Precise Satellite Positioning based on a Search Procedure in the Coordinate Domain

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Key words: GNSS/GPS, precise satellite positioning; GNSS data processing

SUMMARY

The concept of precise satellite positioning, based on an ambiguity function (AF), has been known since the beginning of GPS. In this approach, a search procedure is conducted in the coordinate domain. However, the methods based on a search procedure in an ambiguity domain (e.g., the LAMBDA method) have prevailed over the AF-based methods because of their higher efficiency and better theoretical grounds. Recently, due to the development of the GNSS system, there are more and more satellites available for surveying. In such a case, the computational load considerably rises. The computational load increase is due to the large dimension of the search region in an ambiguity domain. Therefore, it seems reasonable to reconsider the idea of applying the AF-based methods in which a search procedure is moved from the many-dimensional ambiguity domain into the three-dimensional coordinate domain. In such an approach, the computational complexity is robust to the number of satellites. A substantial improvement of the AF-based methods has been developed in the last decade. The Modified Ambiguity Function Approach (MAFA) is based on a search procedure conducted in the coordinate domain. The paper presents the improvements that can be applied to the search procedure. A way of forming a search region is described. A search region encloses the candidate set. The distance between the closest candidates is computed using actual satellites' configuration. The formula of the criterion of seeking the final solution is presented and discussed. The numeric experiment has been designed and performed. The test results demonstrate a good performance of the proposed method.

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1. INTRODUCTION

The evident new opportunities induced from the Global Navigation Satellite Systems (GNSS) development are associated with new challenges in signal processing methods. Besides a new observation, each new satellite also provides a new unknown value, i.e., ambiguity to be estimated. The rise in amounts of parameters increases the computational load. Even the most popular and efficient methods of optimizing the computational process in the classic approach (Teunissen, 1995, 1999; de Jonge and Tiberius, 1996; Liu et al., 1999; Chang et al., 2005; Zhou 2011; Xu, 2012) are sensitive to the number of ambiguities. Xu has shown that if the number of satellites exceeds a specific level, decorrelation techniques do not improve the search procedure efficiency (Xu, 2001). Therefore, we decided to reconsider the concept of searching for integer ambiguities in a coordinate domain. The search space dimension is constant and amounts to three in such a case. Thus, the search space dimension does not depend on the number of satellites. The computational load of a search procedure does not rise with increasing the number of satellites. Searching for a fixed solution in the coordinate domain dates back to the beginning of satellite navigation system development. It utilizes the Ambiguity Function (AF) method in the computation process. The AF method of processing the GPS carrier phase observations was proposed by Counselman and Gourevitch (1981) and then by Remondi (1984). The AF method has been substantially improved and presented in the form of the Modified Ambiguity Function Approach (MAFA) by Cellmer et al. (2010). The MAFA method has been described in many papers, for example, (Cellmer 2012 2013 2015; Cellmer et al. 2010 2017 2018 2021; Kwasniak et al. 2016 2017; Baselga 2010; Nowel et al. 2018; Wang et al. 2007; Wang et al. 2019; Wu et al. 2019, Zhao et al., 2022). The present contribution takes advantage of some solutions used in the MAFA method: defining the search region and setting an optimal search step (Cellmer et al. 2018, 2021). The search region is here the approximate position's error ellipsoid. The optimal search step's length derivation is based on the Voronoi cells (VC) concept and an actual satellites' configuration. According to Hassibi and Boyd (1998), the Voronoi cell of the point X_0 in a lattice is the set of points in space closer to X_0 than to any other point in the lattice. Xu (2006) used the VC to analyze an ambiguity resolution problem. In Teunissen's integer equivariant estimation theory, the VC is called an "in-pull region," where distance accounts for the non-Euclidean metric (Teunissen, 1999, 2003). This meaning of the VC is assumed in our work. The search step's length resulting from the method proposed by Cellmer et al. (2021) is the longest distance between neighboring candidates that one at the same time guarantees we do not skip any VC included in the search region. The inputs for a search procedure proposed in this contribution are approximate position and its covariance matrix. Both magnitudes are obtained from a float solution. In the next section, the classic form of the mixed integer-real least squares estimation math model is introduced to familiarize readers with the notation used in this work. The third section is devoted to the main topic of this work, i.e., the search procedure in a coordinate domain (SinCD). Sole results from a float solution are used in this procedure as input. Two types of computational experiments are described in the fourth section. They are based on real and simulated data. The results of those experiments illustrate the effects of the proposed method. The conclusions are presented in the last section.

2. THE COMPUTATIONAL PROCESS OF PRECISE POSITIONING

2.1 Math model

The math model of precise satellite positioning is (Hoffman-Wellenhof et al., 2008; Leick et al., 2015):

$$\mathbf{y} = \mathbf{Aa}+\mathbf{Bb}+\mathbf{e} \tag{1}$$

where **y** is the data vector, **a** is the integer ambiguity vector, **b** is the real parameter vector, **A** and **B** are the corresponding design matrices, and **e** is the noise vector. The **y** vector consists of double differenced (DD) carrier phase observations and/or code observations. The **b** vector consists of the baseline components. It can also contain other parameters, e.g., atmospheric corrections. The three stages of the classic approach of precise satellite positioning can be distinguished:

1) The unconstrained least-squares (LS) solution ("float solution").

2) The integer ambiguity resolution. This stage also includes a validation of the results.

3) The solution with integer ambiguities. This solution is called the "fixed solution."

In the first stage, the integer nature of ambiguities is discarded. The LS estimates of the **a** and **b** vectors are obtained from:

$$\begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{\hat{a}} & \mathbf{Q}_{\hat{a}\hat{b}} \\ \mathbf{Q}_{\hat{b}\hat{a}} & \mathbf{Q}_{\hat{b}} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{\mathbf{T}}\mathbf{P}_{y}\mathbf{y} \\ \mathbf{B}^{\mathbf{T}}\mathbf{P}_{y}\mathbf{y} \end{bmatrix} \tag{2}$$

where: $\begin{bmatrix} \mathbf{Q}_{\hat{a}} & \mathbf{Q}_{\hat{a}\hat{b}} \\ \mathbf{Q}_{\hat{b}\hat{a}} & \mathbf{Q}_{\hat{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{\mathbf{T}}\mathbf{P}_{y}\mathbf{A} & \mathbf{A}^{\mathbf{T}}\mathbf{P}_{y}\mathbf{B} \\ \mathbf{B}^{\mathbf{T}}\mathbf{P}_{y}\mathbf{A} & \mathbf{B}^{\mathbf{T}}\mathbf{P}_{y}\mathbf{B} \end{bmatrix}^{-1}$ and $\mathbf{P}_{\mathbf{y}}$ is the observation weight matrix.

The LS estimator of the integer ambiguity vector is defined as:

$$\breve{\mathbf{a}} = \arg\min_{\mathbf{z}\in \mathbf{Z}^{n}} \left\| \hat{\mathbf{a}} - \mathbf{z} \right\|^{2}_{\mathbf{Q}_{\hat{a}}} \tag{3}$$

The expression at the arg min(.) operator denotes a weighted norm of the ($\hat{\mathbf{a}}-\mathbf{z}$) vector:

$$\left\| \hat{\mathbf{a}} - \mathbf{z} \right\|^{2}_{\mathbf{Q}_{\hat{a}}} = \left(\hat{\mathbf{a}} - \mathbf{z}\right)^{\mathrm{T}} \mathbf{Q}_{\hat{a}}^{-1} \left(\hat{\mathbf{a}} - \mathbf{z}\right) \tag{4}$$

The solution domain of formula (3) has a discrete character. Therefore, the search procedure is applied in the second stage. The ambiguity resolution step also includes the validation of the results. The last step in precise positioning is a "fixed solution," which incorporates the integer ambiguities obtained in the previous step:

$$\breve{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}}\mathbf{Q}_{\hat{\mathbf{a}}}^{-1}\left(\hat{\mathbf{a}} - \breve{\mathbf{a}}\right) \tag{5}$$

The critical part of this three-step precise positioning is ambiguity resolution using a search procedure conducted in *n*-dim ambiguity space in the classic approach. The concept of conducting the search procedure in the 3-dim coordinate space is presented below.

2.2 The search procedure in a coordinate domain

The conditional solution (5) derived from model (2) describes the change of the **b** vector resulting from the modification of the **a** vector. The method proposed in this work utilizes the relationship of type (5) but with swapped the **a** and **b** vectors and corresponding cofactor matrices. The conditional solution of the **a** vector resulting from the change of the **b** vector is:

$$\breve{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{Q}_{\hat{\mathrm{a}}\hat{\mathrm{b}}}\mathbf{Q}_{\hat{\mathrm{b}}}^{-1}\left(\hat{\mathbf{b}} - \breve{\mathbf{b}}\right) \tag{6}$$

The formula (6) is applied in the proposed method as an initial phase of a search procedure. The process starts with forming a search region in coordinate space. Subsequently, a grid of candidates inside this space is established, then each candidate is tested. The proposed method assumes that the float solution is obtained the same way as the traditional approach. Next, the search region, as the confidence region of the float solution, is established. In the 3-dim coordinate space, this region takes the form of an error ellipsoid of an approximate position from the float solution. The error ellipsoid axes lengths are computed from the formulas:

$$r_x = \sqrt{\frac{\chi_c^2}{\mu_x}}, r_y = \sqrt{\frac{\chi_c^2}{\mu_y}}, r_z = \sqrt{\frac{\chi_c^2}{\mu_z}} \tag{7}$$

where $\chi_{\mathbf{c}}^2$ is the critical value of the χ^2 distribution with 3 degrees of freedom, and μ_x, μ_y, μ_z are eigenvalues of the $\mathbf{Q}_{\hat{\mathbf{b}}}^{-1}$ matrix. The initial candidates (IC) grid is formed inside the search space. The distances between the IC have to be sufficiently small so as not to omit any of the VC located inside the search region. On the other hand, the distance between neighboring IC should be as long as possible so that the number of IC is small. Cellmer et al. (2021) described the rules of forming the search region and setting the grid of IC inside it, which account above conditions. Figure 1 shows the example of an arrangement of ICs inside the error ellipsoid. The next step of the data processing is the essence of the proposed method. Let us name the ICs with $\mathbf{b}_{IC}$. For *i*-th $\mathbf{b}_{IC}$ we compute the corresponding ambiguity vector:

$$\mathbf{a}_{ICi} = \hat{\mathbf{a}} - \mathbf{Q}_{\hat{\mathrm{a}}\hat{\mathrm{b}}}\mathbf{Q}_{\hat{\mathrm{b}}}^{-1}\left(\hat{\mathbf{b}} - \mathbf{b}_{ICi}\right) \tag{8}$$

Each ambiguity vector obtained by (8) is rounded to the nearest integer vector:

$$\mathbf{a}_{FCi} = \boldsymbol{round}\left(\mathbf{a}_{ICi}\right) \tag{9}$$

The integer vector $\mathbf{a}_{FCi}$ corresponds to the *i*-th final candidate. The final candidates are tested, and this one, which minimizes criterion:

$$\mathbf{a}_{FC\min} = \arg\min_{\mathbf{z}\in \boldsymbol{Z}^n} \left\|\hat{\mathbf{a}} - \mathbf{a}_{FCi}\right\|^2_{\mathbf{Q}_{\hat{a}}} \tag{10}$$

takes part in computing the final fixed solution from formula (5).

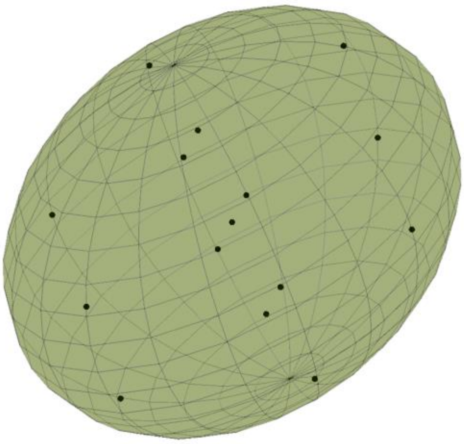

Fig. 1 The example of the initial candidates' arrangement inside the error ellipsoid.

The computational process flow chart is depicted in Figure 2. The first three steps of the computation process were discussed in (Cellmer et al., 2018; 2021).

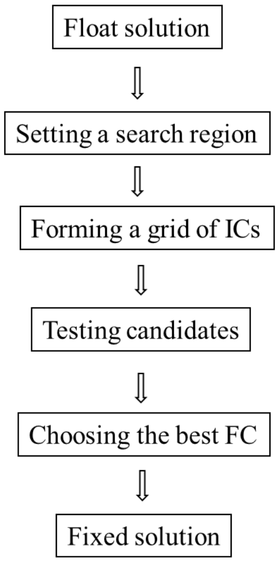

Fig. 2 Computational process of precise positioning based on a search procedure in coordinate domain

The primary issue of the proposed method is included in the fourth step of the computational process: ICs testing. This step comprises operations (8) and (9), and also computing the norm on the right hand of formula (10). Note the number of ICs can be greater than the number of FCs. On the other hand, a few ICs may fall into the same VC and consequently give the same FC. This is the cost of reducing the dimension of a search space and has no significant impact on a computational load providing a search step length is set correctly.

3. NUMERICAL EXPERIMENT

The subject of the experiment is to test the performance of the new search procedure. The analysis compares the new method with the classic one implemented in the LAMBDA method. In the latter approach, a search for a fixed solution conducts in an ambiguity domain. The very short baseline (1.62 m) was chosen for the tests to avoid any impact of atmospheric delays as extraneous factors irrelevant in comparative analysis. Such a short baseline lets to take the assumption that all atmospheric delays are eliminated by forming double differences of observations. Only the single-frequency GPS signals were processed in the experiment. The data used in this test was already utilized in another experiment, described by Cellmer et al. (2021). The data was collected in two permanently operating EUREF stations: WTZR and WTZZ (Germany), on 12, May, 2019. The interval between consecutive epochs was 30 seconds. The WTZR was set as a reference station and the WTZZ as a rover. The precise position of the rover was computed based on the 24h-long session. This position was used as a reference for comparisons. Next, the whole observation set was divided into short observation sessions. The 2000 subsequent epochs from the beginning of the 24h-long session were adopted as the starting epochs for the successive short sessions. The tests were carried out for sessions of various lengths: from a single-epoch solution to 30 minutes sessions. There were 2000 separate sessions tested for each session length. There were seven satellites in view in most sessions. The approximate position (float solution) in each session was obtained using the DD code and phase observations. The error ellipsoid of the approximate position was formed using the covariance matrix from a float solution and setting the confidence level to 0.999. The final precise position was estimated using two approaches. The first way is the method proposed in the article and the second way is the LAMBDA method. For simplicity, the first approach is denoted as #1 and the second as #2. The results were compared. In the second approach, the computations were conducted using the Matlab function *LAMBDA_06.m* retrieved from the TU Delft website: https://www.tudelft.nl/citg/over-faculteit/afdelingen/geoscience-remote-sensing/research/lambda/lambda (LAMBDA method, 2021). Table 1 lists the solutions' success rates (SR) in both approaches. The SR values were computed separately for each session length. The ambiguities obtained from 24h session were assumed to be actual. Thus, the tested short-session solutions containing the actual ambiguity values are considered the correct solutions. The SR values are expressed in the percentage of correct solutions among 2000 sessions. The differences in SR ranged from 0 (for 30 min sessions) to 5 (for 10 min sessions). The proposed method achieved higher SR for single-epoch solutions. In other cases, better SR was obtained by the LAMBDA method. Although the percentage of correct solutions is not precisely the same in both approaches, the differences in their values are minor. The mean value of absolute differences in SR is 2.0%.

Table 1 Success rates as a percentage of correct solutions in the samples of 2000 observation sessions

	Session length [minutes]							
	Single-epoch	1	2	5	10	15	20	30
	Success rates [%]							
#1 (SinCD)	45.6	59.8	70.2	85.3	94.8	98.2	98.6	100.0
#2 (LAMBDA)	45.2	60.7	72.4	90.0	99.8	100.0	100.0	100.0

The core element of the analysis is a comparison of the time needed for finding the correct solution in both approaches. Figure 3 depicts the mean computation time for 2000 sessions. Each session length was processed separately. On the X-axis are session lengths expressed in minutes, whereas on the Y-axis is a computation time in seconds. The log scale has been applied on the Y-axis. The computation time in the LAMBDA method amounts to slightly over 10^{-3} seconds for all session lengths.

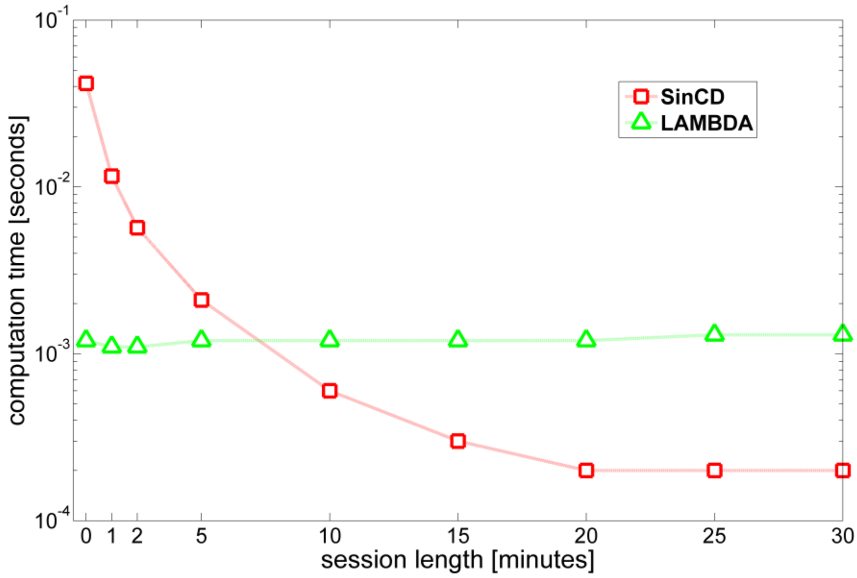

Fig. 3 Sessions' lengths and a computation time in the SinCD and the LAMBDA methods.

The impact of a session length on computation time is significant in the SinCD method. Its value ranges from 0.2*10-3 (for the longest sessions) to 41.8*10-3 in the case of single-epoch. Shorter computation time in the SinCD method than in the LAMBDA is for 10-minute and longer sessions.

4. CONCLUSIONS

The dimension of the search space is non-negligible for search procedure efficiency. The time needed for obtaining a solution raises considerably when the search space dimension is greater. Therefore, the concept of searching for a fixed solution in a coordinate domain instead of in an ambiguity domain was investigated. As a result, the new method was developed and tested. A major advantage of the proposed method is moving a search procedure from multi-dimensional ambiguity space to only three-dimensional coordinate space. Such an operation allows for shortening computation time. It has great importance in the prospect of increasing the number of satellites resulting from developing modern satellite navigation systems. The test results confirmed shortening computation time when using the proposed method and that its advantage over traditional methods unveils in the case of many satellites.

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