.104
291	28.325	28.175	0.150	28.167	0.158

- The correction value according to Multi-quadratic Interpolation

Point Number	Geoid Undulation ( ) (m)	Calculated Geoid Undulation( ) (m) (LS)	V (m)	Calculated Geoid Undulation( ) (m) (LAV)	V (m)
409	28.341	28.35	-0.009	28.34	0.002
413	28.205	28.24	-0.035	28.23	- 0.025
466	28.248	28.39	-0.142	28.38	- 0.132
473	28.113	28.39	-0.277	28.39	- 0.272
472	28.131	28.5	-0.369	28.49	- 0.364
408	28.333	28.32	0.013	28.30	0.029
471	28.142	28.22	-0.078	28.22	- 0.075
469	28.183	28.25	-0.067	28.24	- 0.058
399	28.321	28.29	0.031	28.29	0.034
291	28.325	28.27	0.055	28.26	0.061

- In this study conducted for geoid determination for Samsun region. The polynomial interpolation and multiquadratic interpolation according to LS and LAV method were compared first by using the ellipsoidal and orthometric heights of 20 points.

- After the unknowns parameters in first degree polynomial were found according to both methods. For testing these method another 10 points were used in the research area and residual were found.
- It was found that the multiquadratic interpolation method gave more accurated results.

THANK YOU FOR YOUR LISTENING.