XXVII FIG CONGRESS
11–15 SEPTEMBER 2022
Warsaw, Poland

Volunteering for the future – Geospatial excellence for a better living

Trajectory evaluation using repeated rail-bound measurements

Gereon Tombrink, Ansgar Dreier, Lasse Klingbeil, Heiner Kuhlmann

ORGANISED BY FIG SGP

PLATINUM SPONSORS esri Trimble.

Mobile Mapping


https://researchleap.com/


https://www.diamondaircraft.com


https://grunder.ch

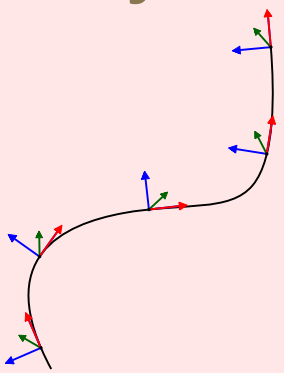

https://leica-geosystems.com

Importance of Trajectory Estimation

Precision? Accuracy?

Trajectory

$$\overrightarrow{\mathbf{p}_i} = \begin{bmatrix} t_x \\ t_y \\ t_z \\ \phi \\ \theta \\ \psi \end{bmatrix}$$



+

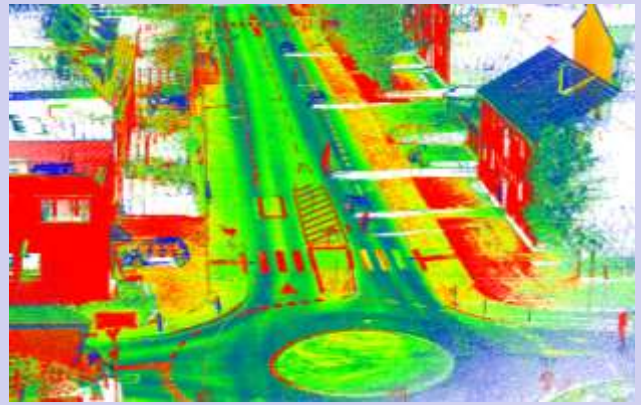
Object Acquisition

https://zofre.de

http://www.riegl.com/

→

Point Cloud

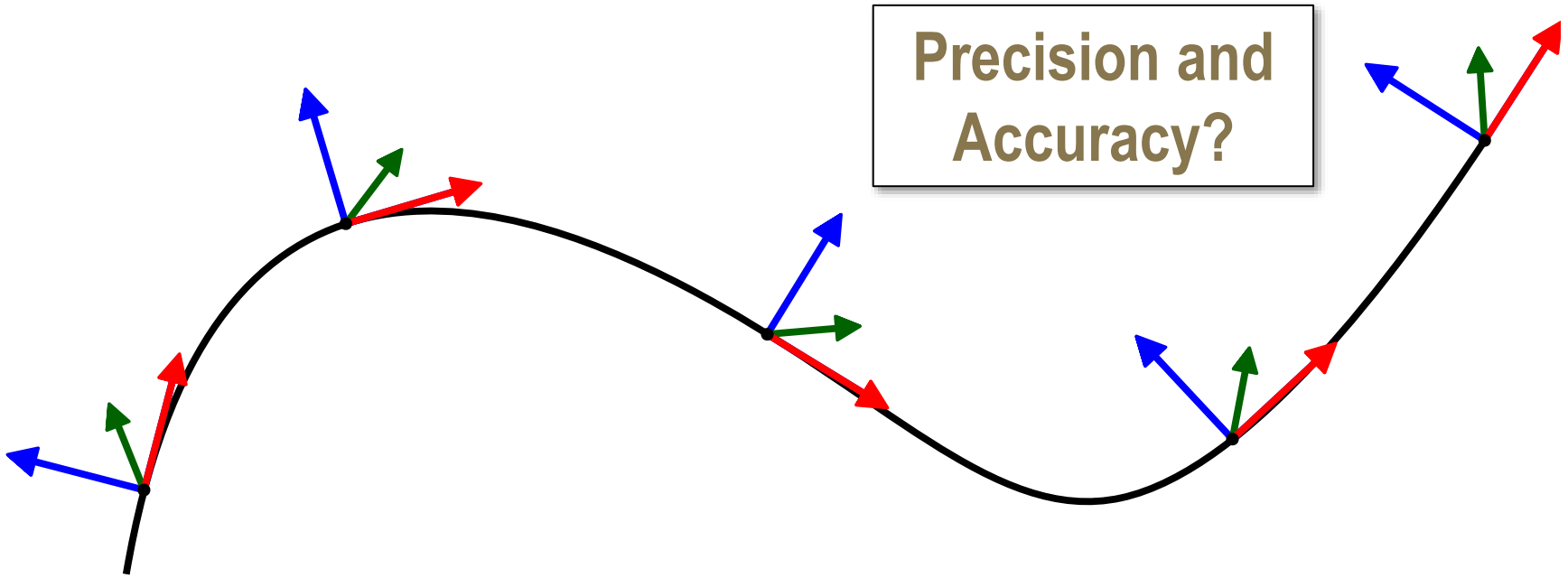


$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + R_n^e(L,B) \cdot R_b^n(\phi,\theta,\psi) \cdot \left(\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + R_s^b(\alpha,\beta,\gamma) \cdot \begin{bmatrix} 0 \\ d \cdot \sin(b) \\ d \cdot \cos(b) \end{bmatrix} \right) = \begin{bmatrix} X_e \\ Y_e \\ Z_e \end{bmatrix}$$

Trajectory Evaluation – Goal

Trajectory: Position and Orientation over time

$$\overrightarrow{\mathbf{p}_i} = \begin{bmatrix} t_x \\ t_y \\ t_z \\ \phi \\ \theta \\ \psi \end{bmatrix}$$



Trajectory Evaluation – Approaches

Theoretical Approaches

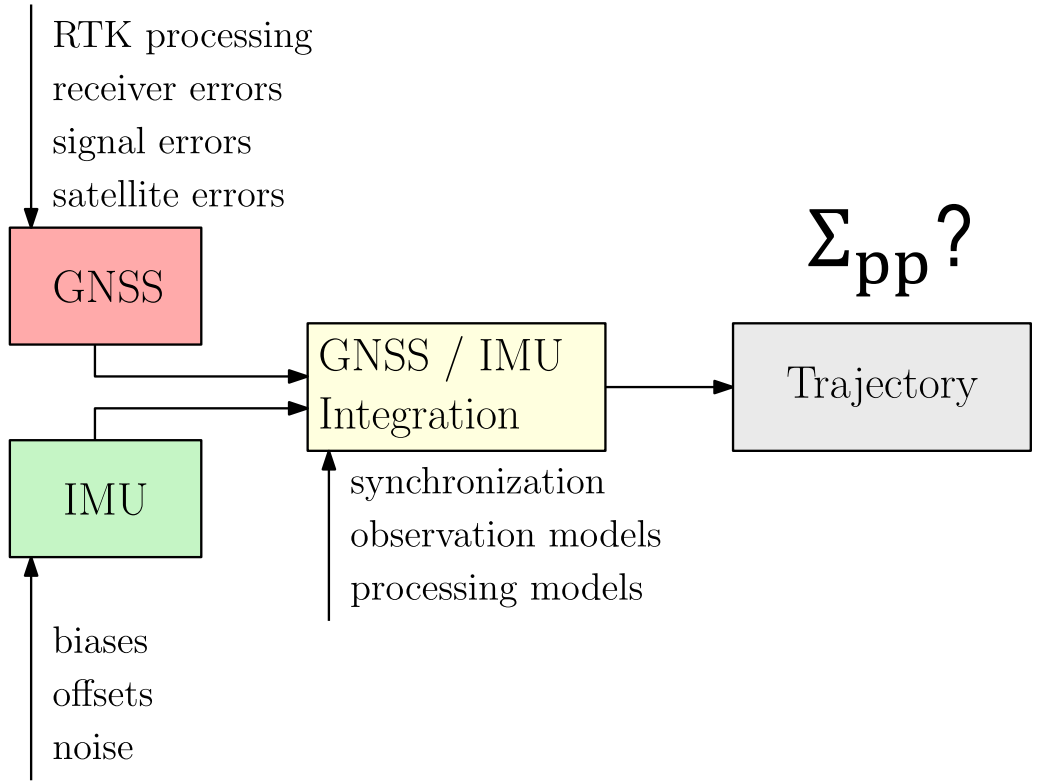
Empirical Approaches

Trajectory Evaluation – Theoretical Approaches

- Variance-covariance propagation
- Monte-Carlo Simulation

Challenges:

- Combination of multiple sensors
- Closed-source filter-algorithms



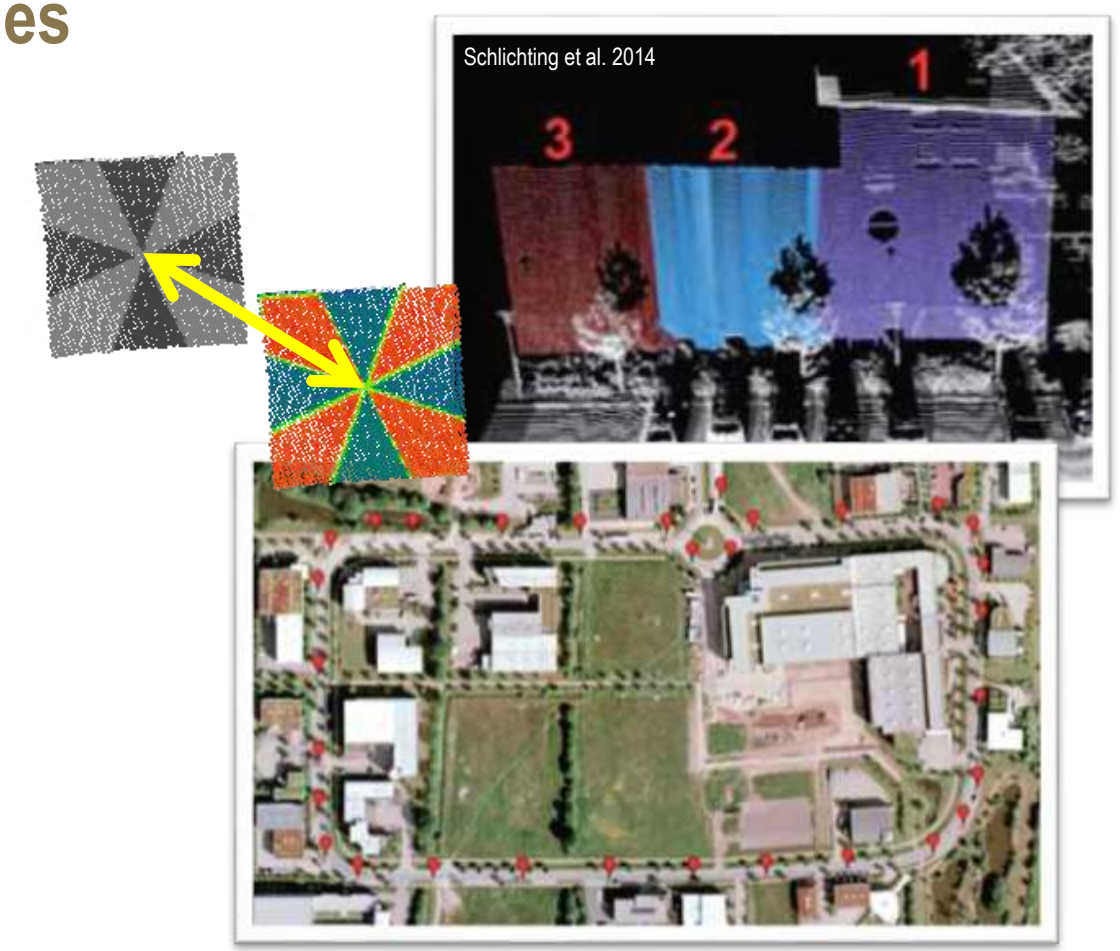
Trajectory Evaluation – Empirical approaches

Indirect methods

- Point-cloud based
- Control point / plane extraction

Disadvantage:

- Isolated trajectory evaluation difficult



Trajectory Evaluation – Empirical approaches

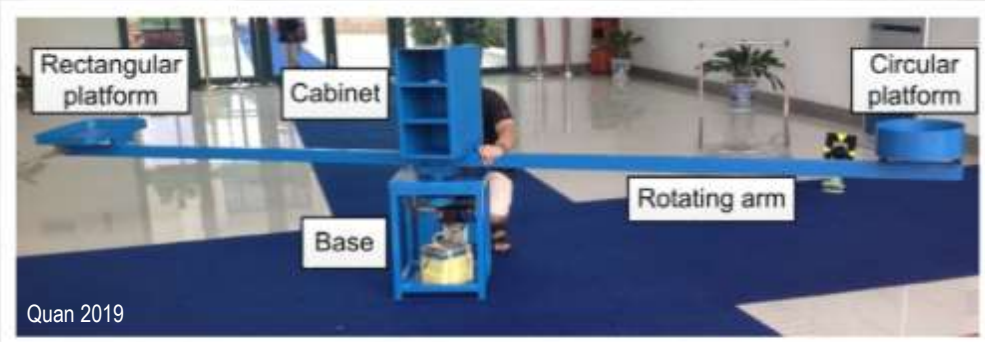
Direct methods

- Pose-by-pose comparison
- Reference: other GNSS solutions / total station

Until now:

- No repetition / repetition with constraints
- Analysis limited to positions


Stephenson et al. 2011



Quan 2019

Trajectory Evaluation – Empirical approaches

Direct m

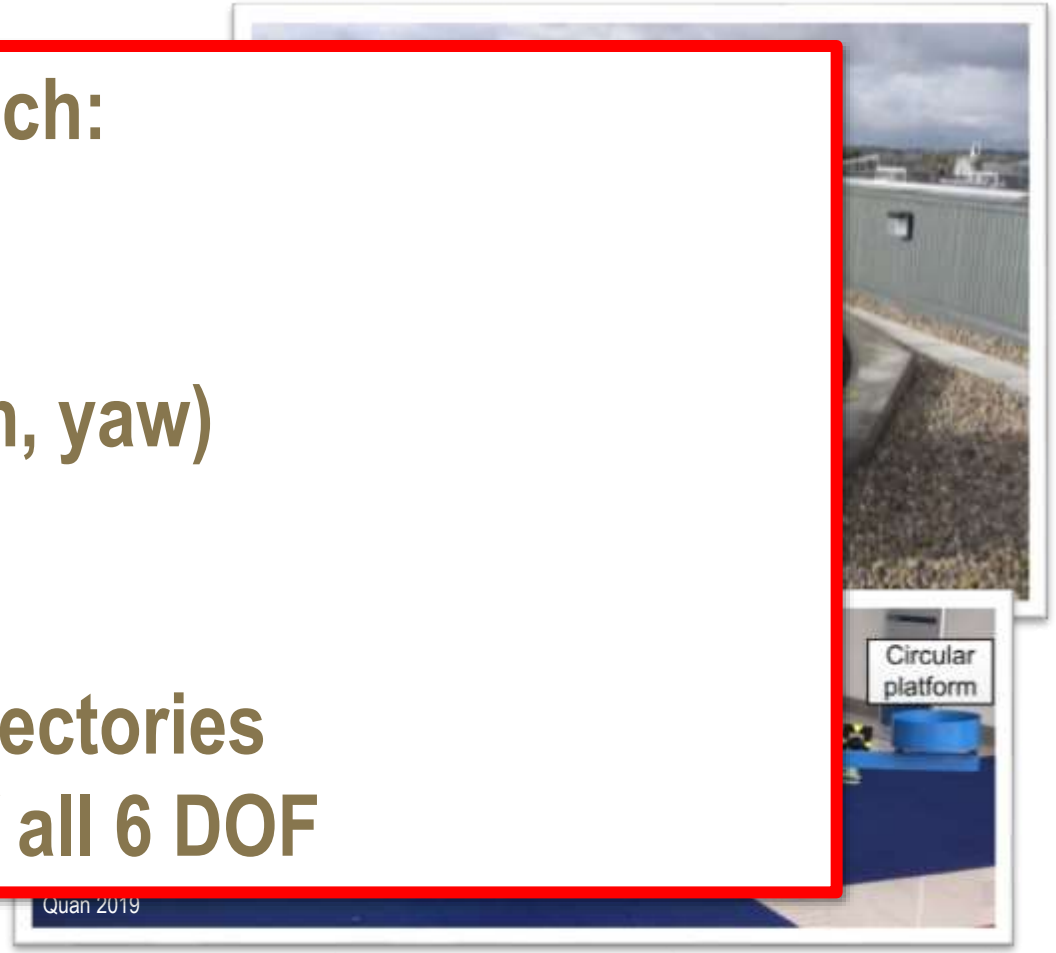
- Pose-
- Refer

Until nov

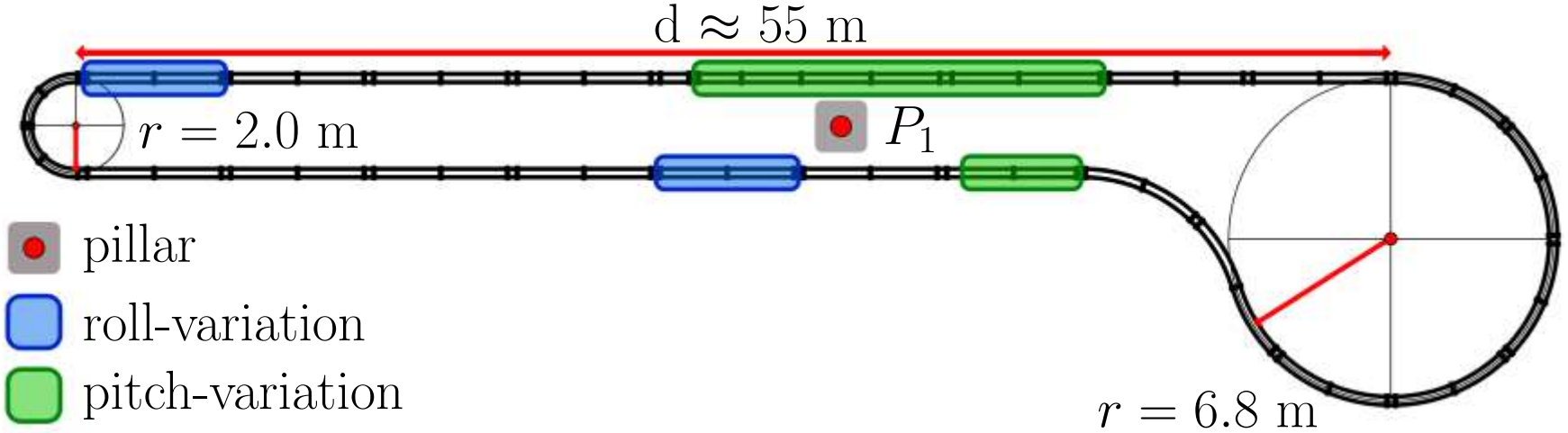
- No re
- Analysis limited to positions

Our Approach:

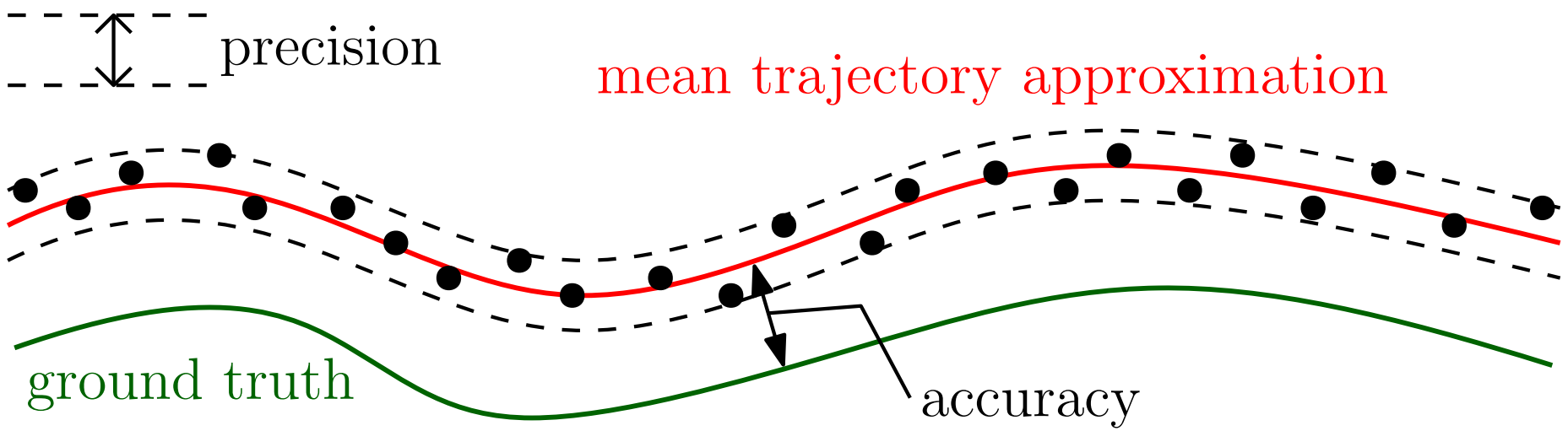
1. **Rail track**
   - 140 m in length
   - All 6 DOF (x, y, z, roll, pitch, yaw)
2. **Own methodology**
   - Evaluation of repeated trajectories
   - Precision and Accuracy of all 6 DOF



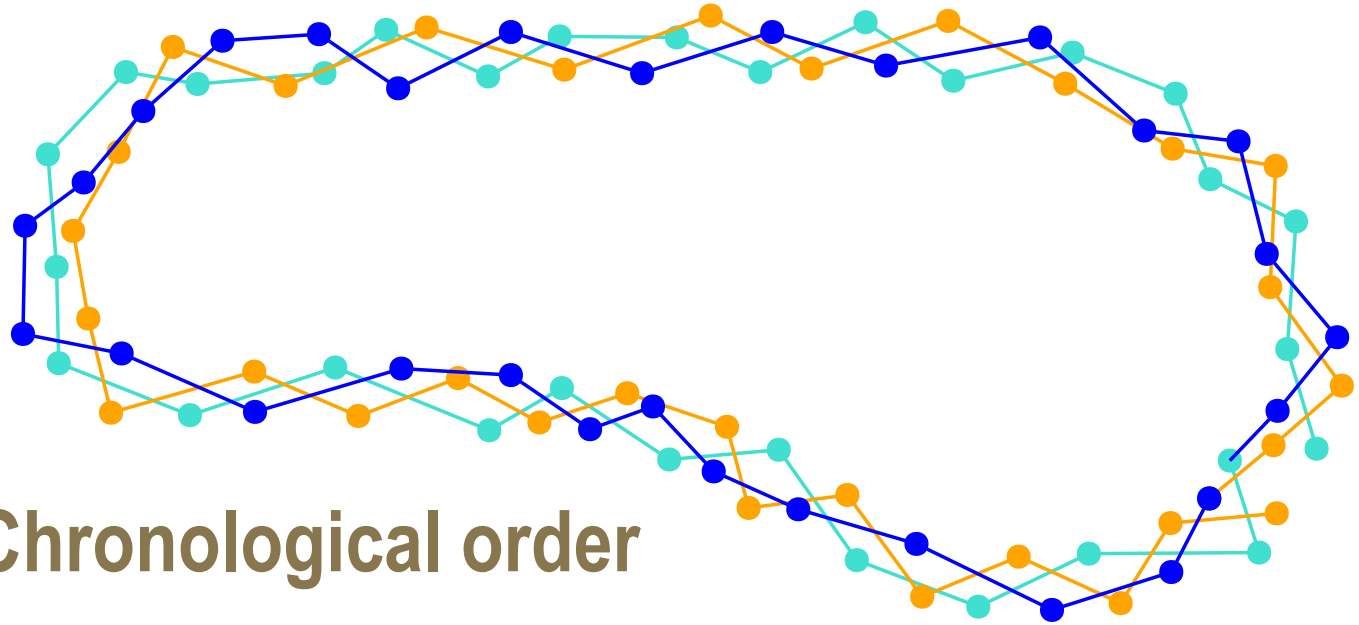
Rail Track at the University of Bonn



Trajectory Evaluation – Goal: Precision and Accuracy



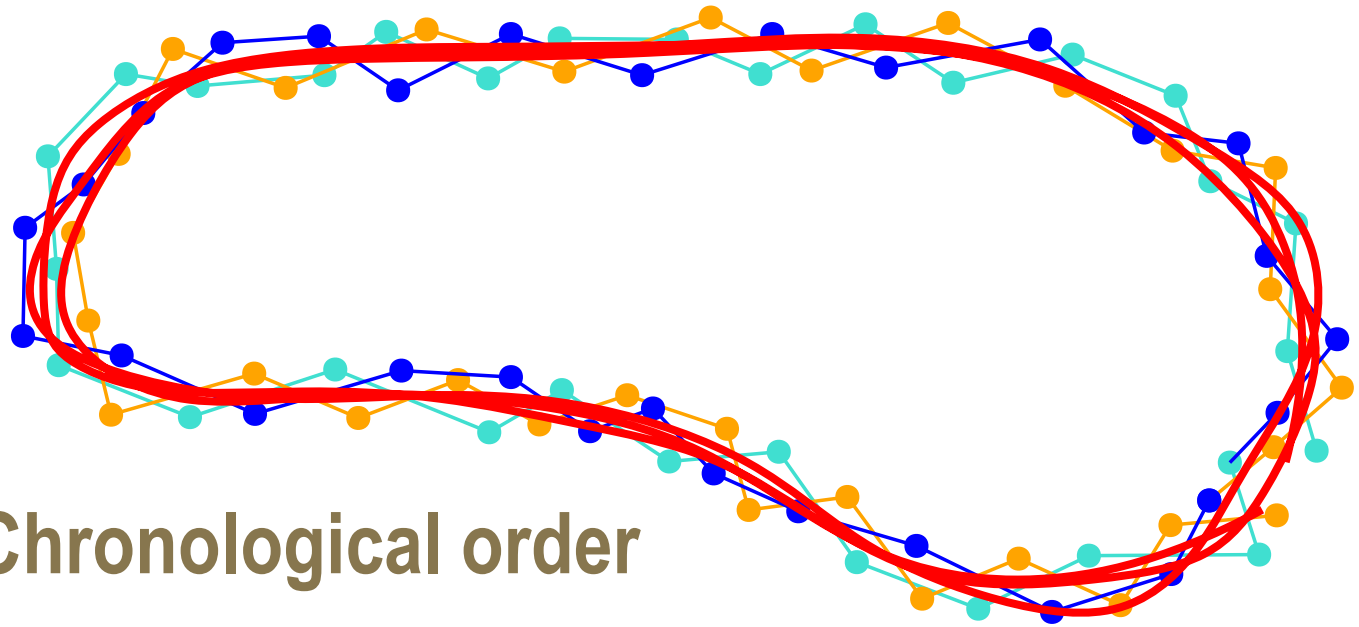
Trajectory Evaluation using repeated rail-bound measurements



Chronological order

- Lap 1
- Lap 2
- Lap 3

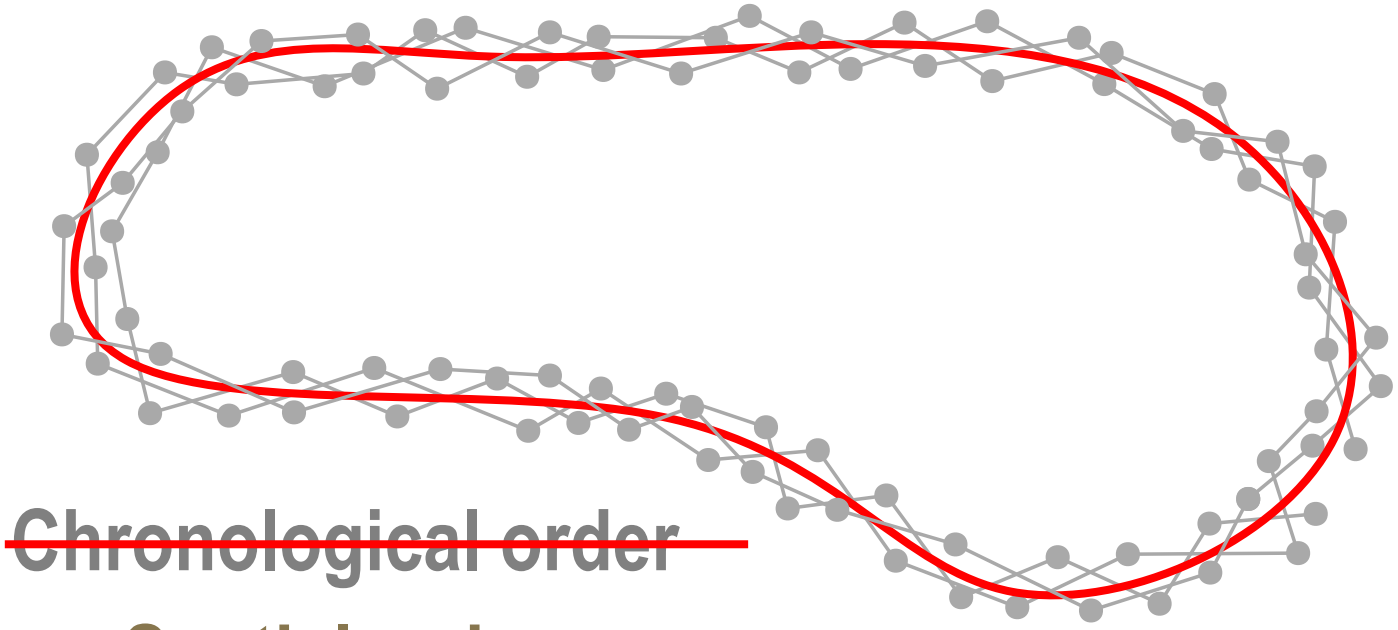
Trajectory Evaluation using repeated rail-bound measurements



Chronological order

- Lap 1
- Lap 2
- Lap 3
- Mean

Trajectory Evaluation using repeated rail-bound measurements

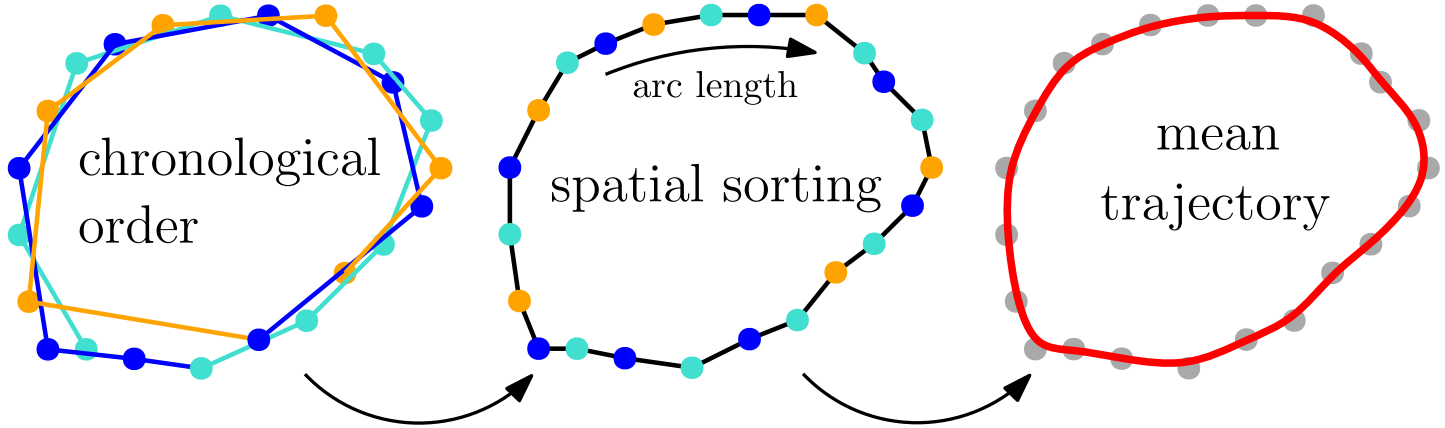


~~Chronological order~~

Spatial order

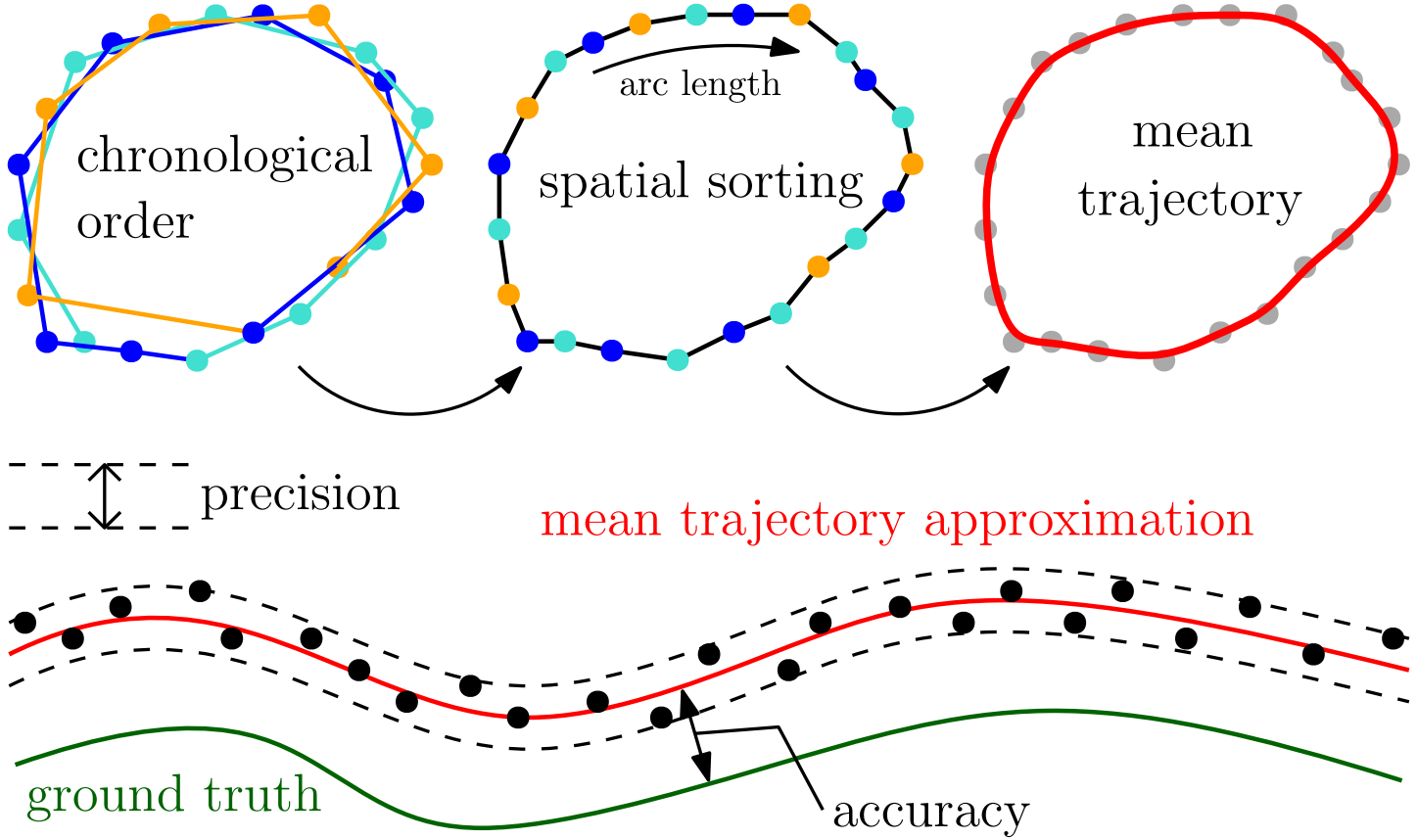
- Lap 1
- Lap 2
- Lap 3
- Mean

Proposed Methodology – Overview



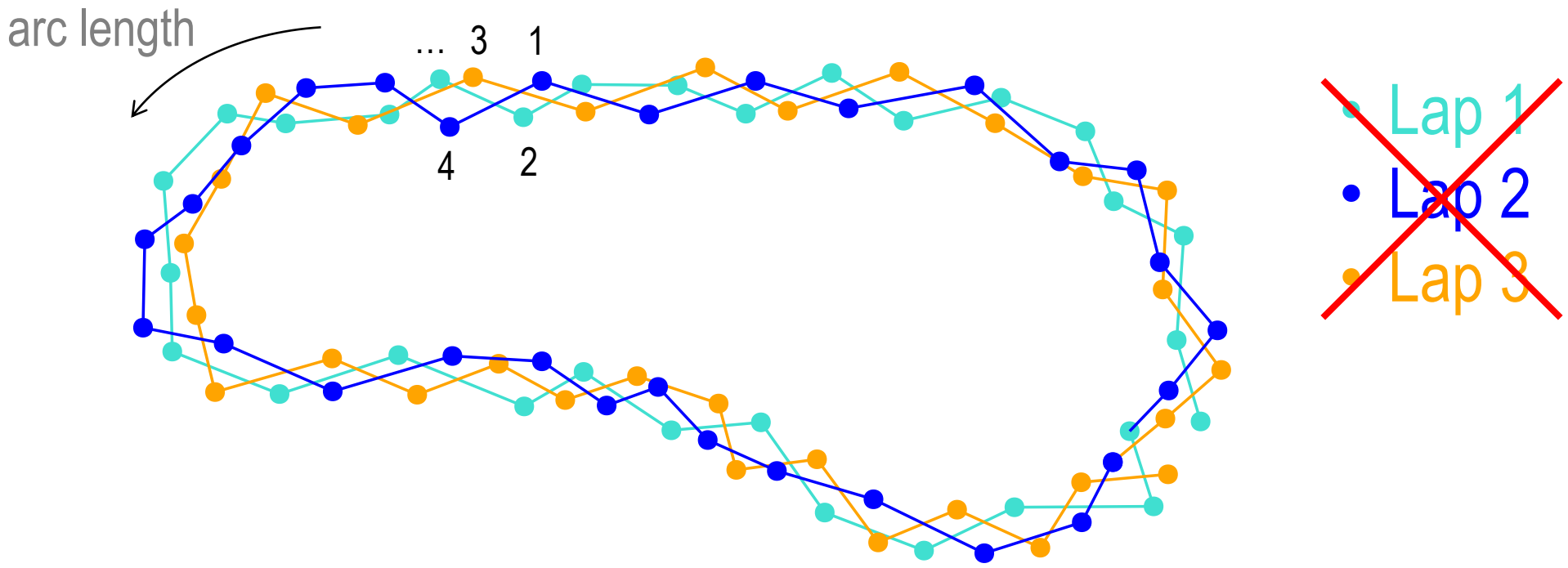
1. Spatial sorting
2. Approximation
3. Quality measures

Proposed Methodology – Overview

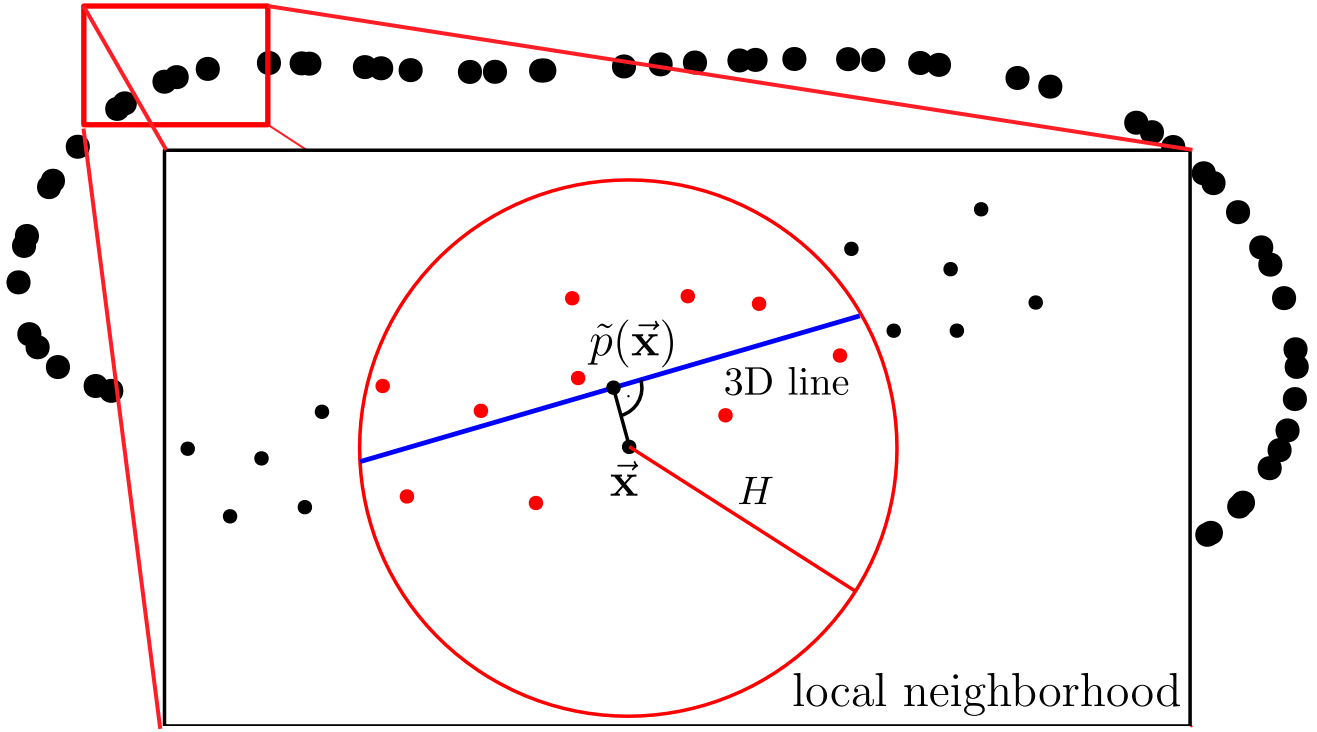


1. **Spatial sorting**
2. **Approximation**
3. **Quality measures**

Proposed Methodology – Spatial Sorting



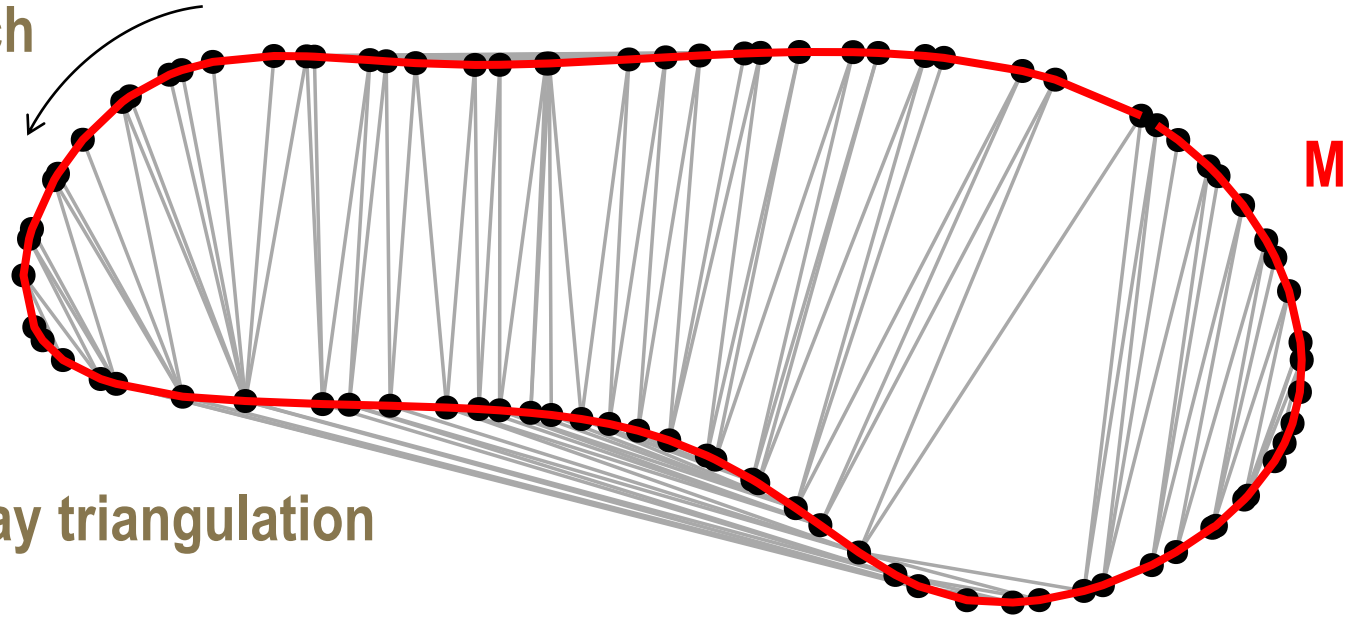
Proposed Methodology – Spatial Sorting



Smoothing using Moving-Least-Squares Approximation

Proposed Methodology – Spatial Sorting

breadth-first-search



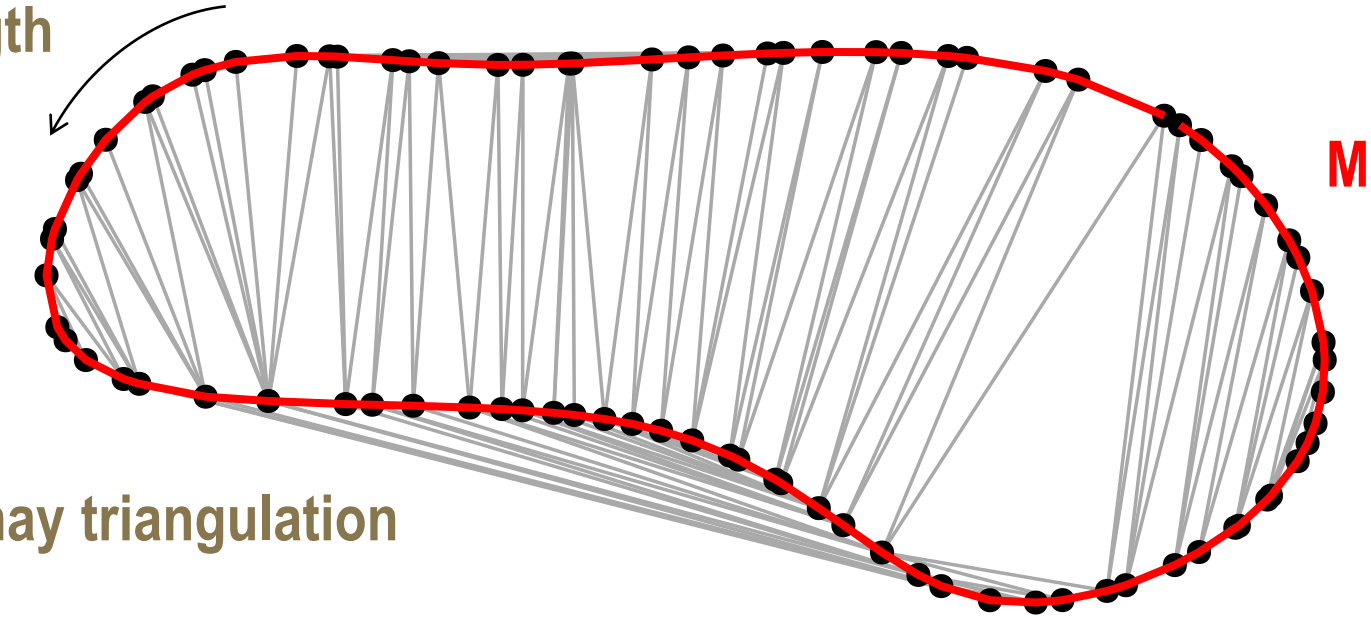
Delaunay triangulation

Sorting using
Minimum-Spanning-Tree

Proposed Methodology – Spatial Sorting

arc length

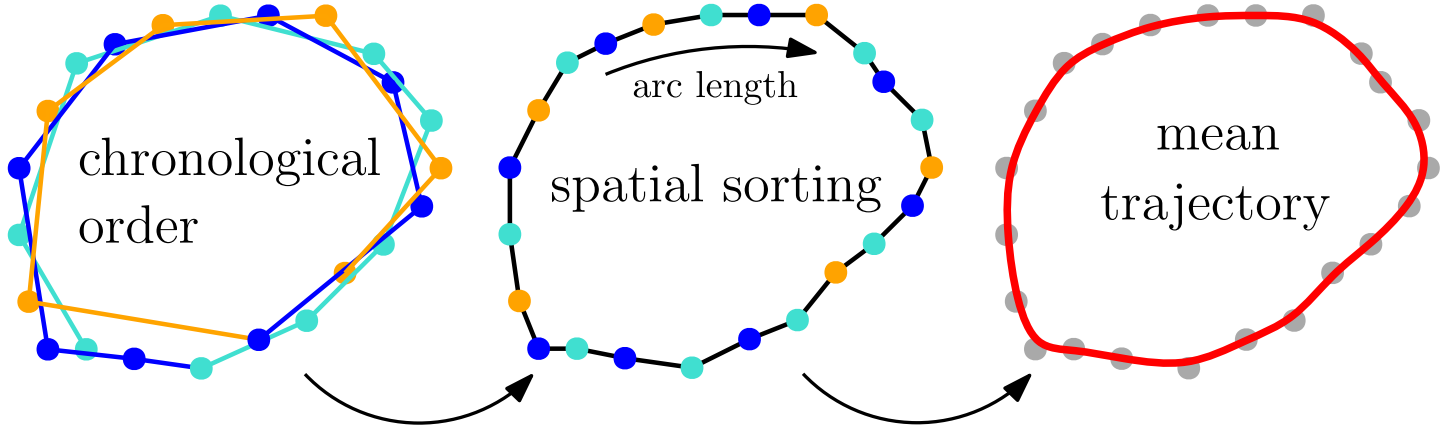
$$l_{arc} = \sum_{i=1}^{n-1} \|p_{i+1} - p_i\|$$

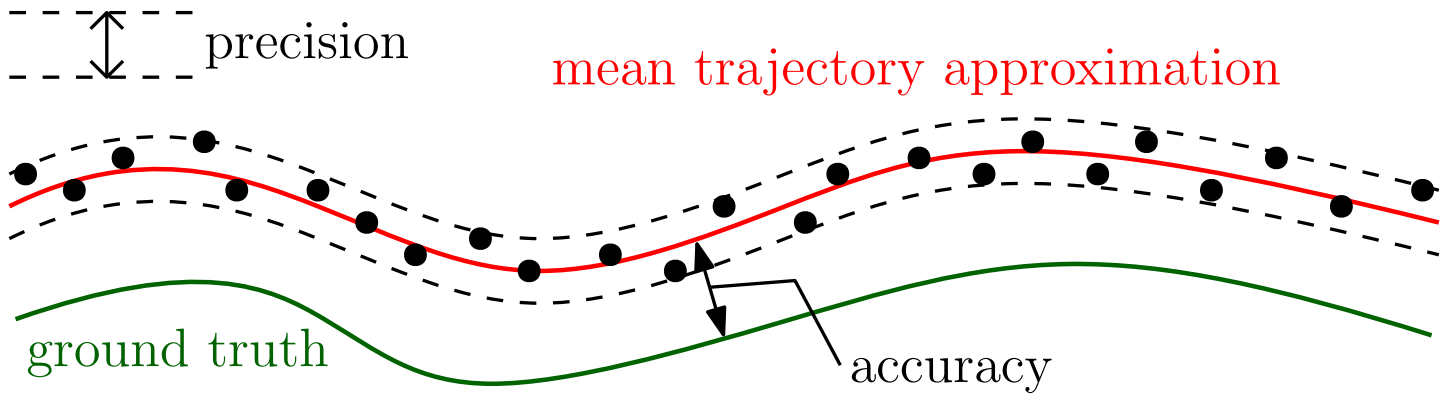


Delaunay triangulation

Sorting using
Minimum-Spanning-Tree

Proposed Methodology – Overview



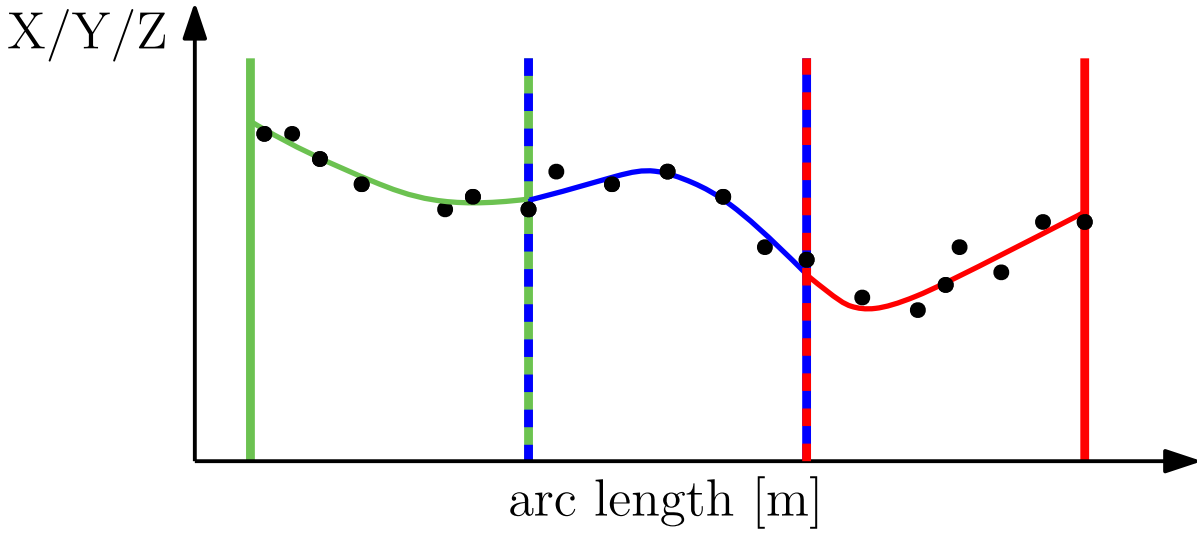


1. Spatial sorting ✓
2. Approximation
3. Quality measures

Proposed Methodology – Approximation

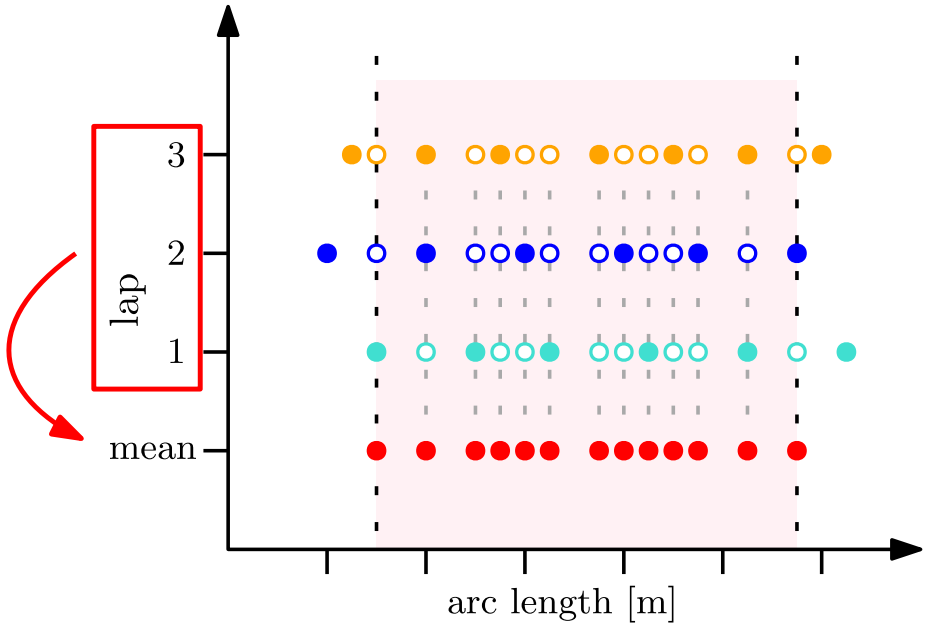
Positions

Cubic Approximation

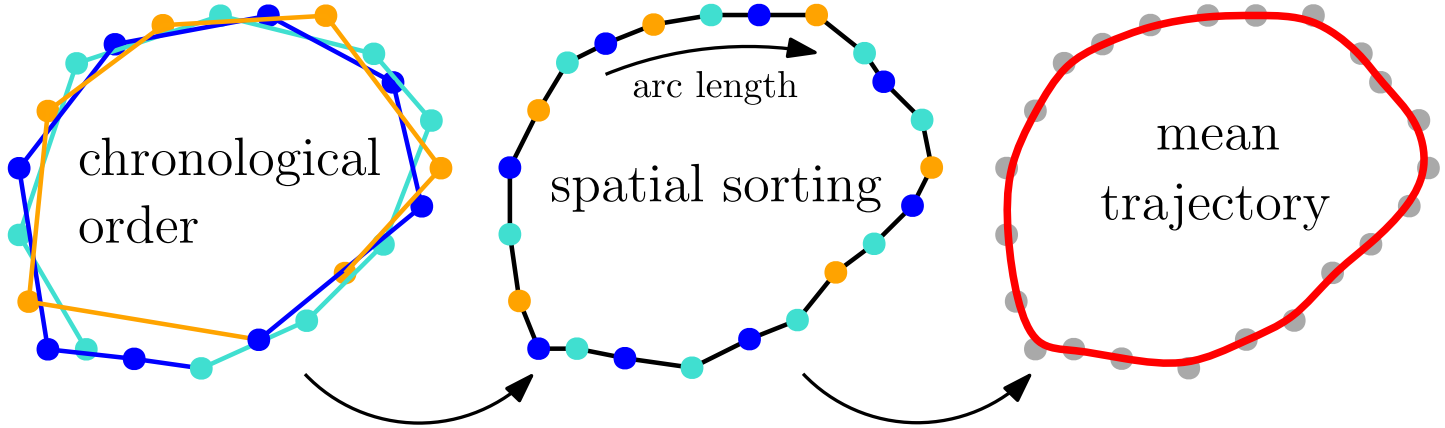


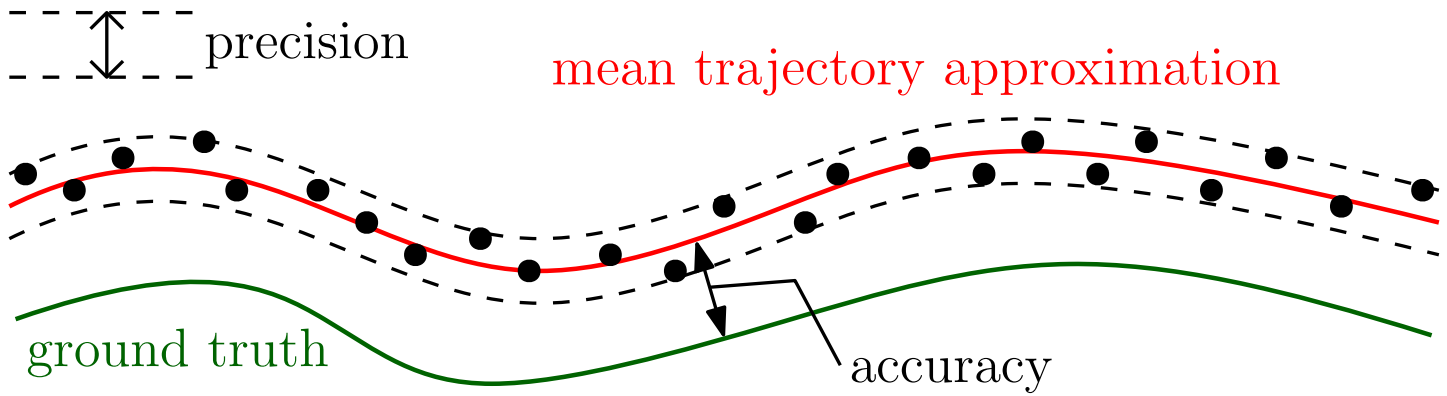
Rotations

Interpolation + Averaging



Proposed Methodology – Overview

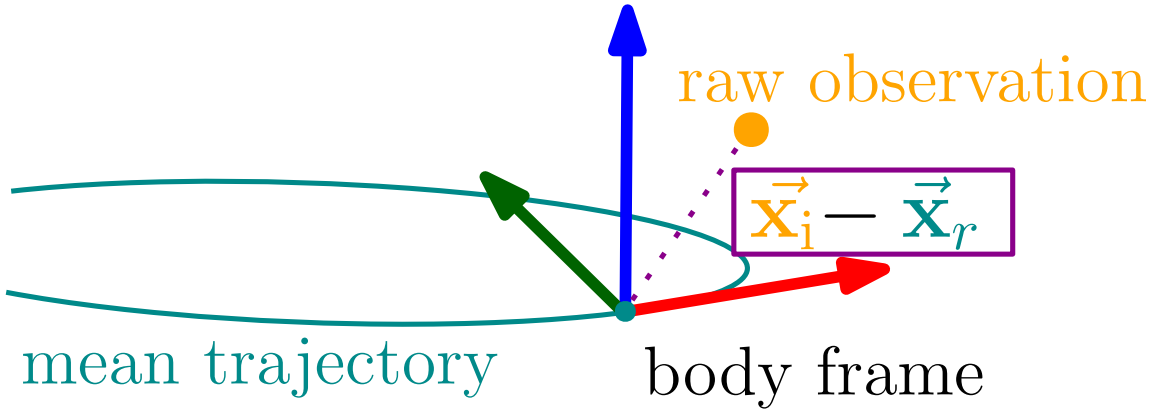




1. Spatial sorting ✓
2. Approximation ✓
3. **Quality measures**

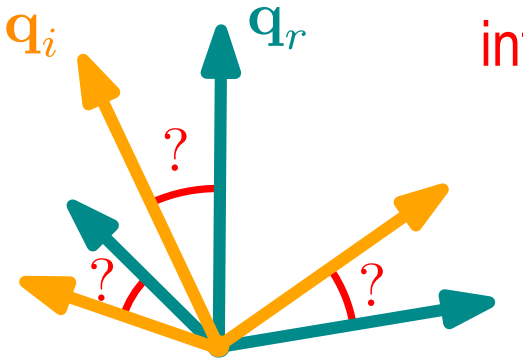
Proposed Methodology – Quality Measures

Positions



- Deviations are relative to the body frame
- Cross-track deviations

Rotations

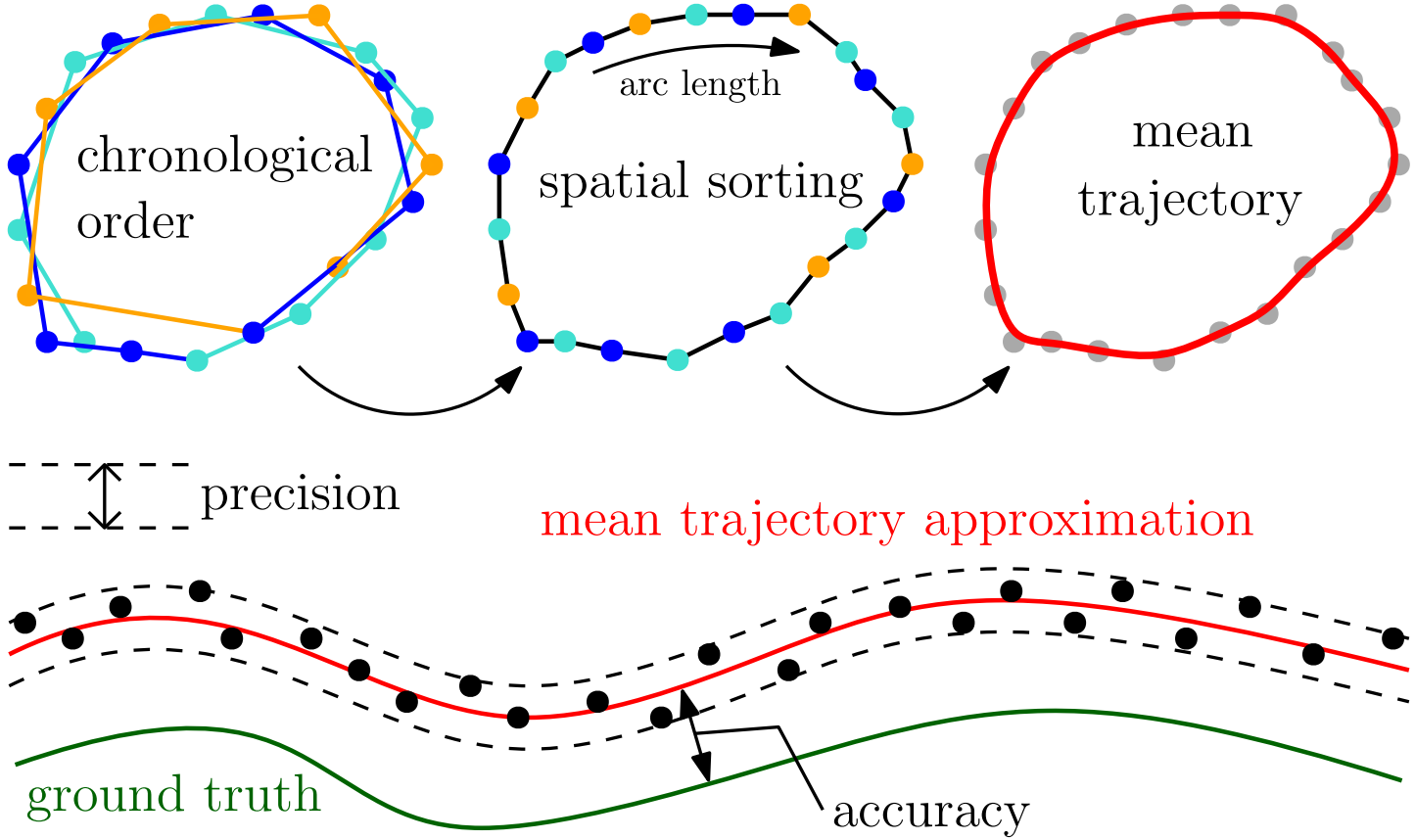


Differences of previously interpolated rotations

$$q_r^i = q_i \cdot q_r^{-1}$$

- Conversion to roll, pitch, yaw

Proposed Methodology – Overview

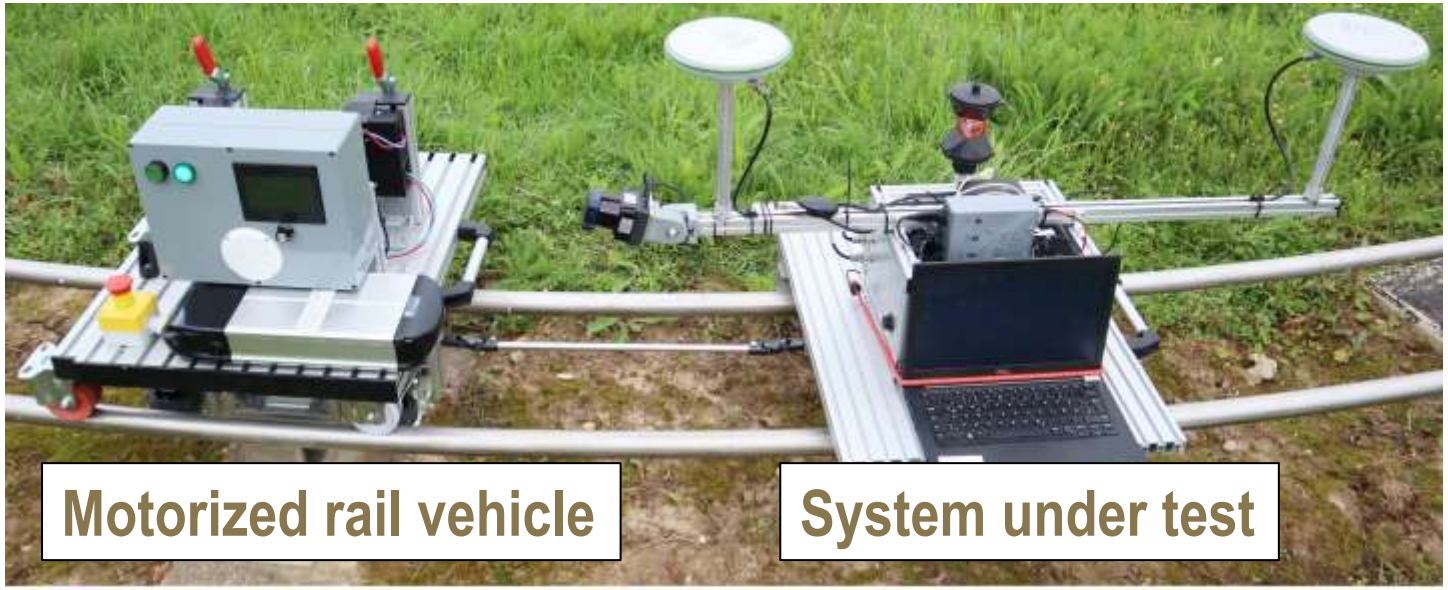


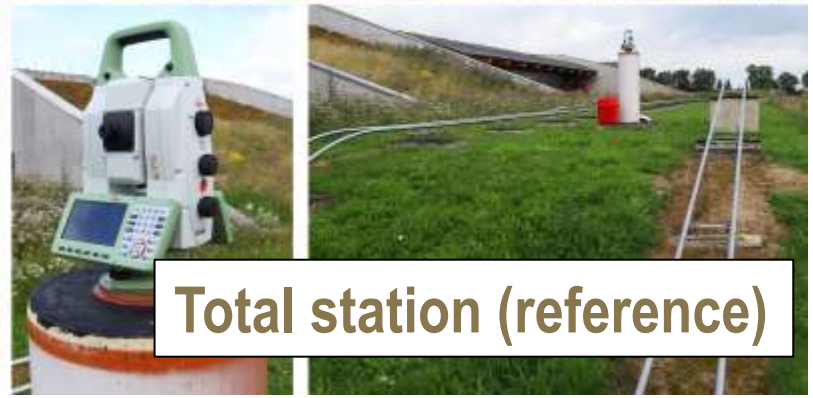
1. Spatial sorting ✓
2. Approximation ✓
3. Quality measures ✓

Experiments

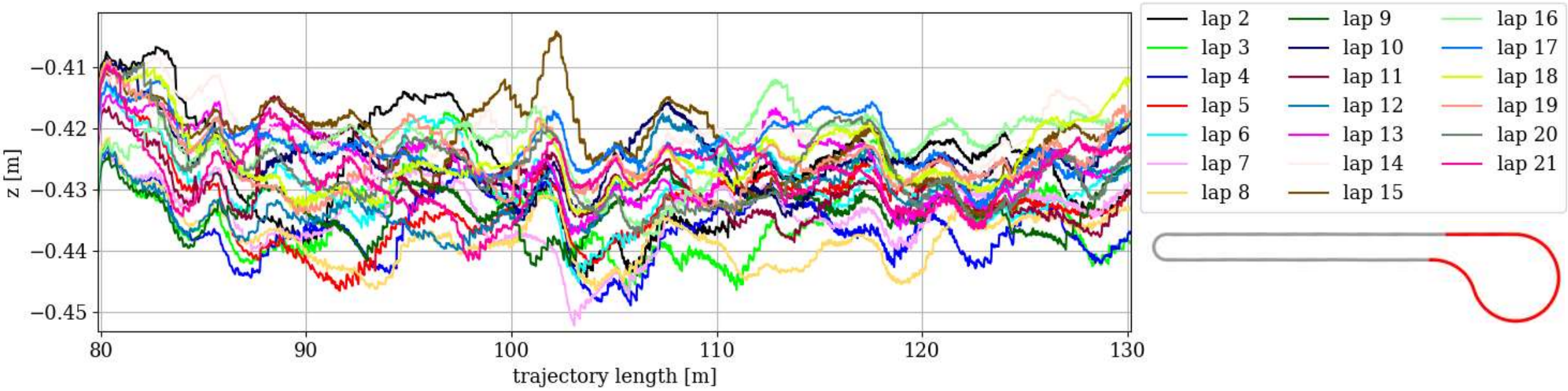
- Motorized track vehicle + trailer
- INS from SBG-Systems
- Leica TS60 for kinematic tracking

→ **20+ laps, ~ 30 mins**

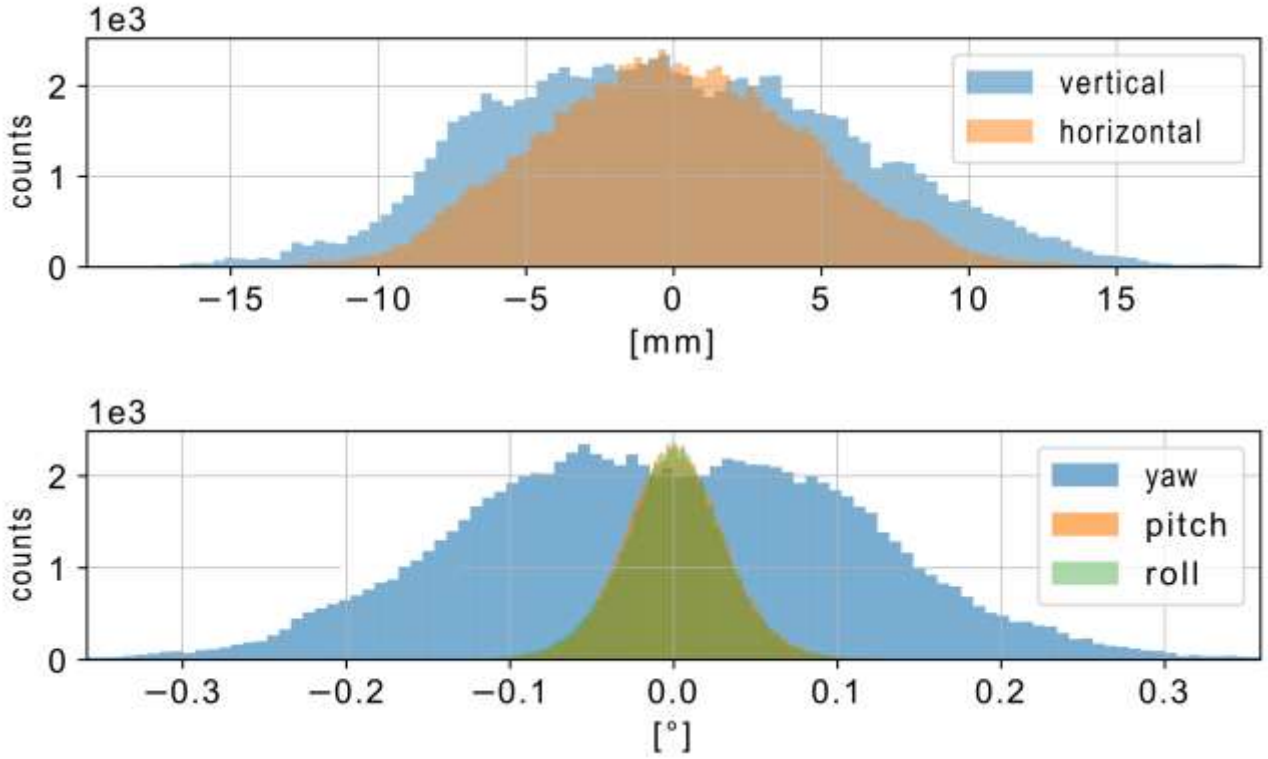




Results – Z-Component



Results – Precision

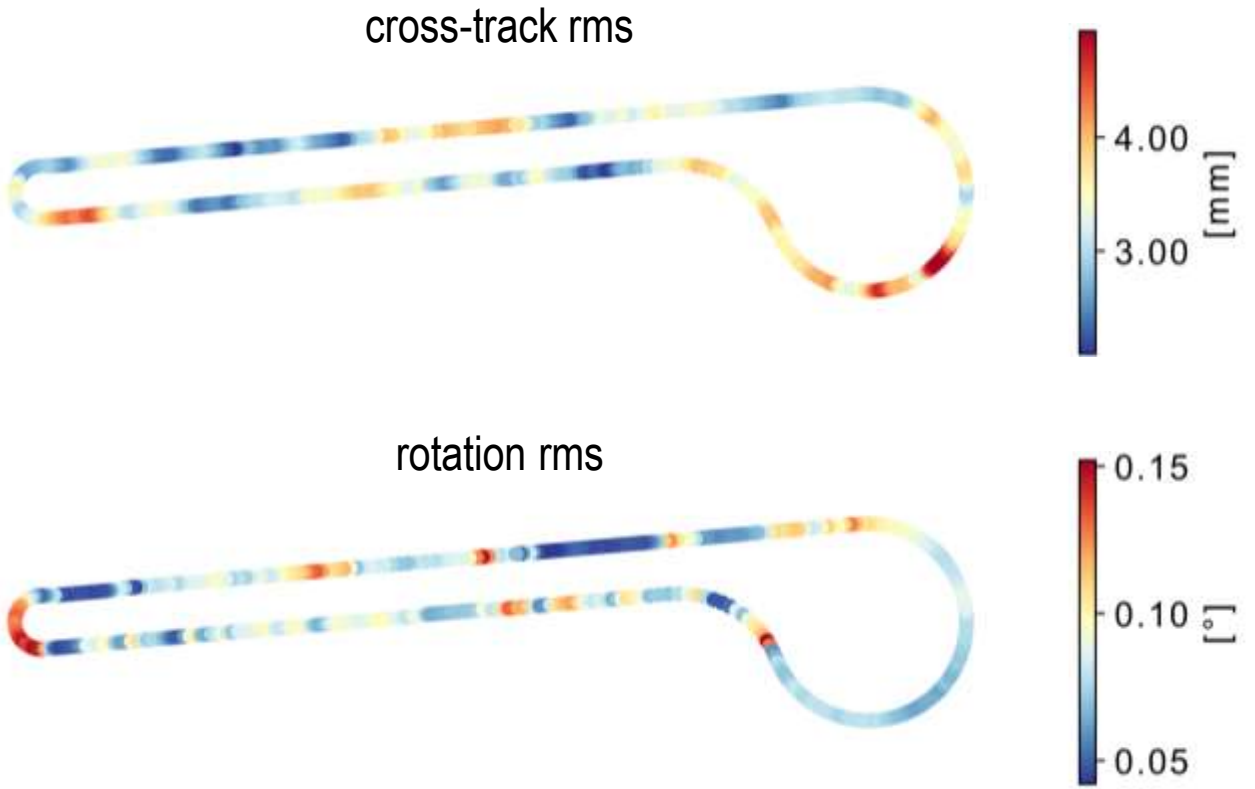


- Deviations of raw trajectory compared to mean trajectory
- Manufacturer's specifications met
- Approximately normally distributed

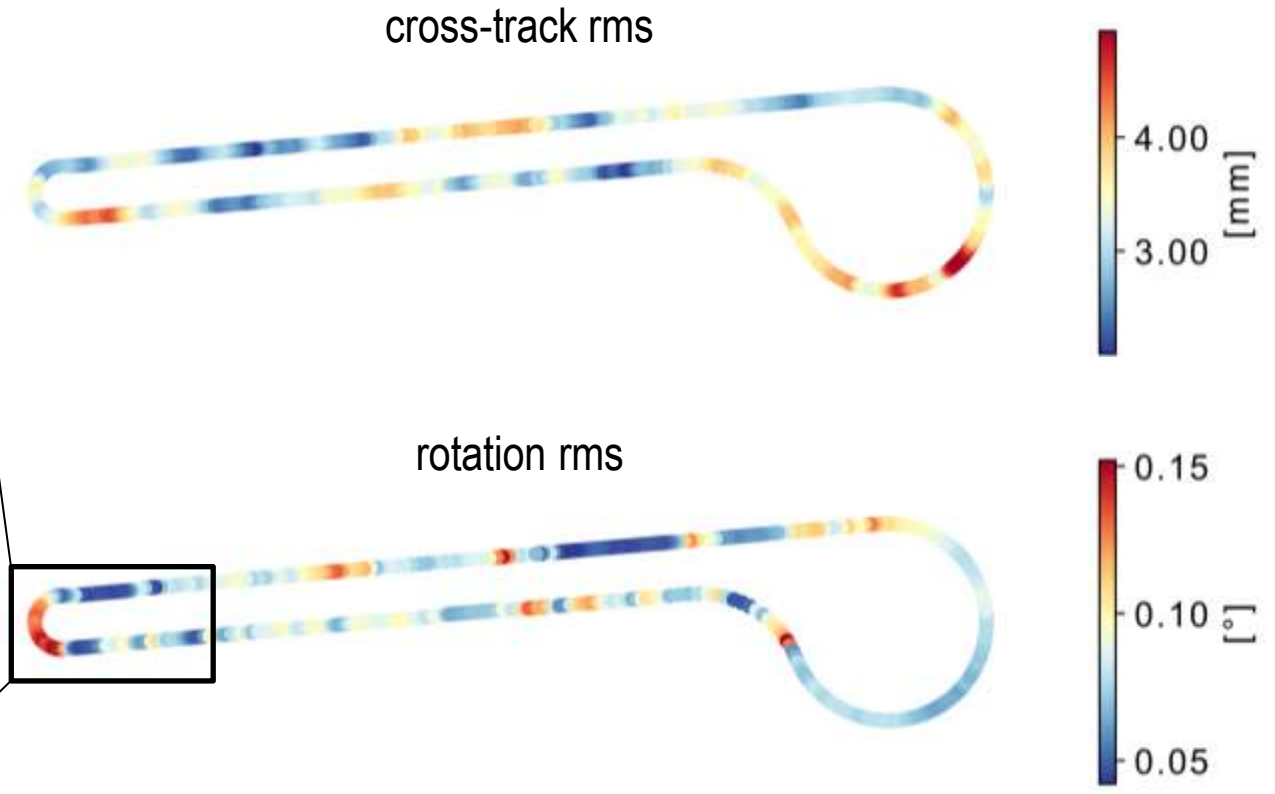
… while single laps show strong systematics

Results – Root Mean Square Error

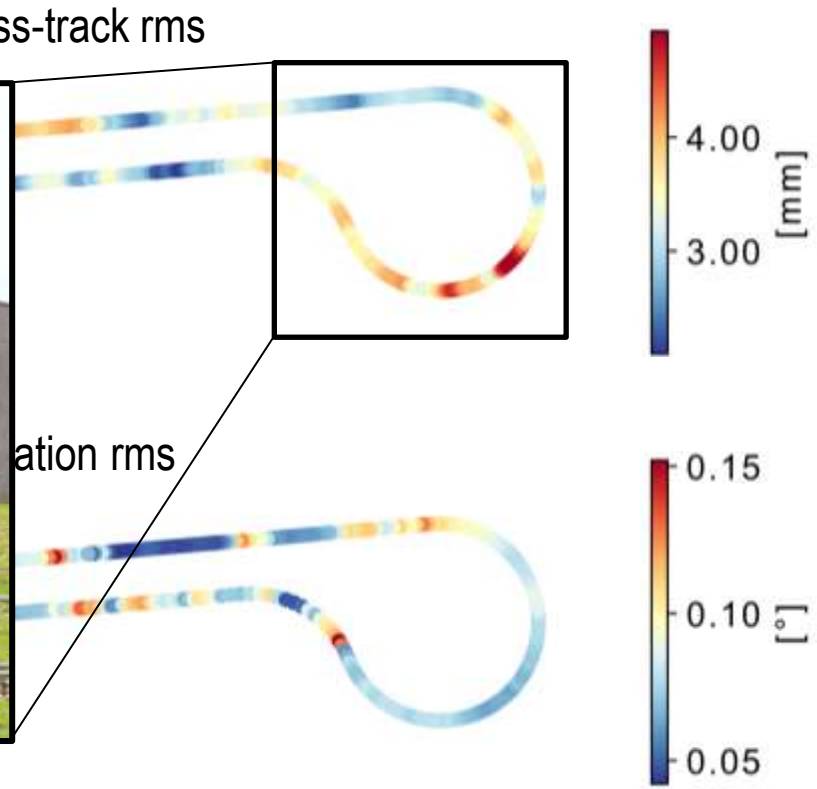
- Root Mean Square Error (RMS) computed using sliding window (0.5 m)
- Spatial deviation analysis
  - ➢ Location dependent irregularities can be revealed



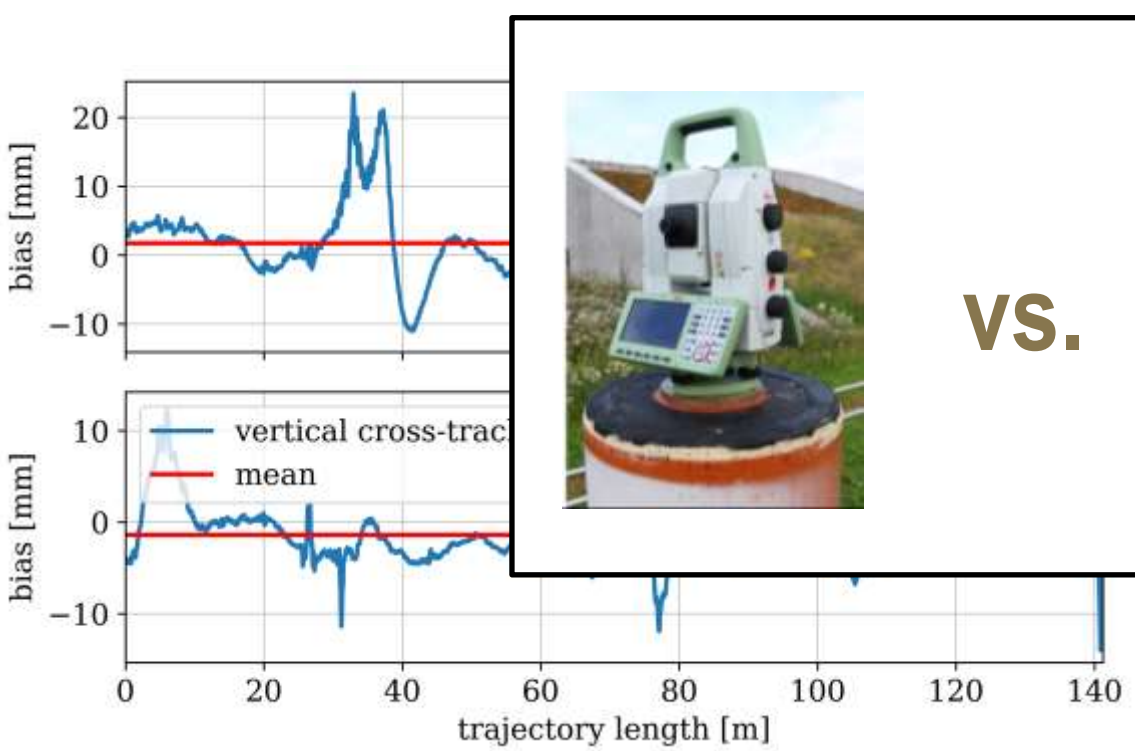
Results – Root Mean Square Error



Results – Root Mean Square Error

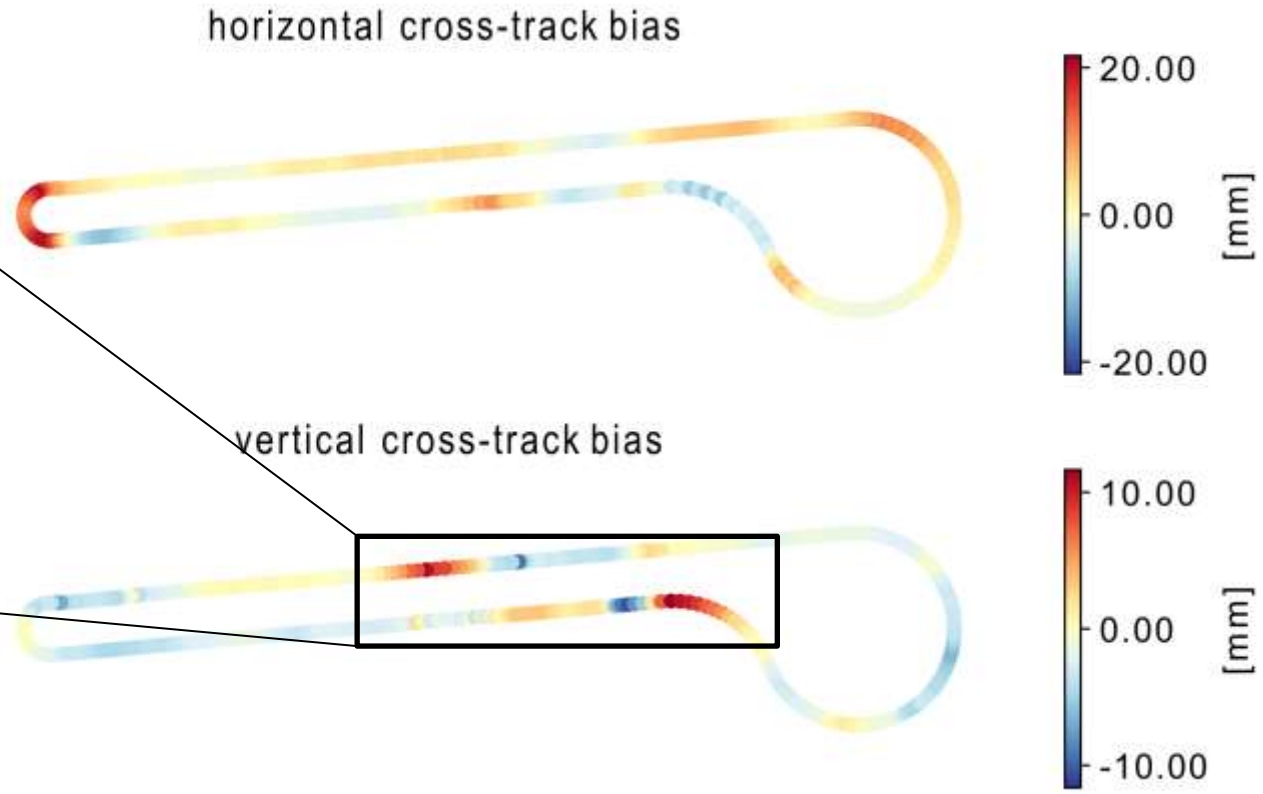


Results – Accuracy



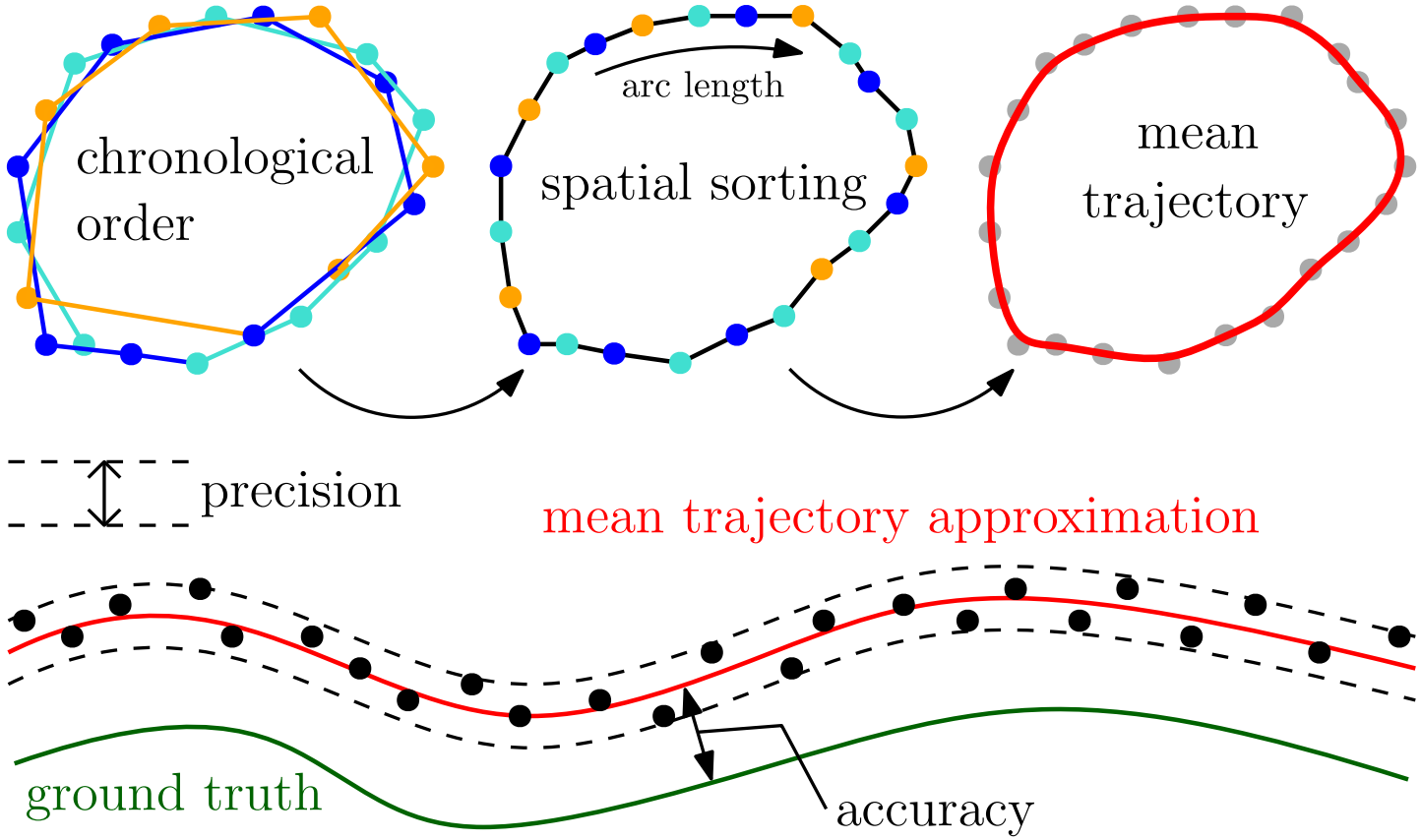
VS.

Results – Accuracy



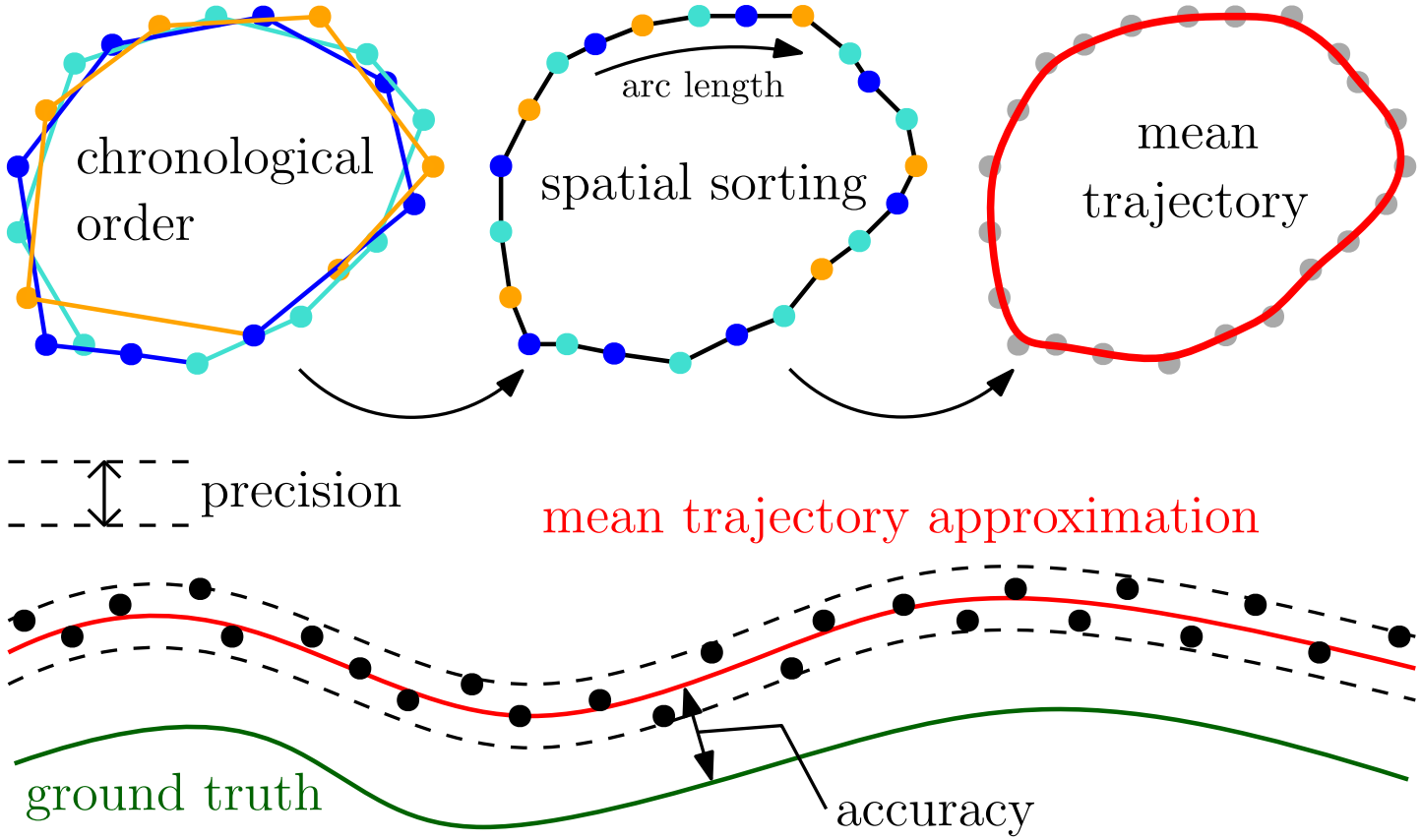
Summary

- Focus: Approach to evaluate navigation sensors
- Verification of the methodology with real sensor data
- Enables detailed analysis of the trajectory estimation quality

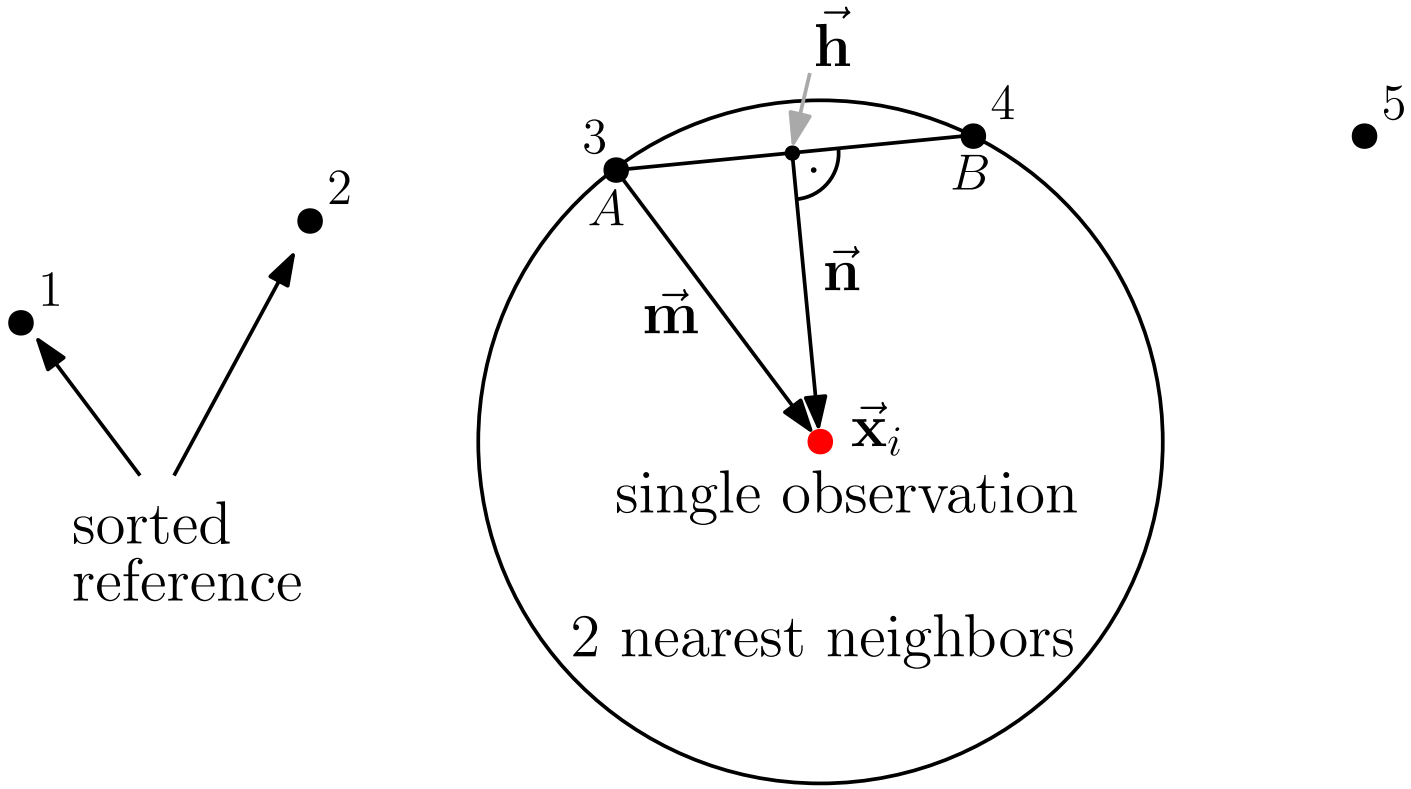


Outlook

- Sensor-Synchronization for along-track-deviations
- Static measurement of the track
  - Higher accuracy of reference
  - Rotation accuracy analysis
- Algorithm evaluation using the results of our methodology



Position deviations – Alternative approach



Accuracy – Spatial Relationship of sensors