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Precise Satellite Positioning based on a Search Procedure in the Coordinate Domain

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Introduction

- The evident new opportunities induced from the Global Navigation Satellite Systems (GNSS) development are associated with new challenges in signal processing methods. Besides a new observation, each new satellite also provides a new unknown value, i.e., ambiguity to be estimated. The rise in amounts of parameters increases the computational load.
- Therefore, we decided to reconsider the concept of searching for integer ambiguities in a coordinate domain. The search space dimension is constant and amounts to three in such a case.
- Thus, the search space dimension does not depend on the number of satellites. The computational load of a search procedure does not rise with increasing the number of satellites.
- Searching for a fixed solution in the coordinate domain dates back to the beginning of satellite navigation system development. It utilizes the Ambiguity Function (AF) method in the computation process.

The math model of precise satellite positioning is:

$$\mathbf{y} = \mathbf{A}\mathbf{a}+\mathbf{B}\mathbf{b}+\mathbf{e}$$

where **y** is the data vector, **a** is the integer ambiguity vector, **b** is the real parameter vector, **A** and **B** are the corresponding design matrices, and **e** is the noise vector.

In the initial phase the integer nature of ambiguities is discarded. The least-squares (LS) estimates of the **a** and **b** vectors are obtained from:

$$\begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{\hat{a}} & \mathbf{Q}_{\hat{a}\hat{b}} \\ \mathbf{Q}_{\hat{b}\hat{a}} & \mathbf{Q}_{\hat{b}} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{T}\mathbf{P}_{y}\mathbf{y} \\ \mathbf{B}^{T}\mathbf{P}_{y}\mathbf{y} \end{bmatrix}$$

The LS estimator of the integer ambiguity vector is defined as:

$$\breve{\mathbf{a}} = \arg\min_{\mathbf{z}\in Z^{n}} \left\|\hat{\mathbf{a}} - \mathbf{z}\right\|^{2}_{\mathbf{Q}_{\hat{a}}}$$

The expression at the arg min(.) operator denotes a weighted norm of the vector:

$$\left\|\hat{\mathbf{a}} - \mathbf{z}\right\|^{2}_{\mathbf{Q}_{\hat{a}}} = \left(\hat{\mathbf{a}} - \mathbf{z}\right)^{T} \mathbf{Q}_{\hat{a}}^{-1} \left(\hat{\mathbf{a}} - \mathbf{z}\right)$$

The last step in precise positioning is a "fixed solution," which incorporates the integer ambiguities obtained in the previous step:

$$\breve{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{b}\hat{a}}\mathbf{Q}_{\hat{a}}^{-1}\left(\hat{\mathbf{a}} - \breve{\mathbf{a}}\right)$$

The search procedure in a coordinate domain

The method proposed in this work utilizes the relationship of type $\breve{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}}\mathbf{Q}_{\hat{\mathbf{a}}}^{-1}(\hat{\mathbf{a}} - \breve{\mathbf{a}})$ but with swapped the **a** and **b** vectors and corresponding cofactor matrices. The conditional solution of the a vector resulting from the change of the **b** vector is:

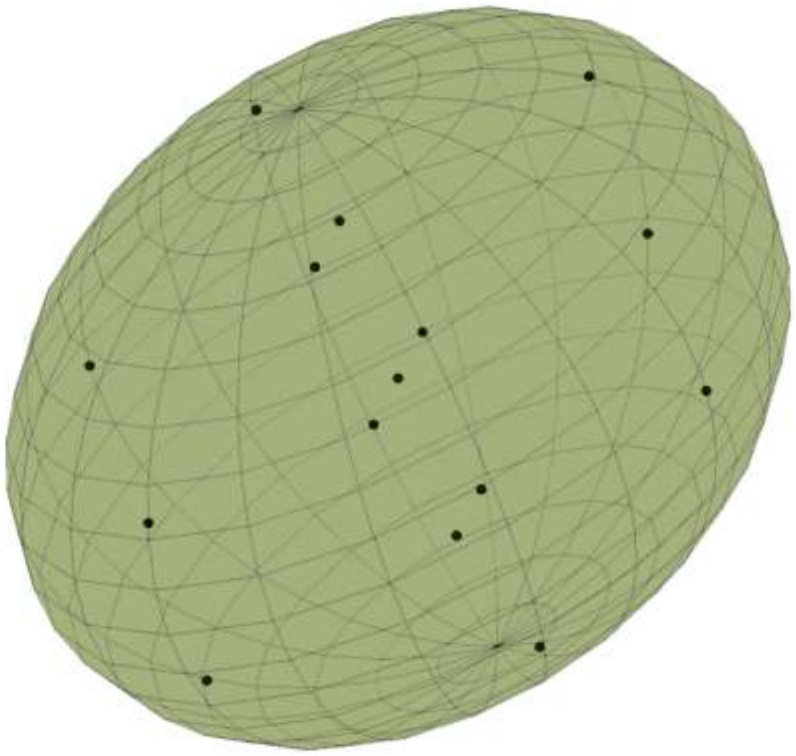
$$\breve{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{Q}_{\hat{a}\hat{b}}\mathbf{Q}_{\hat{b}}^{-1}\left(\hat{\mathbf{b}} - \breve{\mathbf{b}}\right)$$

This formula is applied in the proposed method as an initial phase of a search procedure. In the 3-dim coordinate space, search region takes the form of an error ellipsoid of an approximate position from the float solution. The error ellipsoid axes lengths are computed from the formulas:

$$r_x = \sqrt{\frac{\chi_c^2}{\mu_x}}, r_y = \sqrt{\frac{\chi_c^2}{\mu_y}}, r_z = \sqrt{\frac{\chi_c^2}{\mu_z}}$$

where χ_c^2 is the critical value of the χ^2 distribution with 3 degrees of freedom, and μ_x, μ_y, μ_z are eigenvalues of the $\mathbf{Q}_{\hat{\mathbf{b}}}^{-1}$ matrix. The initial candidates grid is formed inside the search space.

The example of the initial candidates' arrangement inside the error ellipsoid



The distances between the initial candidates (IC) have to be sufficiently small so as not to omit any of the Voronoi cells located inside the search region. On the other hand, the distance between neighboring initial candidates should be as long as possible so that the number of initial candidates is small. The next step of the data processing is the essence of the proposed method. Let us name the initial candidates with bIC. For i-th bIC we compute the corresponding ambiguity vector:

$$\mathbf{a}_{ICi} = \hat{\mathbf{a}} - \mathbf{Q}_{\hat{a}\hat{b}} \mathbf{Q}_{\hat{b}}^{-1} \left(\hat{\mathbf{b}} - \mathbf{b}_{ICi} \right)$$

Each ambiguity vector obtained this way is rounded to the nearest integer vector:

$$\mathbf{a}_{FCi} = \boldsymbol{round}\left(\mathbf{a}_{ICi} \right)$$

The integer vector $\mathbf{a}_{FCi}$ corresponds to the i-th final candidate. The final candidates are tested, and this one, which minimizes criterion:

$$\mathbf{a}_{FC\min} = \arg \min_{\mathbf{z} \in Z^n} \left\| \hat{\mathbf{a}} - \mathbf{a}_{FCi} \right\|_{\mathbf{Q}_{\hat{a}}}^2$$

takes part in computing the final fixed solution.

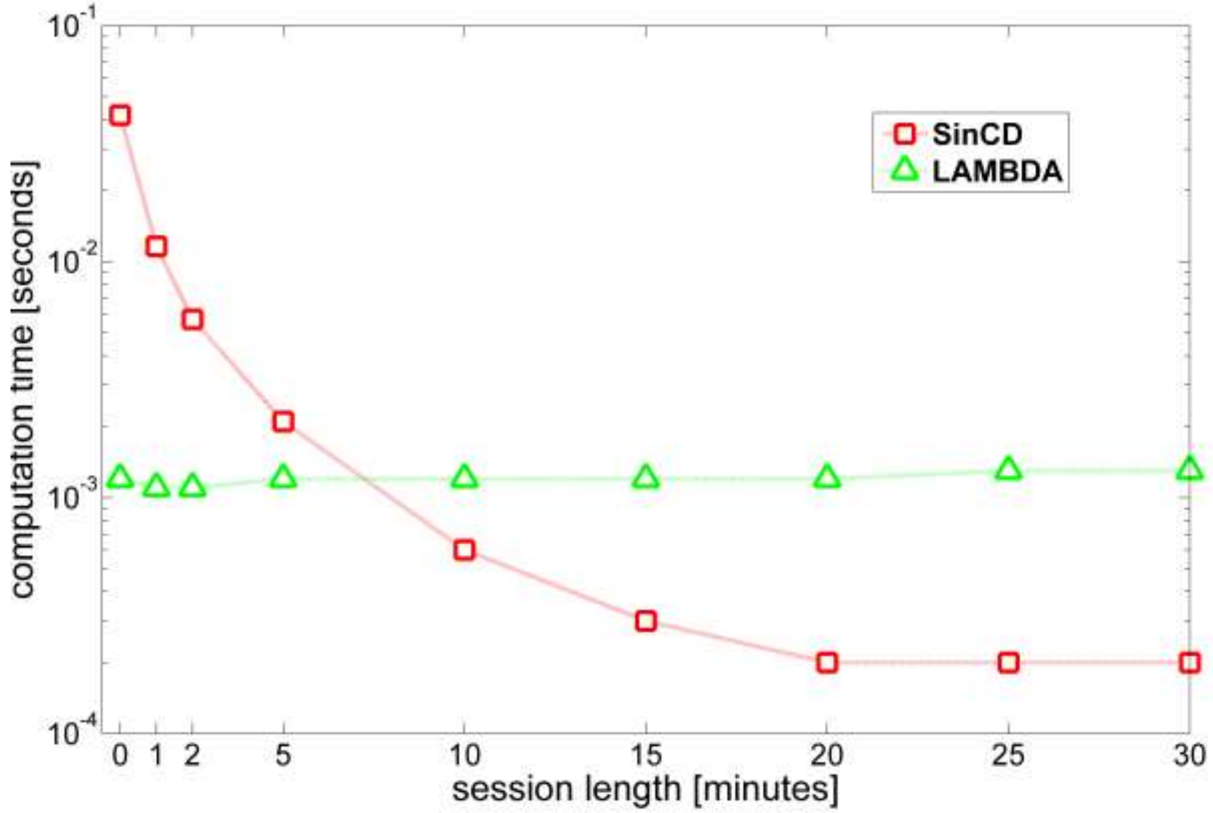
Test description - the search procedure in a coordinate domain (SinCD)

- The very short baseline (1.62 m) was chosen for the tests;
- Only the single-frequency GPS signals were processed in the experiment;
- The data was collected in two permanently operating EUREF stations: WTZR and WTZZ (Germany), on 12, May, 2019;
- The interval between consecutive epochs was 30 seconds;
- The WTZR was set as a reference station and the WTZZ as a rover;
- The precise position of the rover was computed based on the 24h-long session. This position was used as a reference for comparisons;
- hole observation set was divided into short observation sessions. The 2000 subsequent epochs from the beginning of the 24h-long session were adopted as the starting epochs for the successive short sessions;
- The tests were carried out for sessions of various lengths: from a single-epoch solution to 30 minutes sessions;
- There were 7 satellites in view in most sessions;
- The LAMBDA method was used for a comparison.

Success rates as a percentage of correct solutions in the samples of 2000 observation sessions

	Session length [minutes]							
	Single-epoch	1	2	5	10	15	20	30
	Success rates [%]							
#1 (SinCD)	45.6	59.8	70.2	85.3	94.8	98.2	98.6	100.0
#2 (LAMBDA)	45.2	60.7	72.4	90.0	99.8	100.0	100.0	100.0

Sessions' lengths and a computation time in the SinCD and the LAMBDA methods



Conclusions

- The new method was developed and tested.
- A major advantage of the proposed method is moving a search procedure from multi-dimensional ambiguity space to only three-dimensional coordinate space.
- Such an operation allows for shortening computation time.
- It has great importance in the prospect of increasing the number of satellites resulting from developing modern satellite navigation systems.
- The test results confirmed shortening computation time when using the proposed method and that its advantage over traditional methods unveils in the case of many satellites.

Thank you for your attention!