DETERMINATION OF LEAST OBSERVATION TIME CLASSIFIED BY BASELINE GRADE ACCORDING TO GPS SATELLITE COMBINATION

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ABSTRACT

In the 20th century, there was a remarkable progress in survey instrument and technology powered by the development of scientific technologies. Precision positioning is now possible thanks to GPS (Global Positioning System), three dimensional positioning method using satellites. GPS receiver installed in the observation point and at least four satellites determine the three dimensional coordinates, therefore, disposition and number of the satellites and the baseline may affect the accuracy, and observational error often occurs.

This research is to see how earth surface position error changes according to the number and combination of observation satellites through relative location observation method. Least square method is used for position error analysis. It is to analyze what is the minimum observation time when limiting the position error.

1. INTRODUCTION

The most recent, rapid growth of telecommunication - skill, and a space technique bring remarkable development on a surveying instrument, and sector of surveying technique. It accelerates diversification, and a high level of survey such as GPS survey, the technique using the high resolution of satellite image, GPS technique. Especially, an application of GPS surveying instrument is enlarged more and more in the field of survey from new operation method, using instrument, information propulsion, a safety secure, correspondence to internationalization, and protection of nature environment point of view.

GPS stands for Global Positioning System. It sets up three dimension, which you want to observe, the latitude , the longitude , an altitude H, by positioning instrument. The units are degree, minute, second , and meter , and it is divided into point positioning which uses 1 receiver, and relative positioning which uses 2 receiver according to purpose of survey.

Method of base line measurement is that of static measurement applied to detailed control point observation. static observation has a problem with taking a lot of time in practical affair because observed time is not steady according to the distance. Accordingly, this research observes how earth surface position changes according to number, and combination of observation satellites through relative positioning, and analysis of positioning error uses the method of least squares. In this analysis effect, in case of setting limits to positioning error, It is to see how much of least observed time may be applied.

2. RELATIVE POSITIONING OF GPS

To use GPS phase observation formula, survey calculation is carried out with the data that records the number of returned wave phase received by GPS receiver.
The time of starting survey, the number of wave of carrier that is existed between a satellite and GPS receiver. Namely, phase number is called interger number. If you know this, it is possible to calculate a base-line vector at two point intervals by relative positioning.
The problem is that it is hard to count the number of wave because carrier is regular in a type of wave. Therefore, GPS survey calculation is fundamental to what kind of way, and the how little data it needs.
The method of relative positioning to calculate interger number are Single Difference, Double Difference, Triple Difference.

2.1 Single differenece

An error term of a satellite clock is eliminated by calculating phase survey formula of 1 satellite / 2 receiver or an error term of receiver clock is eliminated by calculating phase survey formula of 2 satellite / 1 receiver. An orbit error and tropospheric delay error can be reduced if the distance between two receiver is shorter than height of GPS satellite.

2.2 Double difference

Removing both receiver, and an error term of satellite clock by calculating more than two of single difference, and unknown term will just remain interger number, therefore survey calculation is carried out by an observation equation for 4 of satellites and 3 of double difference.

2.3 Triple difference

Triple difference that subtracts according to continuous time is lower than double difference on an accuracy because of poorness of substance of information. It is used to revise cycle slip which is generated in the middle of surveying. Cycle Slip comes from the case of passing through an obstacle like tree, an active action of ionosphere, or electronic wave obstacle in the area where lots of radio wave emits.

3. COMPUTATION OF OBSERVED VALUE BY THE METHOD OF LEAST SQUARES

3.1 Distance error-linear expression up to GPS satellite

In GPS positioning, To seek location of surveying position, position of coordinate $P_i(x_i, y_i, z_i)$, point P_i is decided as long as you know the distance from 3 of satellites at the least. But reach time should be observed exactly when the distance is measured by means of the reaching time of radio wave for distance calculation.
In general, satellite's clock and receiver's clock are accompanied with a little error. 4 of unknown $x_i, y_i, z_i, time(t)$, and clock error $\Delta\delta$, exist to eliminate them. To solve them, It needs positioning measured from 4 of satellite at the least. Therefore, positioning usually needs the minimum 4 of the satellite, and the relative equation shows as Eqs(1).

If distance from surveying point $P_i(x_i, y_i, z_i)$ to satellite j, $time(t)$ is $\rho_j^i(t)$.

$$\begin{aligned}\rho_i^j(t) &\equiv f(x_i, y_i, z_i) \\ &= \sqrt{\{(x^j(t)-x_i)^2 + (y^j(t)-y_i)^2 + (z^j(t)-z_i)^2\}} \qquad (1)\end{aligned}$$

Supposing, point P is $P_i(x_{io}, y_{io}, z_{io})$ as an approximate value, an approximate distance $\rho_j^i(t)$ between surveying point and satellite is that.

$$\begin{aligned}\rho_{io}^j(t) &\equiv f(x_{io}, y_{io}, z_{io}) \\ &= \sqrt{\{(x^j(t)-x_{io})^2 + (y^j(t)-y_{io})^2 + (z^j(t)-z_{io})^2\}} \qquad (2)\end{aligned}$$

Error Equation shows as Eqs(3).

$$\begin{aligned}x_i &= x_{io} + \Delta x_i \\ y_i &= y_{io} + \Delta y_i \qquad (3) \\ z_i &= z_{io} + \Delta z_i\end{aligned}$$

where,
$\Delta x_i, \Delta y_i, \Delta z_i$ are residual of x_i, y_i, z_i each. $f(x_i, y_i, z_i)$ is evolved Taylor, Eqs(4) because $\Delta x_i, \Delta y_i, \Delta z_i$ are very small amount in Eqs(1).

$$\begin{aligned}&f(x_i, y_i, z_i) \equiv f(x_{io} + \Delta x_i, y_{io} + \Delta y_{io}, z_{io} + \Delta z_i) \\ &= f(x_{io}, y_{io}, z_{io}) + \frac{\partial f(x_{io}, y_{io}, z_{io})}{\partial x_{io}}\Delta x_i + \frac{\partial f(x_{io}, y_{io}, z_{io})}{\partial y_{io}}\Delta y_i + \frac{\partial f(x_{io}, y_{io}, z_{io})}{\partial z_{io}}\Delta z_i + \cdots \quad (4)\end{aligned}$$

This coefficient of development form term 2~4 from Eqs(2) turns as follow.

$$\begin{aligned}\frac{\partial f(x_{io}, y_{io}, z_{io})}{\partial x_{io}} &= -\frac{1}{2} \times 2(x^j(t)-x_{io}) \times \left\{(x^j(t)-x_{io})^2 + (y^j(t)-y_{io})^2 + (z^j(t)-z_{io})^2\right\}^{-\frac{1}{2}} \\ &= -\frac{x^j(t)-x_{io}}{\rho_{io}^j(t)} \qquad (5)\end{aligned}$$

$$\frac{\partial f(x_{io}, y_{io}, z_{io})}{\partial y_{io}} = -\frac{y^j(t)-y_{io}}{\rho_{io}^j(t)} \qquad (6)$$

$$\frac{\partial f(x_{io}, y_{io}, z_{io})}{\partial z_{io}} = -\frac{z^j(t)-z_{io}}{\rho_{io}^j(t)} \qquad (7)$$

Eqs(4) turns Eqs(8) by means of $\rho(t)$ corollary.

$$\rho_i^j(t) = \rho_{io}^j(t) - \frac{x^j(t) - x_{io}}{\rho_{io}^j(t)}\Delta x_i - \frac{y^j(t) - y_{io}}{\rho_{io}^j(t)}\Delta y_i - \frac{z^j(t) - z_{io}}{\rho_{io}^j(t)}\Delta z_i \qquad (8)$$

On survey, supposing distance from surveying point i to satellite j is $R_i^j(t)$. $R_i^j(t)$ turns Eqs(9).

$$R_i^j(t) = \rho_i^j(t) + c\Delta\delta^j(t) - c\Delta\delta_i(t) \qquad (9)$$

where

$\rho_i^j(t)$: geometric distance between satellite and surveying point

c : the velocity of life

$\Delta\delta_i(t)$: clock error of receiver I

$\Delta\delta^j(t)$: clock error of satellite j

Eqs(10) may be gotten from Eqs(9), and (8)

$$\begin{aligned} &R_i^j(t) - \rho_{io}^j(t) - c\Delta\delta^j(t) \\ &= -\frac{x^j(t) - x_{io}}{\rho_{io}^j(t)}\Delta x_i - \frac{y^j(t) - y_{io}}{\rho_{io}^j(t)}\Delta y_i - \frac{z^j(t) - z_{io}}{\rho_{io}^j(t)}\Delta z_i - c\Delta\delta_i(t) \end{aligned} \qquad (10)$$

Revision of satellite clock is monitored by a tracking station, and transmissive coefficient in the navigational message from the satellite can be revised by clock corrective equation. On account of this, value of the left side in Eqs(10) may be calculated. While, 4 of unknown $\Delta x_i, \Delta y_i, \Delta z_i, \Delta\delta_i(t)$ exist on the right side. It needs more than 4 navigational message from satellite to get them. Eqs(10) explains,

$$\begin{aligned} &l^j = R_i^j(t) - \rho_{io}^j(t) - c\Delta\delta^j(t) \\ &a_{xi}^j = -\frac{x^j(t) - x_{io}}{\rho_{io}^j(t)}, a_{yi}^j = -\frac{y^j(t) - y_{io}}{\rho_{io}^j(t)}, a_{zi}^j = -\frac{z^j(t) - z_{io}}{\rho_{io}^j(t)} \end{aligned}$$

and, it can be expressed as follow.

$$l^j = a_{xi}^j \cdot \Delta x_i + a_{yi}^j \cdot \Delta y_i + a_{zi}^j \cdot \Delta z_i - c \cdot \Delta\delta_i(t) \qquad (11)$$

3.2 Approximation of positioning value by the method of least squares

Because linear expression is possible $\Omega(\Delta x_i, \Delta y_i, \Delta z_i, \Delta\delta_i(t)))$ are spreaded out as Eqs(12).

$$\Omega(\Delta x_i, \Delta y_i, \Delta z_i, \Delta\delta_i(t)) = \sum_{j=1}^{n}\left\{l^j - a_{xi}^j \cdot \Delta x_i - a_{yi}^j \cdot \Delta y_i - a_{zi}^j \cdot \Delta z_i - c\Delta\delta_i(t)\right\}^2 \qquad (12)$$

The method of least squares seeks $\Delta x_i, \Delta y_i, \Delta z_i, \Delta\delta_i(t)$ by minimizing $\Omega(\Delta x_i, \Delta y_i, \Delta z_i, \Delta\delta_i(t))$ and assume a part differential $\Delta x_i, \Delta y_i, \Delta z_i, \Delta\delta_i(t)$ as 0, Thus

$$\frac{\partial\Omega(\Delta x_i,\Delta y_i,\Delta z_i,\Delta\delta_i(t))}{\partial\Delta x_i}$$
$$= -2\sum_{j=1}^{n} a_{xi}^{j}(l^{j} - a_{xi}^{j}\cdot\Delta x_i - a_{yi}^{j}\cdot\Delta y_i - a_{zi}^{j}\cdot\Delta z_i + c\Delta\delta x_i(t)) = 0 \qquad (13)$$

$$\frac{\partial\Omega(\Delta x_i,\Delta y_i,\Delta z_i,\Delta\delta_i(t))}{\partial\Delta y_i}$$
$$= -2\sum_{j=1}^{n} a_{yi}^{j}(l^{j} - a_{xi}^{j}\cdot\Delta x_i - a_{yi}^{j}\cdot\Delta y_i - a_{zi}^{j}\cdot\Delta z_i + c\Delta\delta x_i(t)) = 0 \qquad (14)$$

$$\frac{\partial\Omega(\Delta x_i,\Delta y_i,\Delta z_i,\Delta\delta_i(t))}{\partial\Delta z_i}$$
$$= -2\sum_{j=1}^{n} a_{zi}^{j}(l^{j} - a_{xi}^{j}\cdot\Delta x_i - a_{yi}^{j}\cdot\Delta y_i - a_{zi}^{j}\cdot\Delta z_i + c\Delta\delta x_i(t)) = 0 \qquad (15)$$

$$\frac{\partial\Omega(\Delta x_i,\Delta y_i,\Delta z_i,\Delta\delta_i(t))}{\partial\Delta\delta_i}$$
$$= 2c\sum_{j=1}^{n}(l^{j} - a_{xi}^{j}\cdot\Delta x_i - a_{yi}^{j}\cdot\Delta y_i - a_{zi}^{j}\cdot\Delta z_i + c\Delta\delta x_i(t)) = 0 \qquad (16)$$

After theorem of Eqs(13)~Eqs(16), regular equation may be Eqs(17)~Eqs(20).

$$\sum_{j=1}^{n}(a_{xi}^{j})^2\Delta x_i + \sum_{j=1}^{n}a_{xi}^{j}a_{yi}^{j}\Delta y_i + \sum_{j=1}^{n}a_{xi}^{j}a_{zi}^{j}\Delta z_i - c\sum_{j=1}^{n}a_{xi}^{j}\Delta\delta_i(t) = \sum_{j=1}^{n}a_{xi}^{j}l^{j} \qquad (17)$$
$$\sum_{j=1}^{n}a_{xi}^{j}a_{yi}^{j}\Delta x_i + \sum_{j=1}^{n}(a_{yi}^{j})^2\Delta y_i + \sum_{j=1}^{n}a_{yi}^{j}a_{zi}^{j}\Delta z_i - c\sum_{j=1}^{n}a_{yi}^{j}\Delta\delta_i(t) = \sum_{j=1}^{n}a_{yi}^{j}l^{j} \qquad (18)$$
$$\sum_{j=1}^{n}a_{xi}^{j}a_{zi}^{j}\Delta x_i + \sum_{j=1}^{n}a_{yi}^{j}a_{zi}^{j}\Delta y_i + \sum_{j=1}^{n}(a_{zi}^{j})^2\Delta z_i - c\sum_{j=1}^{n}a_{zi}^{j}\Delta\delta_i(t) = \sum_{j=1}^{n}a_{xz}^{j}l^{j} \qquad (19)$$
$$\sum_{j=1}^{n}a_{xi}^{j}\Delta x_i + \sum_{j=1}^{n}a_{yi}^{j}\Delta y_i + \sum_{j=1}^{n}a_{zi}^{j}\Delta z_i - c\sum_{j=1}^{n}\Delta\delta_i(t) = \sum_{j=1}^{n}l^{j} \qquad (20)$$

Then, a part of coefficient of $\Delta x_i,\Delta y_i,\Delta z_i,\Delta\delta_i(t)$ is expressed as type of $k_{m,n}(m,n=1\sim 4)$, and Eqs(17)~Eqs(20) can be Eqs(21).

$$\left.\begin{aligned}
k_{11}\Delta x_i + k_{12}\Delta y_i + k_{13}\Delta z_i + k_{14}\Delta\delta_i(t) = g_1\\
k_{21}\Delta x_i + k_{22}\Delta y_i + k_{23}\Delta z_i + k_{24}\Delta\delta_i(t) = g_2\\
k_{31}\Delta x_i + k_{32}\Delta y_i + k_{33}\Delta z_i + k_{34}\Delta\delta_i(t) = g_3\\
k_{41}\Delta x_i + k_{42}\Delta y_i + k_{43}\Delta z_i + k_{44}\Delta\delta_i(t) = g_4
\end{aligned}\right\} \qquad (21)$$

From the forth dimensional simultaneous equation, $\Delta x_i,\Delta y_i,\Delta z_i,\Delta\delta_i(t)$ and x_i, y_i, z_i in Eqs(3) will be determined.

4. THE BETHOD OF SURVEYING

This research is that the control point being used at Pukyoung National University in Pusan was used, and observed point within the compass of distance 1km, 2.5km, 5km, 7.5km, 10km, 15km, 20km, was selected. as shown Figure 1.
The time of survey were 30, 60, 90, and 120 minutes and acquisition space of the data was 30 sec.
The distances of rapid static positioning were selected 1km, 2.5km, 5km, 7.5km, 10km, 15km as Method of static positioning, and the time of survey were 5, 10, 15, and 20 minutes and acquisition space of the data was 5 sec.
To combine GPS satellite, considering in case of being getting over 4each of satellites, Since it is possible to see selection of r units from n units as the combination on selecting r units without considering the order of n value from different satellite, the number of satellite which can be used may be increased.
For example, ${}_4C_4 = 1, {}_5C_4 = 5, {}_6C_4 = 15,$ and ${}_7C_4 = 35, {}_8C_4 = 70,$ ${}_9C_4 = 126,$ ${}_{10}C_4 = 210$. In this research, We limit satellite combinations to total 7each, because of the situation of survey place, even we use 2 frequency GPS which is 8 channel. Shown Table 1.

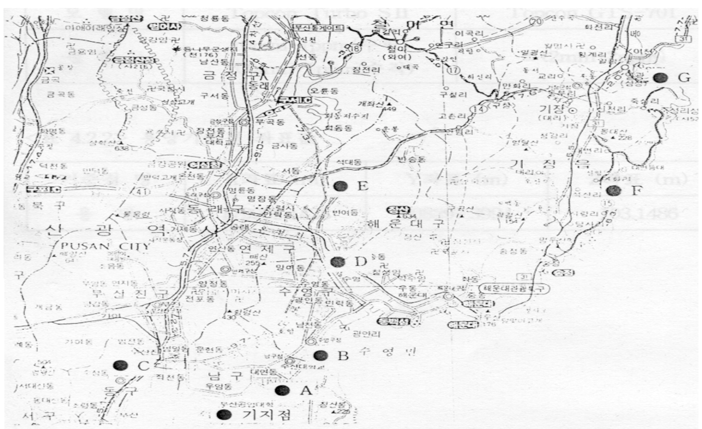

Fig.1 A Map of GPS observation points

Table 1. The number of combinations of satellite with the number of satellite.

r n	4	5	6	7
4	1	5	15	35
5	-	1	6	21
6	-	-	1	7
7	-	-	-	1

Observed period of GPS is 0.5 Sidereal Day (about 11hours and 58 minutes), so It is 4 minutes faster in a day. We apply static positioning or rapid static positioning belong to relative positioning to observe the same time everyday, and altitude of receiver is 15 degree.
Accuracy is estimated as comparison of dimension of absolute observed error, and relative accuracy of distance X, and Y by relative positioning with 2 GPS receivers.
Received data is calculated by the method of least squares according to computer process.

5. ANALYSIS OF OBSERVED VALUE

5.1 Result of distance observation according to the number of satellite in static positioning

Allowable scope error limits absolute observed error to 5mm, and relative accuracy is shown Table 2.

Table 2. Control point relative observation allowable error

Division	Control Point			
	First Grade	Second Grade	Third Grade	Fourth Grade
Baseline	30~37(km)	10(km)	5(km)	2.5(km)
Accuracy	1/1,000,000	1/500,000	1/200,000	1/100,000

If absolute observed error is limited to 5mm following satellite combinations in distance in static positioning, at the time of that satellite is 4 each, satisfied observation more than 120minutes within shorter than 2.5km, and when the number of satellite are 5, 6 is sufficient over 30 min within 1km, over 60min within 2.5km, and over 120min within 5km. In the case of more than 7, the time of observation needs over 30min within 1km, 60min within 2.5km, and 120min within 10km.

Table 3. Results of GPS observation using 4 GPS Satellites

Station	Time (min)	Baseline	Error of Plane X	Y	$\sqrt{X^2+Y^2}$	Z	3-D	$\frac{\sqrt{X^2+Y^2}}{L}$
A (1km)	30	1185.4811	0.0117	0.0060	0.0131	0.0060	0.0145	1/90495
	60	1185.4809	0.0037	0.0035	0.0051	0.0049	0.0071	1/232447
	90	1185.4813	0.0040	0.0046	0.0061	0.0046	0.0076	1/194341
	120	1185.4813	0.0035	0.0024	0.0035	0.0065	0.0073	1/338712
B (2.5km)	30	2355.1828	0.0189	0.0105	0.0216	0.0051	0.0222	1/109037
	60	2355.1881	0.0052	0.0060	0.0079	0.0062	0.0101	1/298125
	90	2355.1880	0.0067	0.0040	0.0078	0.0071	0.0106	1/301947
	120	2355.1871	0.0033	0.0032	0.0046	0.0091	0.0102	1/511997
C (5km)	30	5004.1092	0.0295	0.0143	0.0328	0.0134	0.0354	1/152564
	60	5004.1037	0.0077	0.0070	0.0104	0.0107	0.0150	1/481164
	90	5004.1001	0.0086	0.0052	0.0100	0.0109	0.0148	1/500410
	120	5004.0992	0.0045	0.0042	0.0062	0.0116	0.0131	1/807113
D (7.5km)	30	7417.9642	0.0349	0.0168	0.0387	0.0155	0.0417	1/191679
	60	7417.9592	0.0071	0.0115	0.0135	0.0138	0.0193	1/549478
	90	7417.9609	0.0121	0.0073	0.0141	0.0127	0.0190	1/526097
	120	7417.9660	0.0053	0.0051	0.0074	0.0135	0.0154	1/1002426
E (10km)	30	10667.7208	0.0372	0.0181	0.0414	0.0175	0.0449	1/257674
	60	10667.7197	0.0087	0.0120	0.0148	0.0190	0.0233	1/720792
	90	10667.7324	0.0127	0.0123	0.0177	0.0129	0.0219	1/602697
	120	10667.7182	0.0057	0.0055	0.0079	0.0152	0.0171	1/1350344
F (15km)	30	14832.9864	0.0404	0.0201	0.0451	0.0151	0.0490	1/328891
	60	14832.9771	0.0103	0.0124	0.0161	0.0199	0.0256	1/921303
	90	14832.9800	0.0101	0.0148	0.0179	0.0096	0.0203	1/828659
	120	14832.9780	0.0062	0.0062	0.0088	0.0167	0.0189	1/1685696
G (20km)	30	20344.3478	0.0384	0.0450	0.0592	0.0254	0.0644	1/343655
	60	20344.3350	0.0119	0.0098	0.0153	0.0159	0.0221	1/1329665
	90	20344.3381	0.0115	0.0118	0.0165	0.0135	0.0213	1/1232990
	120	20344.3380	0.0069	0.0065	0.0095	0.0178	0.0202	1/2141508

Table 4. Results of GPS observation using 5 GPS Satellites

Station	Time (min)	Baseline	Error of Plane X	Y	$\sqrt{X^2+Y^2}$	Z	3-D	$\frac{\sqrt{X^2+Y^2}}{L}$
A (1km)	30	1185.4813	0.0040	0.0034	0.0052	0.0052	0.0074	1/227977
	60	1185.4823	0.0024	0.0030	0.0038	0.0040	0.0055	1/311969
	90	1185.4798	0.0031	0.0028	0.0042	0.0034	0.0054	1/282257
	120	1185.4798	0.0020	0.0019	0.0028	0.0032	0.0042	1/423396
B (2.5km)	30	2355.1909	0.0053	0.0049	0.0072	0.0047	0.0086	1/327118
	60	2355.1843	0.0033	0.0040	0.0052	0.0037	0.0064	1/452929
	90	2355.1886	0.0029	0.0034	0.0045	0.0041	0.0060	1/523375
	120	2355.1856	0.0026	0.0026	0.0037	0.0042	0.0056	1/636537
C (5km)	30	5004.1001	0.0109	0.0079	0.0135	0.0115	0.0176	1/370674
	60	5004.1050	0.0058	0.0050	0.0077	0.0075	0.0108	1/649864
	90	5004.0991	0.0055	0.0043	0.0070	0.0058	0.0091	1/714871
	120	5004.1007	0.0038	0.0035	0.0052	0.0048	0.0070	1/962327
D (7.5km)	30	7417.9654	0.0096	0.0080	0.0120	0.0130	0.0177	1/618164
	60	7417.9675	0.0054	0.0058	0.0079	0.0085	0.0116	1/938963
	90	7417.9677	0.0050	0.0057	0.0076	0.0070	0.0103	1/976047
	120	7417.9597	0.0048	0.0040	0.0062	0.0059	0.0086	1/1196445
E (10km)	30	10667.7211	0.0095	0.0092	0.0132	0.0143	0.0195	1/808161
	60	10667.7197	0.0063	0.0064	0.0090	0.0097	0.0132	1/1185302
	90	10667.7325	0.0062	0.0060	0.0086	0.0069	0.0110	1/1240434
	120	10667.7187	0.0048	0.0045	0.0066	0.0059	0.0089	1/1616321
F (15km)	30	14832.9822	0.0114	0.0104	0.0154	0.0158	0.0221	1/963181
	60	14832.9790	0.0069	0.0069	0.0098	0.0104	0.0143	1/1513569
	90	14832.9830	0.0097	0.0063	0.0103	0.0087	0.0123	1/1440095
	120	14832.9718	0.0047	0.0049	0.0068	0.0059	0.0090	1/2181319
G (20km)	30	20344.3316	0.0278	0.0158	0.0320	0.0111	0.0339	1/635760
	60	20344.3384	0.0070	0.0081	0.0107	0.0122	0.0162	1/1901340
	90	20344.3428	0.0095	0.0097	0.0136	0.0130	0.0175	1/1495908
	120	20344.3311	0.0061	0.0061	0.0086	0.0077	0.0115	1/2365620

Table 5. Results of GPS observation using 6 GPS Satellites

Station	Time (min)	Baseline	Error of Plane			Z	3-D	$\frac{\sqrt{X^2+Y^2}}{L}$
			X	Y	$\sqrt{X^2+Y^2}$			
A (1km)	30	1185.4831	0.0038	0.0032	0.0048	0.0037	0.0061	1/346936
	60	1185.4833	0.0021	0.0025	0.0033	0.0034	0.0047	1/359237
	90	1185.4790	0.0024	0.0025	0.0035	0.0029	0.0045	1/338798
	120	1185.4797	0.0018	0.0019	0.0026	0.0027	0.0037	1/455954
B (2.5km)	30	2355.1913	0.0042	0.0039	0.0057	0.0042	0.0071	1/413191
	60	2355.1839	0.0025	0.0035	0.0043	0.0032	0.0054	1/547717
	90	2355.1881	0.0027	0.0030	0.0040	0.0030	0.0050	1/588797
	120	2355.1863	0.0022	0.0024	0.0033	0.0030	0.0045	1/713693
C (5km)	30	5004.1087	0.0059	0.0046	0.0075	0.0062	0.0097	1/667214
	60	5004.1043	0.0044	0.0047	0.0064	0.0067	0.0093	1/781891
	90	5004.0994	0.0051	0.0041	0.0065	0.0048	0.0081	1/769861
	120	5004.1005	0.0035	0.0034	0.0049	0.0038	0.0062	1/1021245
D (7.5km)	30	7417.9728	0.0082	0.0072	0.0109	0.0084	0.0138	1/680548
	60	7417.9701	0.0051	0.0056	0.0076	0.0064	0.0100	1/976049
	90	7417.9604	0.0044	0.0049	0.0066	0.0047	0.0081	1/1123933
	120	7417.9590	0.0041	0.0038	0.0056	0.0044	0.0072	1/1324636
E (10km)	30	10667.7234	0.0089	0.0078	0.0121	0.0098	0.0150	1/881630
	60	10667.7154	0.0050	0.0057	0.0079	0.0077	0.0113	1/1350344
	90	10667.7327	0.0046	0.0048	0.0074	0.0049	0.0090	1/1441586
	120	10667.7295	0.0038	0.0039	0.0061	0.0044	0.0076	1/1748808
F (15km)	30	14832.9794	0.0100	0.0086	0.0132	0.0102	0.0166	1/1123711
	60	14832.9804	0.0055	0.0062	0.0083	0.0090	0.0122	1/1787108
	90	14832.9764	0.0055	0.0057	0.0079	0.0055	0.0096	1/1877592
	120	14832.9716	0.0044	0.0046	0.0064	0.0047	0.0080	1/2317662
G (20km)	30	20344.3339	0.0144	0.0110	0.0181	0.0108	0.0211	1/1123996
	60	20344.3430	0.0073	0.0084	0.0111	0.0127	0.0169	1/1832824
	90	20344.3432	0.0077	0.0069	0.0103	0.0088	0.0135	1/1975179
	120	20344.3307	0.0047	0.0050	0.0069	0.0062	0.0092	1/2948454

Table 6. Results of GPS observation using 7 GPS Satellites

Station	Time (min)	Baseline	Error of Plane			Z	3-D	$\frac{\sqrt{X^2+Y^2}}{L}$
			X	Y	$\sqrt{X^2+Y^2}$			
A (1km)	30	1185.4841	0.0036	0.0031	0.0048	0.0033	0.0058	1/246976
	60	1185.4893	0.0021	0.0024	0.0032	0.0030	0.0044	1/370465
	90	1185.4808	0.0022	0.0023	0.0032	0.0028	0.0042	1/370463
	120	1185.4806	0.0017	0.0017	0.0024	0.0026	0.0036	1/493950
B (2.5km)	30	2355.1961	0.0041	0.0038	0.0056	0.0034	0.0065	1/420570
	60	2355.1945	0.0024	0.0031	0.0039	0.0029	0.0049	1/603883
	90	2355.1985	0.0026	0.0027	0.0037	0.0029	0.0047	1/636537
	120	2355.1986	0.0021	0.0024	0.0032	0.0028	0.0043	1/735996
C (5km)	30	5004.1062	0.0054	0.0041	0.0068	0.0058	0.0089	1/735898
	60	5004.1046	0.0038	0.0045	0.0059	0.0050	0.0077	1/848152
	90	5004.1002	0.0046	0.0038	0.0060	0.0046	0.0076	1/834017
	120	5004.1007	0.0033	0.0030	0.0045	0.0037	0.0058	1/1112022
D (7.5km)	30	7417.9706	0.0080	0.0071	0.0107	0.0075	0.0131	1/693268
	60	7417.9135	0.0046	0.0054	0.0071	0.0063	0.0095	1/1044777
	90	7417.9604	0.0042	0.0044	0.0062	0.0044	0.0075	1/1216059
	120	7417.9598	0.0035	0.0036	0.0050	0.0043	0.0067	1/1483592
E (10km)	30	10667.7355	0.0091	0.0079	0.0118	0.0089	0.0143	1/904045
	60	10667.7263	0.0053	0.0058	0.0076	0.0081	0.0108	1/1403648
	90	10667.7411	0.0048	0.0056	0.0066	0.0051	0.0082	1/1616324
	120	10667.7380	0.0043	0.0043	0.0054	0.0046	0.0070	1/1975567
F (15km)	30	14832.9799	0.0098	0.0085	0.0130	0.0087	0.0156	1/1140998
	60	14832.9804	0.0054	0.0061	0.0081	0.0084	0.0117	1/1831232
	90	14832.9757	0.0052	0.0052	0.0074	0.0053	0.0090	1/2004406
	120	14832.9715	0.0042	0.0041	0.0059	0.0046	0.0074	1/2514083
G (20km)	30	20344.3317	0.0114	0.0098	0.0150	0.0084	0.0173	1/1356089
	60	20344.3361	0.0065	0.0080	0.0103	0.0078	0.0128	1/1975178
	90	20344.3406	0.0074	0.0067	0.0100	0.0085	0.0131	1/2034434
	120	20344.3322	0.0045	0.0049	0.0067	0.0056	0.0087	1/3036487

5.2 Result of distance observation according to the number of satellite in rapid static positioning

Absolution observed error is limited to 5mm according to satellite combinations in distance in rapid static positioning, In case of 4 of satellite, the time of observation needs over 20min within 1km. in case of 5 of satellite, the time of observation needs over 10min within 1km, over 20min within 2.5km, and in the case of 6 of satellite, the time of observation needs over 15min within 1km,2.5km, over 20min within 7.5km.

Table 7. Results of GPS observation using 4 GPS Satellites

Station	Time (min)	Baseline	Error of Plane			Z	3-D	$\frac{\sqrt{X^2+Y^2}}{L}$
			X	Y	$\sqrt{X^2+Y^2}$			
A (1km)	5	1185.4684	0.0247	0.0206	0.0321	0.0377	0.0495	1/36930
	10	1185.4739	0.0061	0.0049	0.0078	0.0097	0.0125	1/151984
	15	1185.4752	0.0098	0.0064	0.0108	0.0083	0.0136	1/109766
	20	1185.4862	0.0038	0.0039	0.0054	0.0050	0.0074	1/219524
B (2.5km)	5	2355.2518	0.2412	0.3105	0.3932	0.6100	0.7257	1/5990
	10	2355.2346	0.0070	0.0087	0.0097	0.0122	0.0122	1/242808
	15	2355.2002	0.0066	0.0086	0.0108	0.0161	0.0194	1/218074
	20	2355.2083	0.0054	0.0087	0.0102	0.0094	0.0139	1/230903
C (5km)	5	5004.2770	0.2531	0.5087	0.5682	0.6980	0.8907	1/8807
	10	5004.1454	0.4405	0.0299	0.4415	0.1366	0.4621	1/11334
	15	5003.6980	0.0077	0.0086	0.0115	0.0149	0.0188	1/435104
	20	5004.0872	0.0081	0.0071	0.0108	0.0130	0.0154	1/463341
D (7.5km)	5	7418.0451	0.3924	0.6634	0.7699	0.7012	1.0407	1/9320
	10	7418.0339	0.4639	0.3210	0.5641	0.0038	0.8263	1/8977
	15	7417.9745	0.0090	0.0085	0.0126	0.0137	0.0186	1/589728
	20	7417.9649	0.0081	0.0096	0.0126	0.0138	0.0186	1/588727
E (10km)	5	10667.8001	0.5803	0.8199	1.0195	0.7307	1.2463	1/10464
	10	10667.3490	0.2898	0.4931	0.6296	0.4994	0.8028	1/16971
	15	10667.6463	0.0102	0.0105	0.0146	0.0144	0.0205	1/730861
	20	10667.6131	0.0128	0.0109	0.0167	0.0121	0.0206	1/638779
F (15km)	5	14832.8603	0.6873	0.9440	1.1677	0.8698	1.4555	1/12703
	10	14833.0425	0.4181	0.5056	0.6560	0.5685	0.8681	1/22613
	15	14833.0090	0.0114	0.0140	0.0183	0.0157	0.0241	1/810547
	20	14832.9798	0.0131	0.0114	0.0174	0.0168	0.0241	1/852470

Table 8. Results of GPS observation using 5 GPS Satellites

Station	Time (min)	Baseline	Error of Plane			Z	3-D	$\frac{\sqrt{X^2+Y^2}}{L}$
			X	Y	$\sqrt{X^2+Y^2}$			
A (1km)	5	1185.5081	0.0073	0.0061	0.0095	0.0084	0.0127	1/124790
	10	1185.4851	0.0049	0.0040	0.0063	0.0046	0.0078	1/188172
	15	1185.4793	0.0029	0.0026	0.0039	0.0035	0.0052	1/303969
	20	1185.4864	0.0031	0.0028	0.0042	0.0041	0.0059	1/282259
B (2.5km)	5	2355.2077	0.0072	0.0074	0.0103	0.0111	0.0151	1/228661
	10	2355.2042	0.0050	0.0063	0.0080	0.0097	0.0126	1/294401
	15	2355.2028	0.0042	0.0040	0.0058	0.0057	0.0081	1/406069
	20	2355.2008	0.0027	0.0030	0.0040	0.0044	0.0059	1/588800
C (5km)	5	5004.0886	0.0107	0.0154	0.0188	0.0130	0.0228	1/266175
	10	5004.0862	0.0075	0.0084	0.0113	0.0090	0.0144	1/442839
	15	5004.0806	0.0055	0.0052	0.0076	0.0074	0.0106	1/658433
	20	5004.0886	0.0040	0.0054	0.0067	0.0063	0.0092	1/746879
D (7.5km)	5	7417.9882	0.0123	0.0176	0.0215	0.0138	0.0255	1/345023
	10	7417.9748	0.0094	0.0086	0.0127	0.0112	0.0170	1/584093
	15	7417.9675	0.0044	0.0055	0.0070	0.0075	0.0103	1/1059710
	20	7417.9640	0.0042	0.0048	0.0064	0.0047	0.0079	1/1159057
E (10km)	5	10667.5854	0.0084	0.0254	0.0271	0.5103	0.8244	1/393638
	10	10667.6387	0.0104	0.0107	0.0149	0.0119	0.0191	1/715949
	15	10667.6396	0.0073	0.0061	0.0095	0.0098	0.0136	1/1122909
	20	10667.6363	0.0052	0.0048	0.0071	0.0056	0.0090	1/1502494
F (15km)	5	14832.9764	0.0184	0.0265	0.0323	0.0229	0.0395	1/459225
	10	14832.9810	0.0138	0.0126	0.0187	0.0150	0.0240	1/793206
	15	14832.9892	0.0076	0.0068	0.0103	0.0100	0.0144	1/1440096
	20	14832.9766	0.0062	0.0077	0.0099	0.0116	0.0152	1/1498280

Table 9. Results of GPS observation using 6 GPS Satellites

Station	Time (min)	Baseline	Error of Plane			Z	3-D	$\frac{\sqrt{X^2+Y^2}}{L}$
			X	Y	$\sqrt{X^2+Y^2}$			
A (1km)	5	1185.4986	0.0059	0.0051	0.0078	0.0063	0.0100	1/151987
	10	1185.4847	0.0043	0.0034	0.0055	0.0042	0.0069	1/215543
	15	1185.4803	0.0028	0.0026	0.0038	0.0033	0.0051	1/311989
	20	1185.4918	0.0028	0.0025	0.0038	0.0034	0.0051	1/311972
B (2.5km)	5	2355.2076	0.0062	0.0063	0.0088	0.0097	0.0132	1/267637
	10	2355.2040	0.0047	0.0058	0.0075	0.0078	0.0108	1/314027
	15	2355.2037	0.0030	0.0035	0.0046	0.0048	0.0066	1/512001
	20	2355.2006	0.0024	0.0028	0.0037	0.0041	0.0055	1/636541
C (5km)	5	5004.0815	0.0088	0.0088	0.0124	0.0105	0.0163	1/403555
	10	5004.0891	0.0063	0.0063	0.0089	0.0072	0.0115	1/562257
	15	5004.0880	0.0049	0.0049	0.0069	0.0069	0.0093	1/725230
	20	5004.0865	0.0038	0.0053	0.0065	0.0049	0.0082	1/769859
D (7.5km)	5	7417.9824	0.0092	0.0105	0.0139	0.0094	0.0168	1/533668
	10	7417.9757	0.0050	0.0069	0.0085	0.0079	0.0116	1/872703
	15	7417.9682	0.0036	0.0044	0.0057	0.0062	0.0084	1/1301398
	20	7417.9671	0.0033	0.0040	0.0052	0.0042	0.0067	1/1426532
E (10km)	5	10987.6471	0.0124	0.0133	0.0182	0.0170	0.0358	1/596134
	10	10987.6428	0.0092	0.0087	0.0130	0.0074	0.0141	1/888870
	15	10987.6442	0.0052	0.0055	0.0076	0.0055	0.0094	1/1403937
	20	10987.6357	0.0047	0.0041	0.0062	0.0049	0.0080	1/1722586
F (15km)	5	14832.9725	0.0142	0.0166	0.0218	0.0176	0.0280	1/680412
	10	14832.9843	0.0096	0.0103	0.0141	0.0117	0.0183	1/1051985
	15	14832.9875	0.0063	0.0065	0.0091	0.0090	0.0128	1/1629999
	20	14832.9740	0.0055	0.0065	0.0085	0.0084	0.0120	1/1735056

6. CONCLUSION

This research calculates dimension of observed error followed satellite combination in distance to determine least observed time of baseline classified followed GPS satellite combinations, and To get optimum observed time suited to fiducial point survey, the conclusions was taken as below by static positioning, and rapid static positioning.

1) When static positioning, and rapid static positioning belong to relative positioning were used, economic observed time to get sufficient data within sphere of critical allowable error came to reduced phenomenon by increasing the number of satellites or reducing length of base line when combined satellites.

2) When observed by static positioning, and rapid static positioning, practicality was estimated by analyzing the same amount of observed data, and observed error as a base, and accuracy using 2nd control point was confirmed by static positioning that survey for 30min, and It would be better to apply to detailed control point surveying.

3) Observed time of base line followed the number of satellite according to satellite combination of grade could be taken, in case of domestic GPS surveying applying to control point surveying.

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