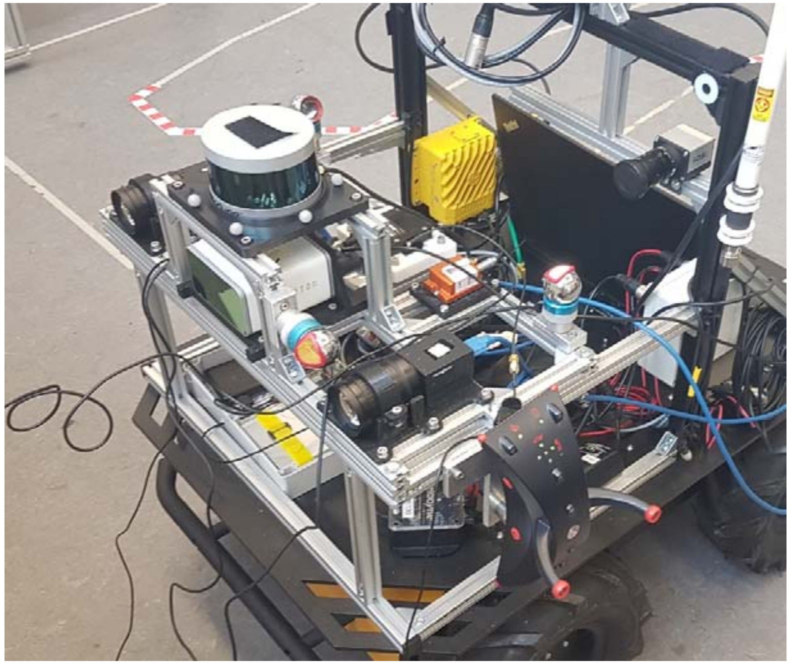
Geodätisches Institut Hannover

DFG Research Training Group i.c.sens (RTG 2159)
Integrity and Collaboration in dynamic sensor networks

Leibniz Universität Hannover

Uncertainty Evaluation for a Kinematic LiDAR-based Multi-Sensor System



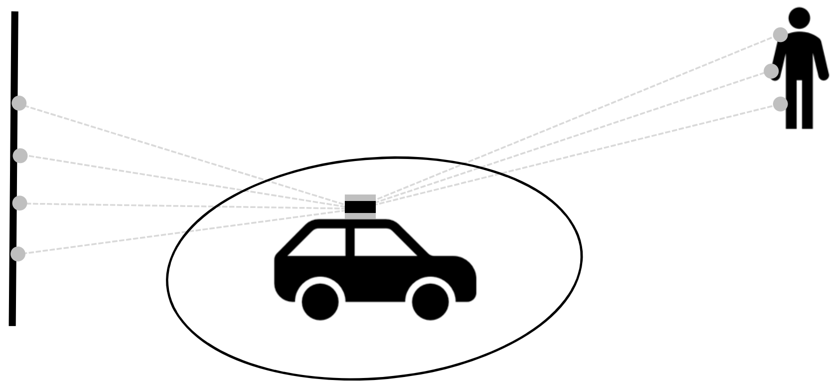
Dominik Ernst
Sören Vogel
Hamza Alkhatib
Ingo Neumann

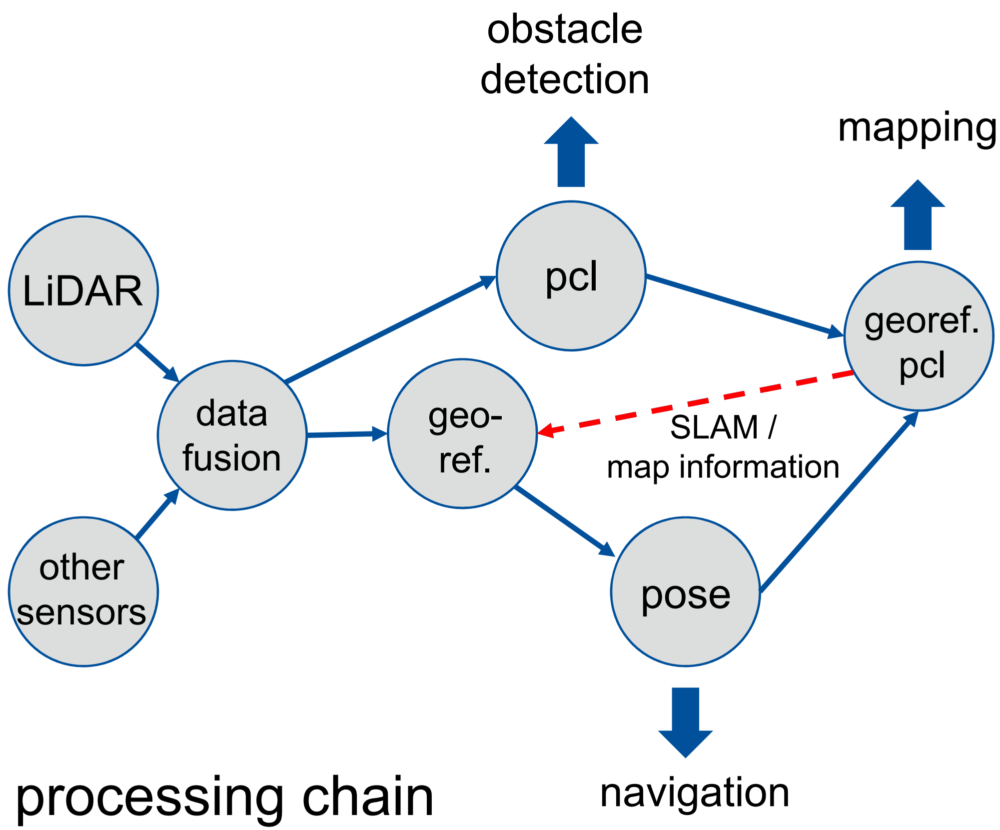
FIG Working Week 2023
Scientific Workshop on Uncertainty and Quality of Multi-Sensor Systems

2023-05-28

Motivation

LiDAR-based MSSs:





Issues related to uncertainty modeling:

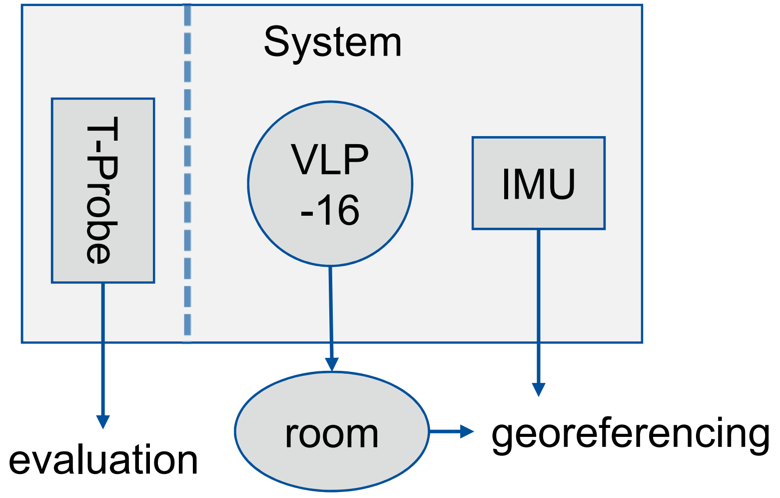
1) Multitude of influences along complete processing chain
2) Feedback loop by using LiDAR observations for localization

Agenda

- Motivation
- Methodology
- Dataset
  - Simulation
  - Real Data
- Results

LiDAR-based Multi-Sensor System

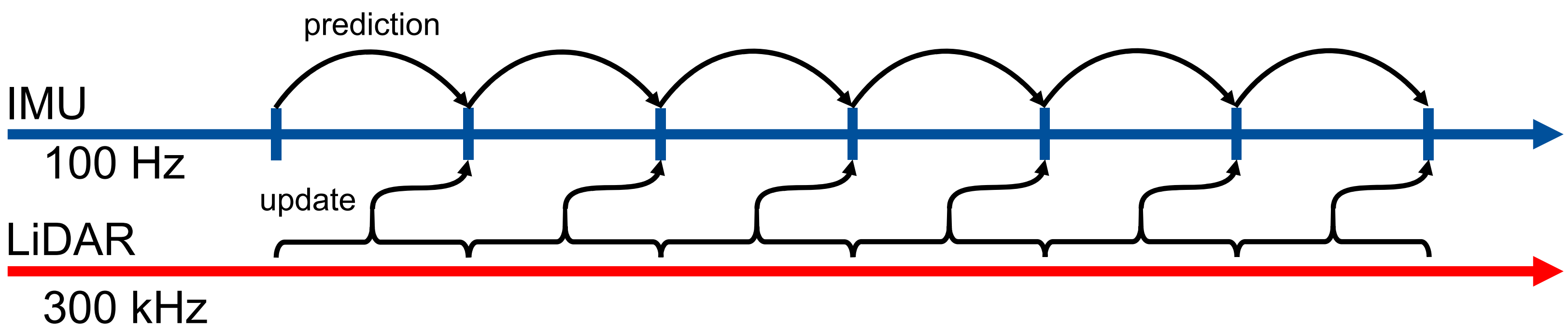
- Minimal multi-sensor system (2 sensors)
  - LiDAR: VLP-16
  - IMU: Microstrain 3DM-GQ4-45
  - Leica T-Probe (for evaluation)



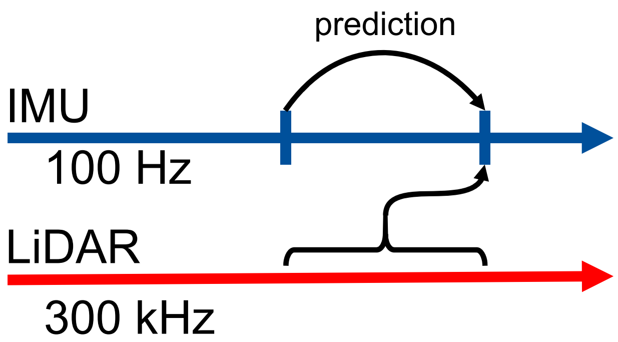
- Principle of position tracking?
  - Prediction by IMU (strapdown)
  - ➢ Localization using known planes in the environment (Vogel, 2020)

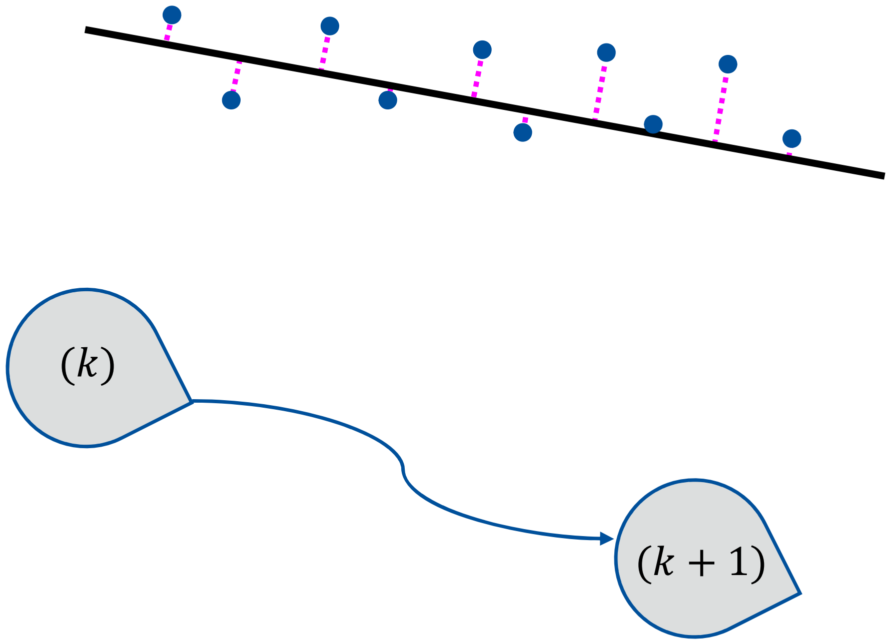
Error State Kalman Filter with implicit Measurement Equations

- States:
  - Pose: translations, orientations (quaternion), velocities
  - Biases for IMU: accelerometer, gyroscope
- ➢ Usage of quaternions: implementation as Error State KF



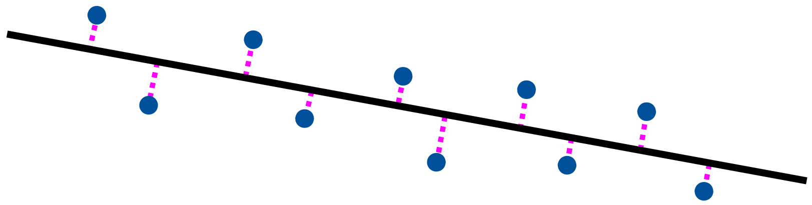
Principle of Position Tracking





Principle of Position Tracking

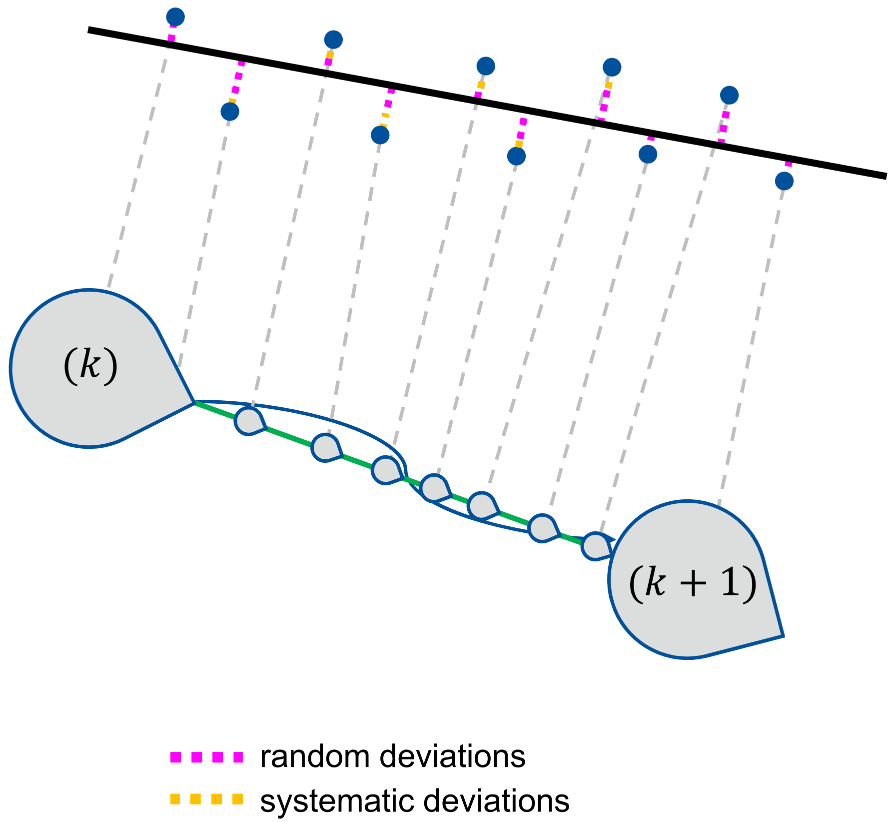
- Prediction by integrating IMU measurements between (k) and $(k+1)$



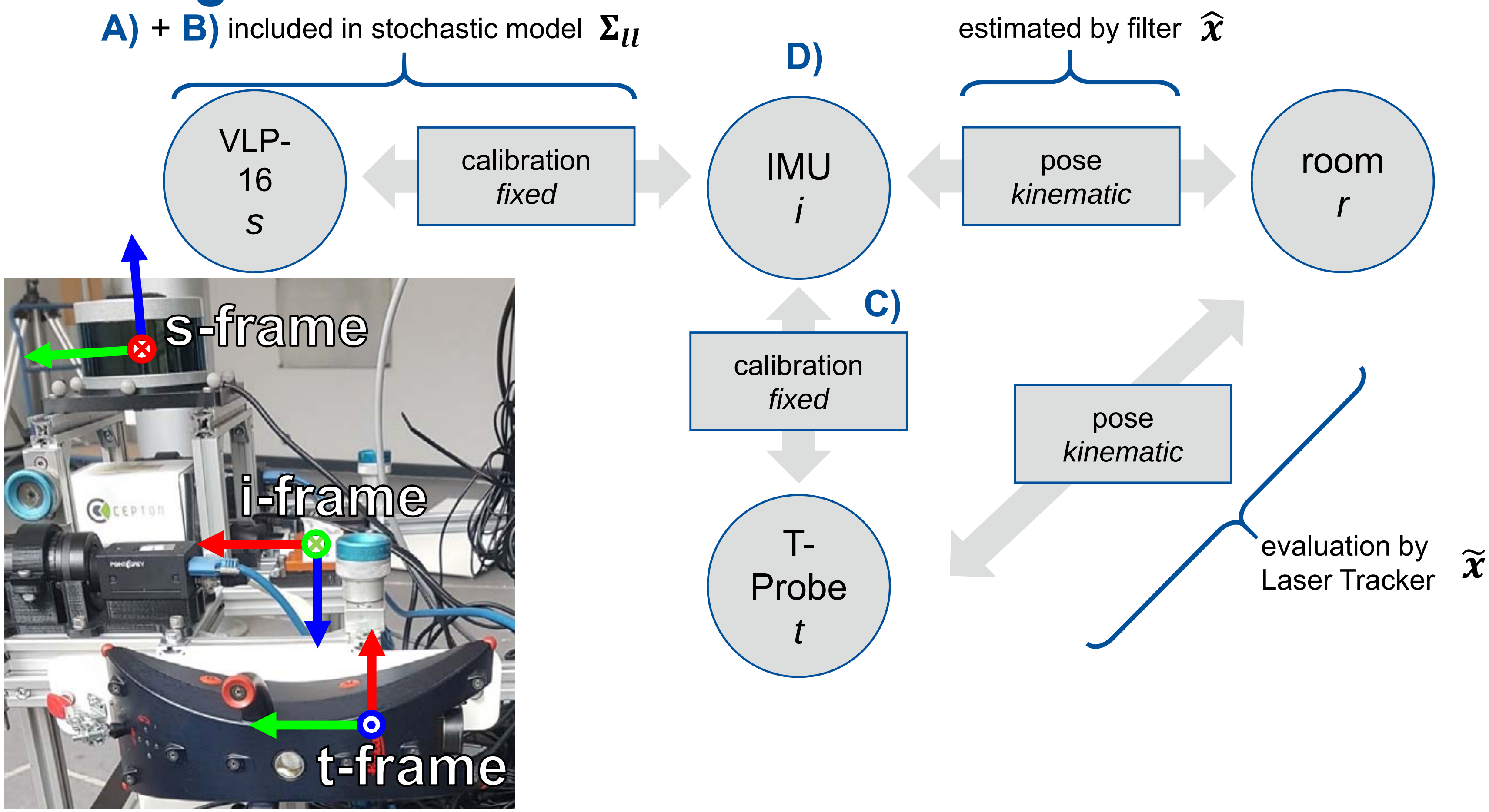
- Update:
  - Transformation of LiDAR points using predicted pose
  - Distance minimization to known planes in environment

Principle of Position Tracking

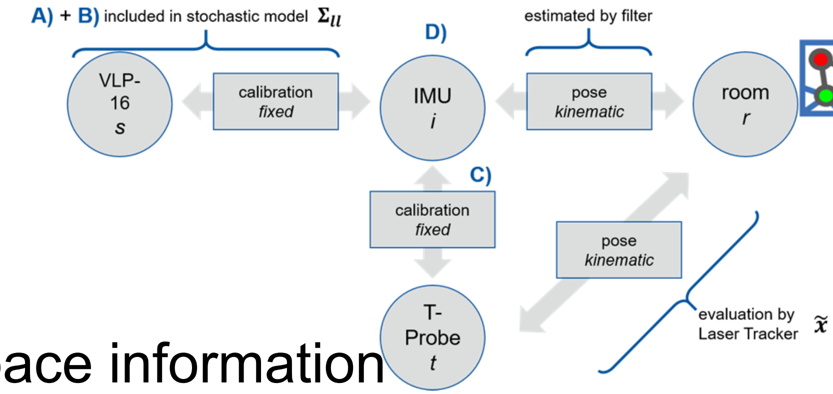
- Usage of linear interpolation to account for rolling shutter
- Timestamps of points for interpolation
  - Translation: simple
  - Orientation: spherical linear interpolation



Processing Chain



Calibrations



A) System calibration using object-space information
usage of LiDAR observations → notable uncertainties (~mm)

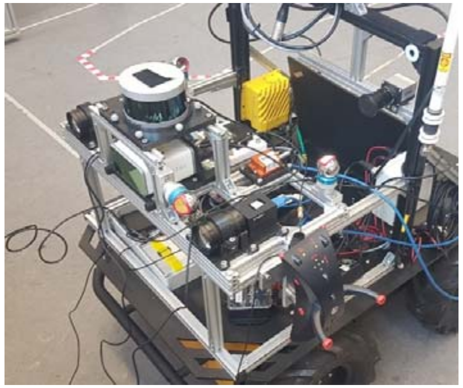
B) System calibration using LT measurements of IMU housing
usage of only LT measurements → small uncertainties (~0.1 mm)

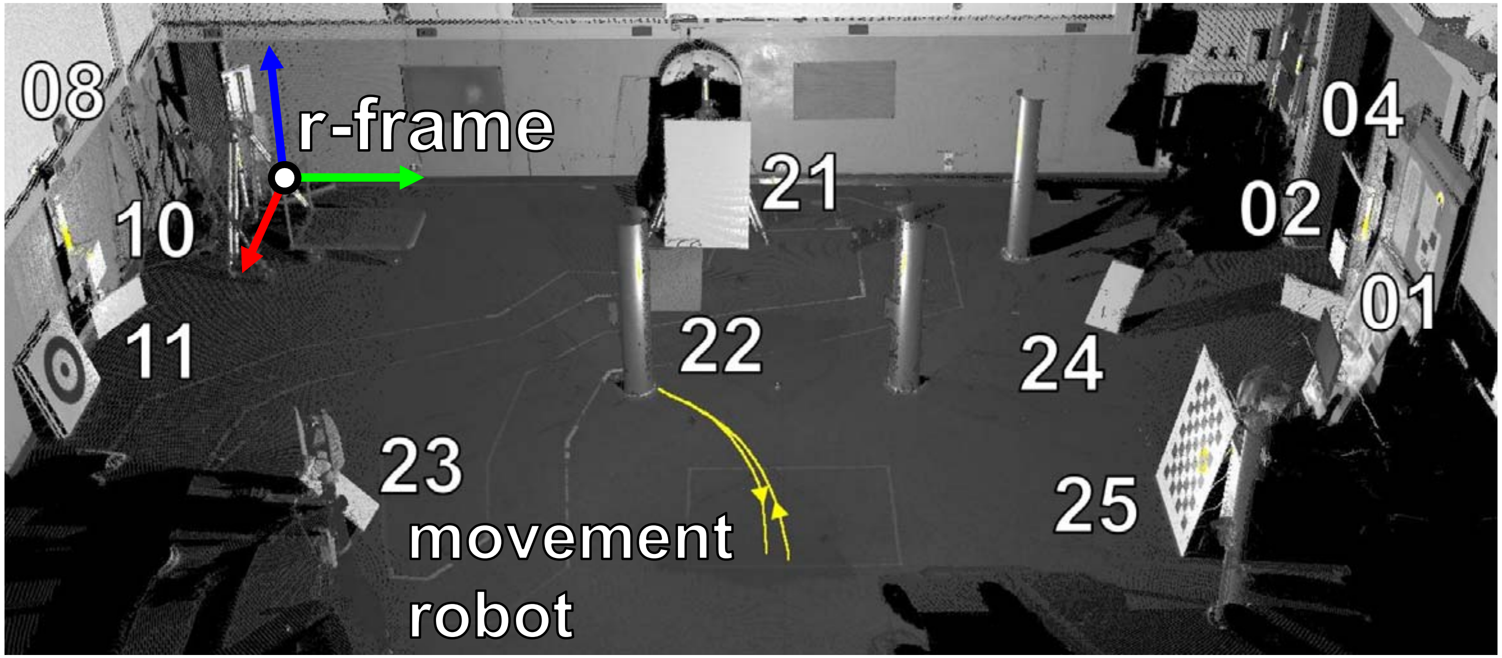
C) System calibration between CCRs and T-Probe
usage of only LT measurements → small uncertainties (~0.1 mm)

D) Computation of Allan variance for IMU noise parameters
short 1-position test (~25s)

Overview: Setup of the Experiment

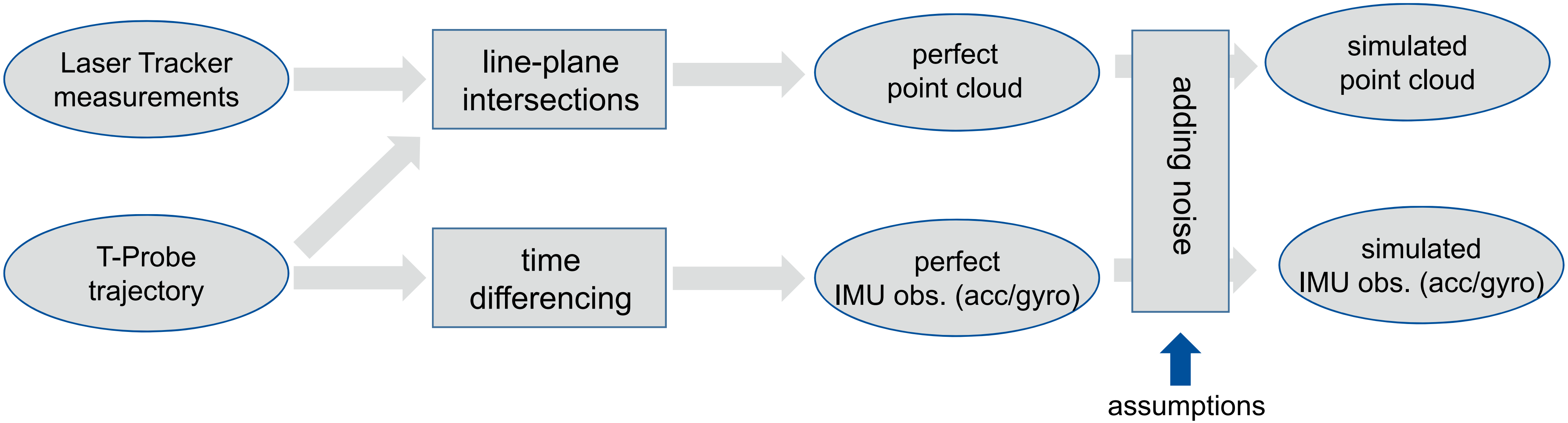
- Laboratory as controlled environment with 11 reference panels
- Reference panels used for calibration
- ➢ Assumptions for LiDAR measurement uncertainty



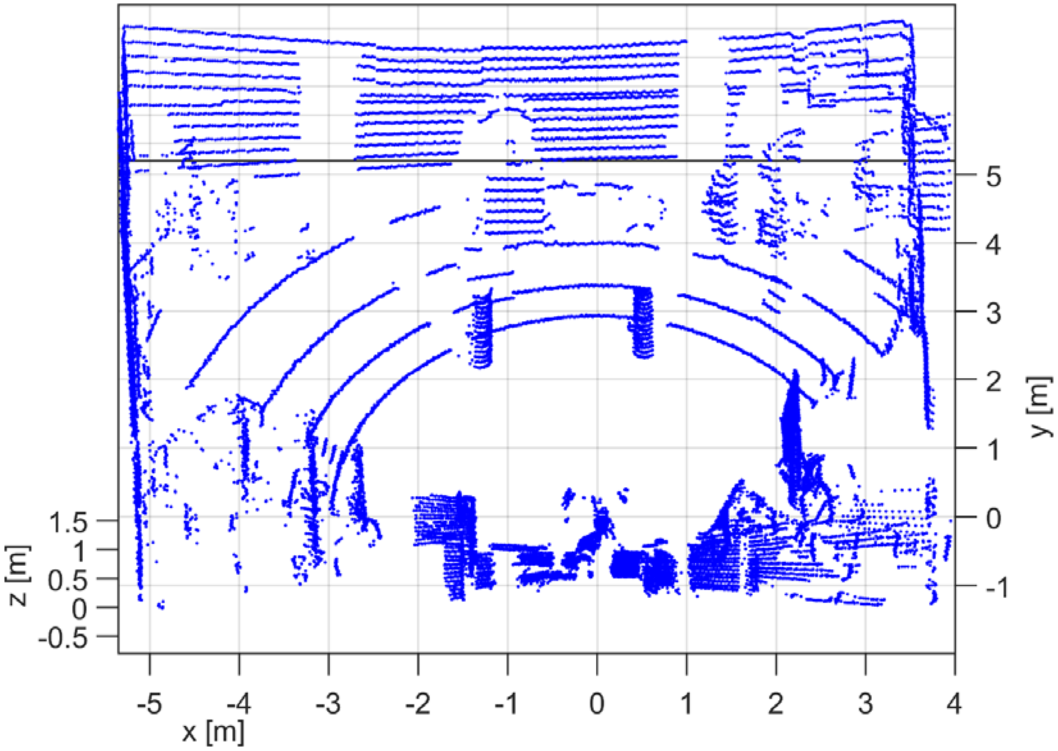


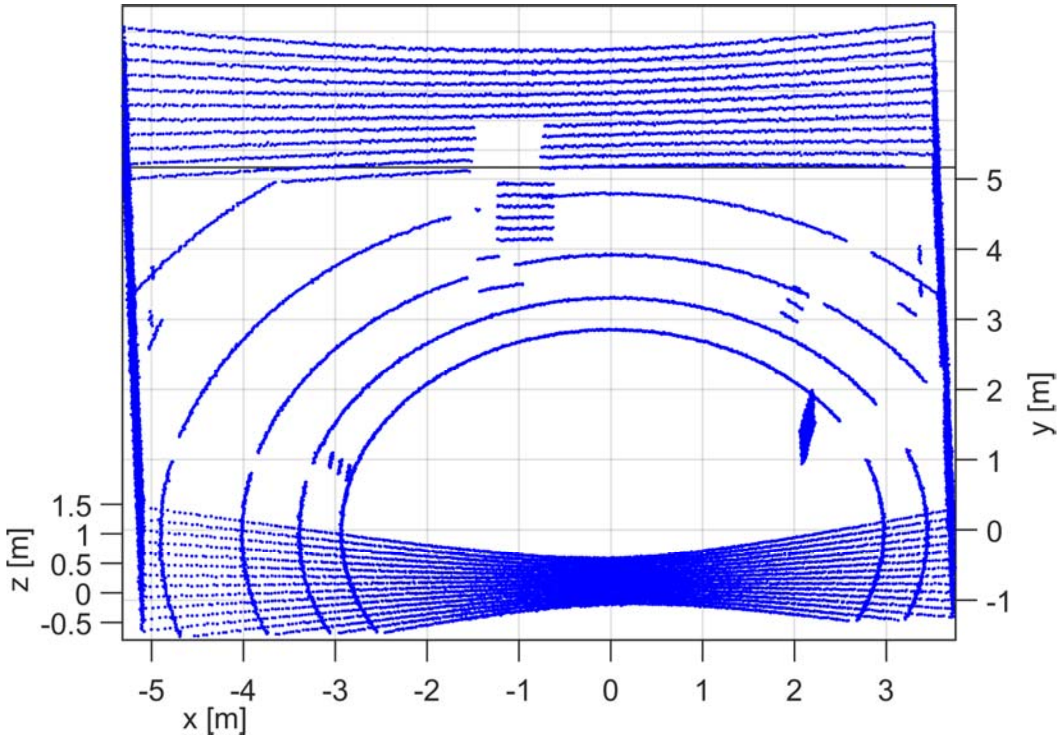
Closed Loop Simulation

- 2 Reasons:
  - Check implementation of filter
  - Check if assumptions lead to expected results

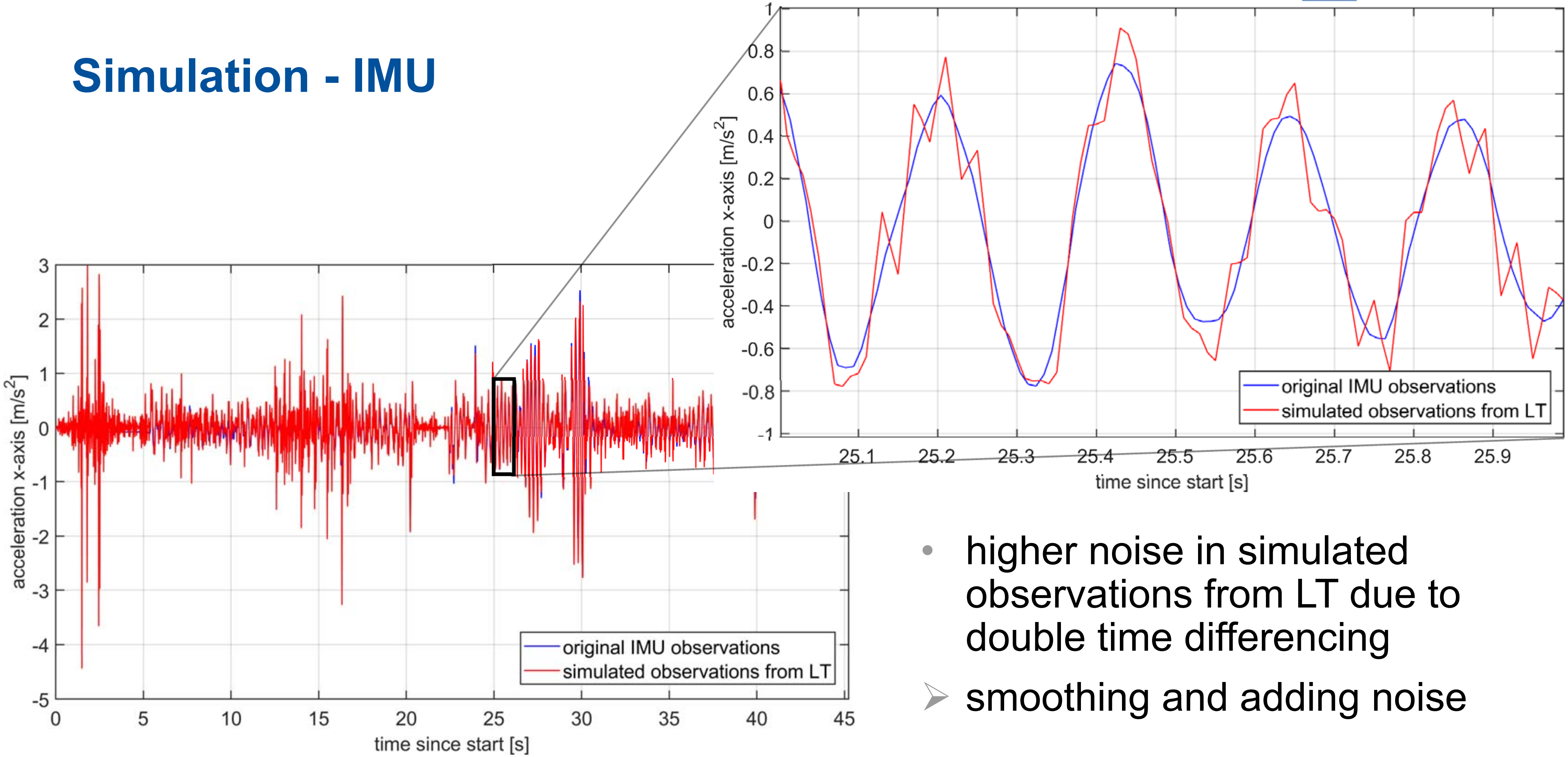


Simulation - LiDAR





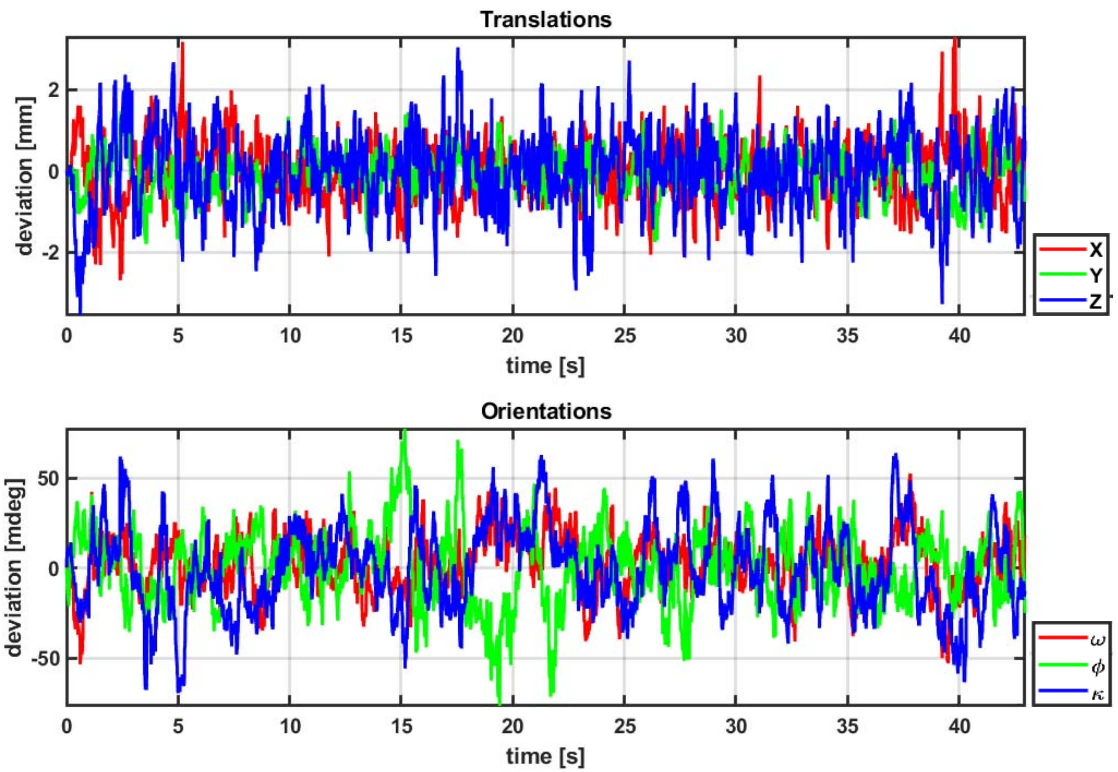
Simulation - IMU



- higher noise in simulated observations from LT due to double time differencing
  - ➢ smoothing and adding noise

Simulation Results – Accuracy

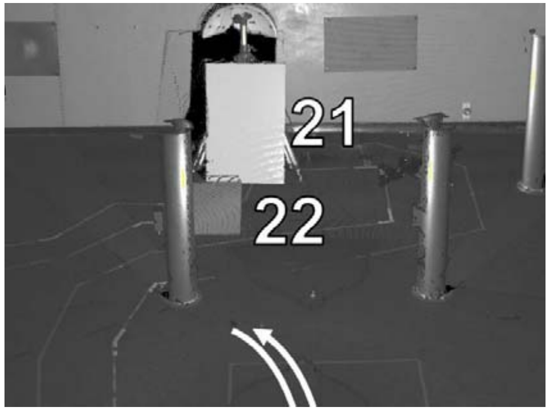
- Deviation from true trajectory $\tilde{\boldsymbol{x}} - \hat{\boldsymbol{x}}$
- RMSE:
  - Translations: 0.76 mm
  - Orientations: 19.94 mdeg

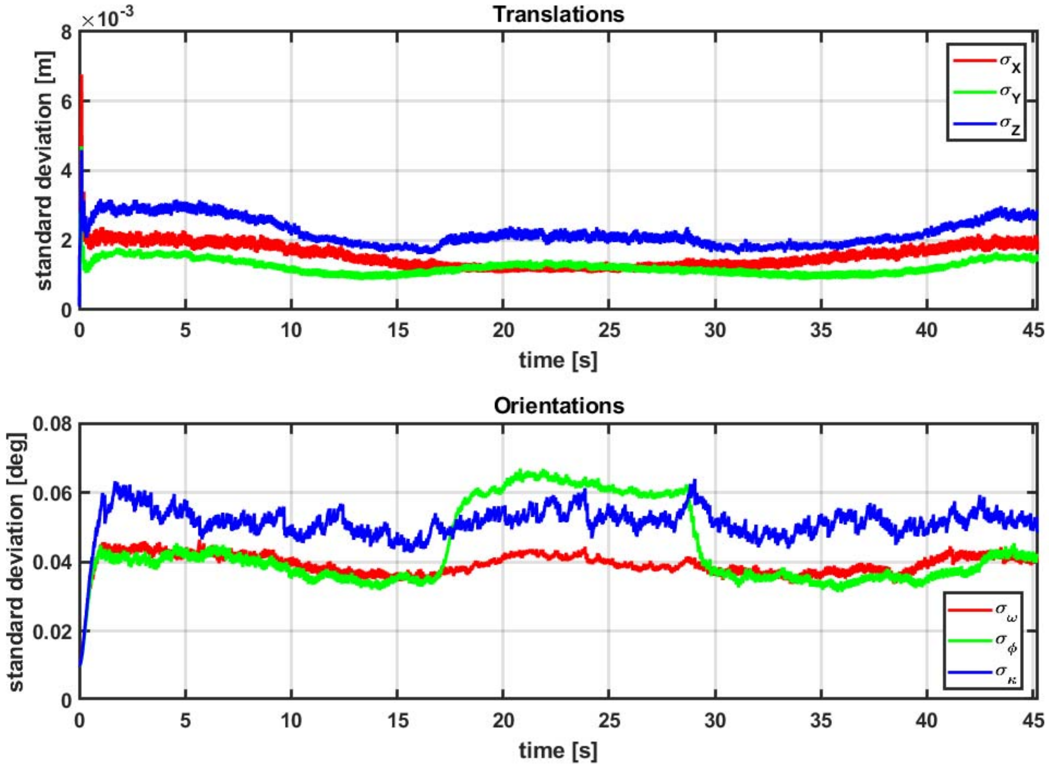


Simulation Results – Estimated Standard Deviations

- Estimated standard deviations for elements of the MSS pose $\hat{\boldsymbol{\sigma}}$

- Notable change with visibility of panels



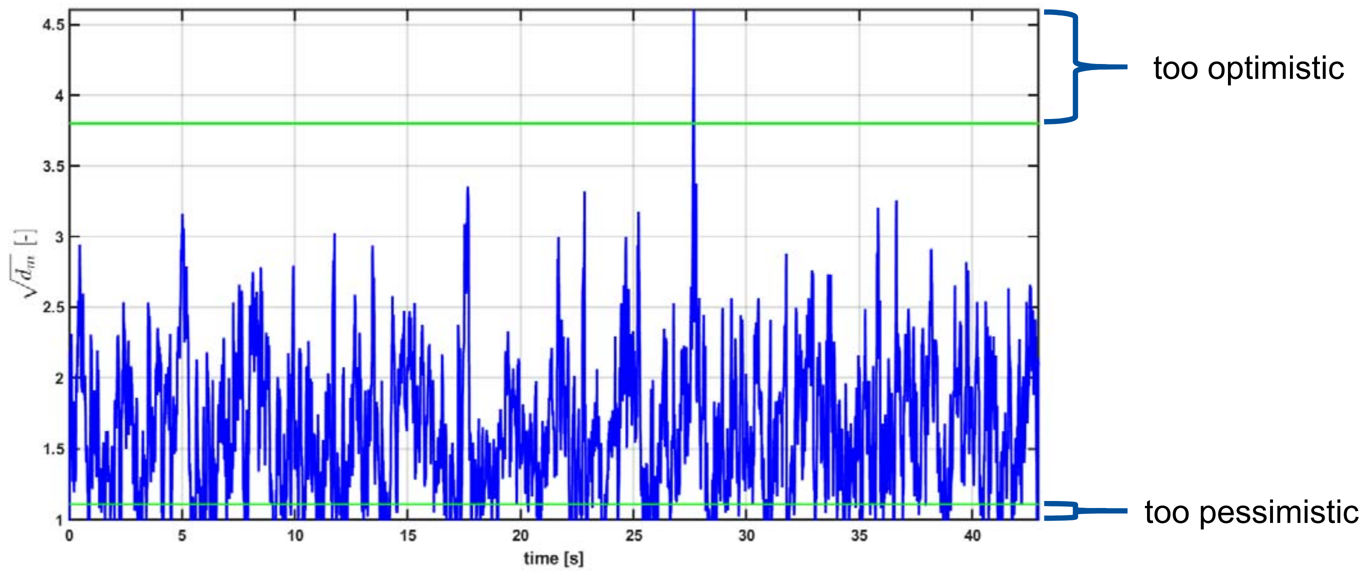


Simulation Results – Consistency Test

- Hypothesis test for pose deviations using Mahalanobis distance:

$$d_M = (\tilde{\boldsymbol{x}} - \hat{\boldsymbol{x}})^T \, \boldsymbol{\Sigma}_{\hat{x}\hat{x}}^{-1} \, (\tilde{\boldsymbol{x}} - \hat{\boldsymbol{x}})$$

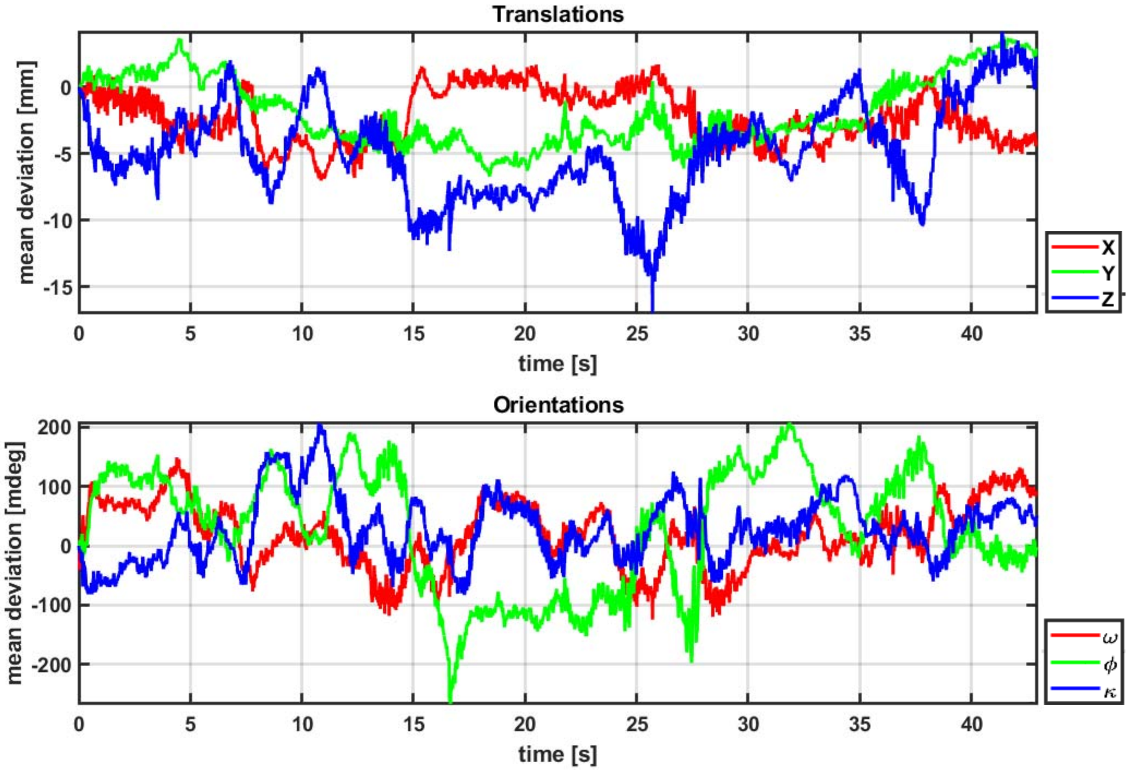
- Tested against χ^2-distribution (in green) (Bar-Shalom et al., 2004)
- Square root of d_M and χ^2-interval shown for better interpretability



Results – Accuracy of Referencing

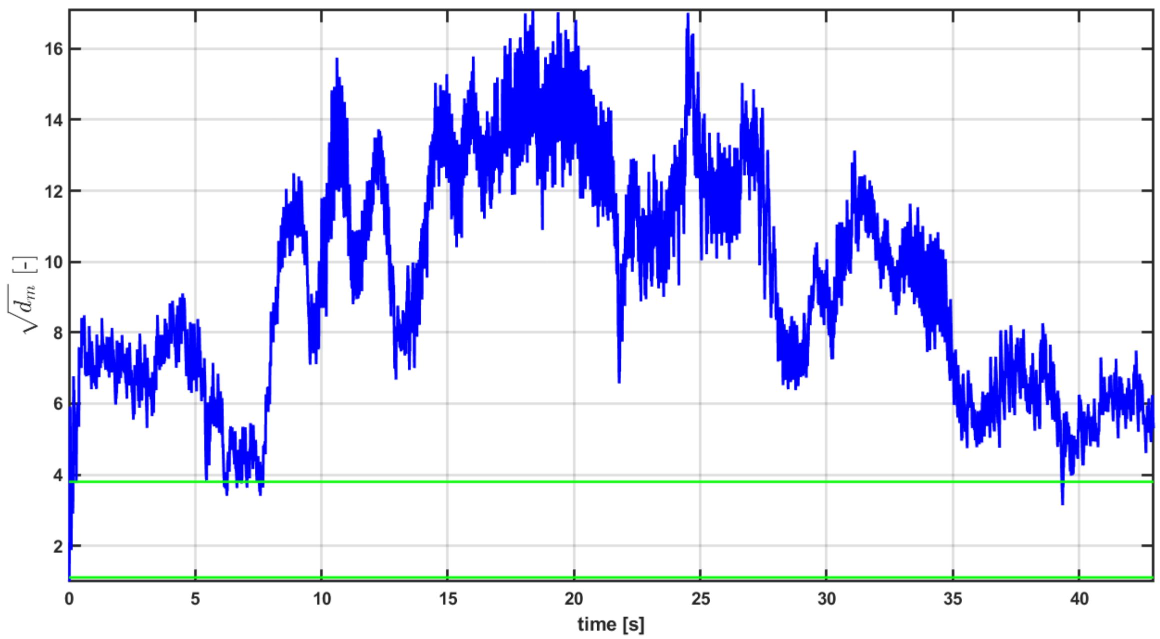
- Deviation from T-Probe trajectory $\tilde{\boldsymbol{x}} - \hat{\boldsymbol{x}}$

- RMSE:
  - Translations: 4.30 mm
  - Orientations: 77.29 mdeg

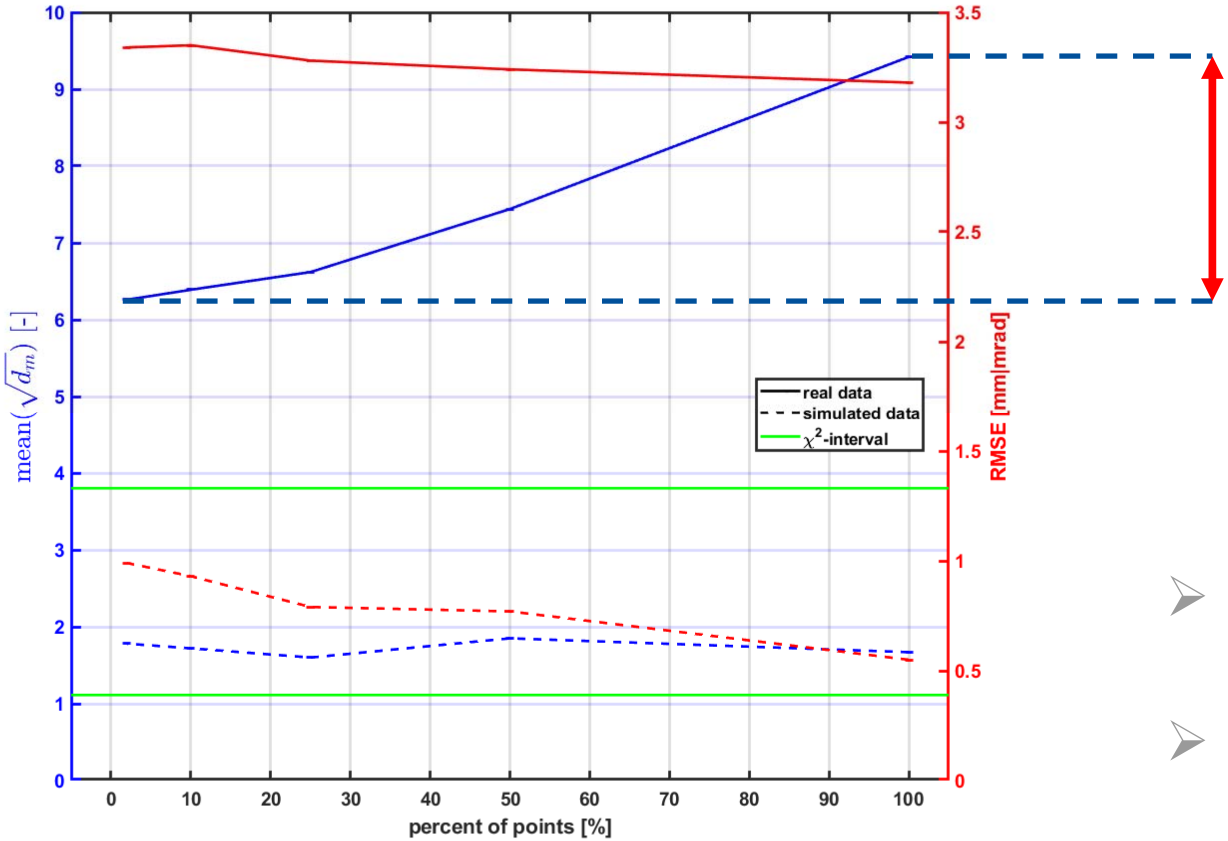


Results – Consistency Test

- Deviations too high for estimated standard deviations
- Inconsistency due to incorrect assumptions



Influence of subsampling of processed points



- standard deviations decrease with $\sqrt{n}$-law
- assumptions of LiDAR and IMU incorrect/incomplete

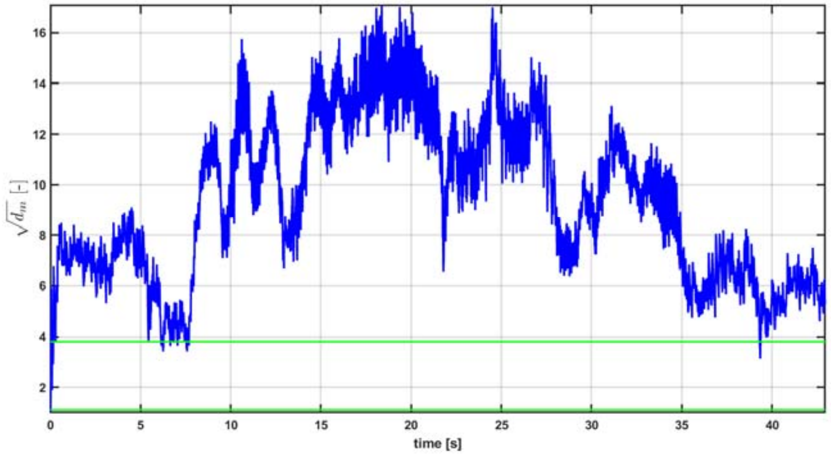
- single run → differences from subsampling
- low increase of deviation with fewer points processed

Conclusion

- Evaluating the estimated pose uncertainties of a filter
  - Uncertainty modeling by variance propagation

- Remaining gap between accuracy and estimated uncertainty, due to:
  - Remaining systematic deviations in the point cloud
  - Unknown correlations of the points in the point cloud
  - (Probably) insufficient IMU calibration
  - Offset in the time synchronization (slow movement → minor influence)
  - Remaining deviations of the Laser Tracker measurements

Summary



- Presented an Error State Kalman Filter with implict measurement equations
- Evaluation of algorithm on simulated data
  - RMSE: 0.76 mm / 19.94 mdeg
- Application to real data set
  - RMSE: 4.30 mm / 77.29 mdeg
- Remaining inconsistency caused by incomplete knowledge (about LiDAR observations)

contact:

Dominik Ernst
ernst@gih.uni-hannover.de

DFG Research Training Group i.c.sens (RTG 2159)
Integrity and Collaboration in dynamic sensor networks

Appendix

- States and Prediction
- Update Step
- Values Stochastic Model

States and Prediction

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{t}_{\mathrm{b}}^{1\,T} & \boldsymbol{v}_{\mathrm{b}}^{1\,T} & \boldsymbol{q}_{\mathrm{b}}^{1\,T} & \boldsymbol{a}_b^T & \boldsymbol{\omega}_b^T \end{bmatrix}^T$$

$$\delta\boldsymbol{x} = \begin{bmatrix} \delta\boldsymbol{t}_{\mathrm{b}}^{1\,T} & \delta\boldsymbol{v}_{\mathrm{b}}^{1\,T} & \delta\boldsymbol{\theta}_{\mathrm{b}}^{1\,T} & \delta\boldsymbol{a}_b^T & \delta\boldsymbol{\omega}_b^T \end{bmatrix}^T$$

$$\boldsymbol{\omega}_o = \boldsymbol{\omega} + \boldsymbol{\omega}_b + \boldsymbol{\omega}_n,$$

$$\boldsymbol{a}_o = \boldsymbol{a} + \boldsymbol{a}_b + \boldsymbol{a}_n + \left(\boldsymbol{R}_{\mathrm{b}}^{1}\right)^T \boldsymbol{g}_{1}$$

$$\delta\dot{\boldsymbol{t}} = \delta\boldsymbol{v},$$

$$\delta\dot{\boldsymbol{v}} = -\boldsymbol{R}_{\mathrm{b}}^{1}\left[\hat{a}\right]_{\times}\delta\boldsymbol{\theta}_{\mathrm{b}}^{1} - \boldsymbol{R}_{\mathrm{b}}^{1}\delta\boldsymbol{a}_b + \boldsymbol{g}_{1} - \boldsymbol{R}_{\mathrm{b}}^{1}\boldsymbol{a}_n,$$

$$\delta\dot{\boldsymbol{\theta}} = -\boldsymbol{R}_{\mathrm{b}}^{1}\delta\omega_b - \boldsymbol{R}_{\mathrm{b}}^{1}\omega_n,$$

$$\delta\dot{\boldsymbol{a}}_b = \boldsymbol{a}_w,$$

$$\delta\dot{\boldsymbol{\omega}}_b = \boldsymbol{\omega}_w,$$

$$\boldsymbol{F_x} = \left.\frac{\partial f}{\partial \delta\boldsymbol{x}}\right|_{\boldsymbol{x},\boldsymbol{u}_m} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{I}\Delta\mathrm{t} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} & -\left[\boldsymbol{R}_{\mathrm{b}}^{1}\hat{\boldsymbol{a}}\right]_{\times}\Delta\mathrm{t} & -\boldsymbol{R}_{\mathrm{b}}^{1}\Delta\mathrm{t} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & -\boldsymbol{R}_{\mathrm{b}}^{1}\Delta\mathrm{t} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}$$

$$\boldsymbol{Q}_i = \begin{bmatrix} \boldsymbol{\sigma}_{\tilde{a}_n}^2\Delta\mathrm{t}^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_{\tilde{\omega}_n}^2\Delta\mathrm{t}^2\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\sigma}_{a_w}^2\Delta\mathrm{t}\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\sigma}_{\omega_w}^2\Delta\mathrm{t}\boldsymbol{I} \end{bmatrix}$$

Update Step

$$\boldsymbol{h}^{(k,m)} = \boldsymbol{h}\left(\boldsymbol{x}_{+}^{(k,m)}, \boldsymbol{\ell}^{(k,m)}\right)$$

$$\boldsymbol{A}^{(k,m)} = \left.\frac{\partial \boldsymbol{h}}{\partial \delta \boldsymbol{x}}\right|_{\boldsymbol{x}_{+}^{(k,m)}, \boldsymbol{\ell}^{(k,m)}} = \left.\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}_{+}^{(k,m)}, \boldsymbol{\ell}^{(k,m)}} \cdot \left.\frac{\partial \boldsymbol{x}}{\partial \delta \boldsymbol{x}}\right|_{\boldsymbol{x}_{+}^{(k,m)}}$$

$$\boldsymbol{B}^{(k,m)} = \left.\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{\ell}}\right|_{\boldsymbol{x}_{+}^{(k,m)}, \boldsymbol{\ell}^{(k,m)}}$$

$$\boldsymbol{O}^{(k,m)} = \boldsymbol{A}^{(k,m)} \boldsymbol{Q}_{\boldsymbol{xx},-}^{(k)} \left(\boldsymbol{A}^{(k,m)}\right)^T$$

$$\boldsymbol{S}^{(k,m)} = \boldsymbol{B}^{(k,m)} \boldsymbol{Q}_{\boldsymbol{\ell\ell}}^{(k)} \left(\boldsymbol{B}^{(k,m)}\right)^T$$

$$\boldsymbol{Q}_{\boldsymbol{rr}}^{(k,m)} = \boldsymbol{O}^{(k,m)} + \boldsymbol{S}^{(k,m)}$$

$$\boldsymbol{K}^{(k,m)} = \boldsymbol{Q}_{\boldsymbol{xx},-}^{(k)} \left(\boldsymbol{A}^{(k,m)}\right)^T \left(\boldsymbol{Q}_{\boldsymbol{rr}}^{(k,m)}\right)^{-1}$$

$$\boldsymbol{r}^{(k,m)} = \boldsymbol{B}^{(k,m)} \left(\boldsymbol{\ell}^{(k,m)} - \boldsymbol{\ell}^{(k)}\right) + \boldsymbol{A}^{(k,m)} \delta \boldsymbol{x}^{(k,m-1)}$$

$$\boldsymbol{w}^{(k,m)} = \boldsymbol{h}^{(k,m)} + \boldsymbol{r}^{(k,m)}$$

$$\delta \boldsymbol{x}^{(k,m)} = -\boldsymbol{K}^{(k,m)} \boldsymbol{w}^{(k,m)}$$

$$\boldsymbol{x}_{+}^{(k,m+1)} = \boldsymbol{x}_{-}^{(k)} \boxplus \delta \boldsymbol{x}^{(k,m)} = \begin{bmatrix} \boldsymbol{t}_{\mathrm{b}-}^{\mathrm{l}} + \delta \boldsymbol{t}_{\mathrm{b}}^{\mathrm{l}} \\ \boldsymbol{v}_{\mathrm{b}-}^{\mathrm{l}} + \delta \boldsymbol{v}_{\mathrm{b}}^{\mathrm{l}} \\ \boldsymbol{q}\{\delta\boldsymbol{\theta}\} \otimes \boldsymbol{q}_{\mathrm{b}-}^{\mathrm{l}} \\ \boldsymbol{b}_{a-} + \delta \boldsymbol{b}_a \\ \boldsymbol{b}_{\omega-} + \delta \boldsymbol{b}_\omega \end{bmatrix}$$

$$\tau_i = \frac{\mathrm{t}_i - \mathrm{t}^{(k-1)}}{\Delta \mathrm{t}}$$

$$\boldsymbol{t}_{\mathrm{b}\tau_i}^{\mathrm{l}} = (1 - \tau_i)\, \hat{\boldsymbol{t}}_{\mathrm{b}}^{\mathrm{l,\,(k\text{-}1)}} + \tau_i\, \boldsymbol{t}_{\mathrm{b}}^{\mathrm{l,\,(k,m)}},$$

$$\Delta\theta = \arccos\left(\left(\hat{\boldsymbol{q}}_{\mathrm{b}}^{\mathrm{l,\,(k\text{-}1)}}\right)^T \cdot \boldsymbol{q}_{\mathrm{b}}^{\mathrm{l,\,(k,m)}}\right),$$

$$\boldsymbol{q}_{\mathrm{b}\tau_i}^{\mathrm{l}} = \hat{\boldsymbol{q}}_{\mathrm{b}}^{\mathrm{l,\,(k\text{-}1)}} \frac{\sin((1-\tau_i)\Delta\theta)}{\sin(\Delta\theta)} + \boldsymbol{q}_{\mathrm{b}}^{\mathrm{l,\,(k,m)}} \frac{\sin(\tau_i \Delta\theta)}{\sin(\Delta\theta)}$$

$$\boldsymbol{p}_{\mathrm{l},i} = \begin{bmatrix} x_{\mathrm{l}} \\ y_{\mathrm{l}} \\ z_{\mathrm{l}} \end{bmatrix}_i = \boldsymbol{t}_{\mathrm{b}\tau_i}^{\mathrm{l}} + \boldsymbol{R}\left\{\boldsymbol{q}_{\mathrm{b}\tau_i}^{\mathrm{l}}\right\} \begin{bmatrix} x_{\mathrm{b}} \\ y_{\mathrm{b}} \\ z_{\mathrm{b}} \end{bmatrix}_i,$$

$$h_i = h(\boldsymbol{x}, \boldsymbol{\ell}_i, \boldsymbol{a}_{\mathrm{l},j}) = [n_{x,\mathrm{l}}\; n_{y,\mathrm{l}}\; n_{z,\mathrm{l}}\; d_{\mathrm{l}}]_j \cdot \begin{bmatrix} \boldsymbol{p}_{\mathrm{l},i} \\ 1 \end{bmatrix}$$

Values Stochastic Model

LiDAR		
distance measurement σ_d	8.5	mm
horizontal angle σ_α	48.7	mdeg
vertical angle σ_e	29.8	mdeg
IMU for (x,y,z)-axes		
acc. noise density $\sigma_{\tilde{a}_n}$	$(6.16,\ 4.61,\ 3.62) \cdot 10^{-3}$	$m\sqrt{Hz}/s^2$
acc. bias instability σ_{a_w}	$(22.26,\ 20.35,\ 36.36) \cdot 10^{-3}$	m/s^2
gyro. noise density $\sigma_{\tilde{\omega}_n}$	$(0.26,\ 0.34,\ 0.24) \cdot 10^{-3}$	$rad\sqrt{Hz}/s$
gyro. bias instability σ_{ω_w}	$(1.23,\ 1.14,\ 1.19) \cdot 10^{-3}$	rad/s